LIGHT AND SOUND

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A TEXT-BOOK FOR COLLEGES
AND TECHNICAL SCHOOLS

BY

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PREFACE.

The comparative absence of productive scholarship in America is a condition which is quite generally recognized, by us Americans no less than by our friends abroad, and it is a condition greatly to be deplored. It is a never-failing topic for discussion among our scientific men, and most of us have a sort of hopeful attitude of expectancy in the matter as if a King might come along and put things to rights. This is exactly our attitude, it is bred in our bones; but a King is not coming, at least not a King regnant.

The most hopeful view of the situation, as it seems to us, is that which is presented in a recent article in *Science* by David Kinley of the University of Illinois.* Dean Kinley sees, as perhaps our State University Professor is in the best position to see, the vital importance to a democratic society of productive scholarship.

The proper development of science cannot, however, be hoped for solely on the basis of a complete appreciation of the results of scientific research. *It must be founded to some extent upon and grow out of a simple living interest in science on the part of all plain people*, and the authors of this series of elementary texts † on physics believe that they are doing as much for the ultimate development of science in America as any of our research specialists in physics. They, therefore, take this occasion to call attention again to the Preface and Introductory Chapter of their *Elements of Mechanics*, and to express again their conviction that it is vitally important in the teaching of elementary physics to emphasize and rationalize those phases of the subject which are exemplified in daily life and business.

* *Democracy and Scholarship*, by David Kinley, *Science*, October 16, 1908.

† *Elements of Mechanics*, 1907; *Elements of Electricity and Magnetism*, 1908; *Light and Sound*, 1909; and *Heat* (in preparation).

The authors do not insist on the study of the practical things in physics merely because of their practical value, although they believe that this consideration alone should determine the trend of all general instruction in elementary physics and chemistry. A so-called knowledge of elementary science which does not relate to some actual physical condition or thing is to them superlatively contemptible, and they believe that the only physical things that are sufficiently prominent in the mind of a young man to be brought into the field of his science study are the things which have been impressed upon him in everyday life.

THE AUTHORS.

SOUTH BETHLEHEM, PA.,
April 22, 1909.

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PART I.

INTRODUCTION.

LIGHT AND SOUND.

CHAPTER I.

LIGHT AND SOUND DEFINED. VELOCITY.

1. **Sensory nerves.** — The sensory nerves of the human body lead from regions near the surface of the body to the central organs of the nervous system. The outer ends of these nerves are exposed in such a way as to be excited or set into commotion by physical disturbances in the region surrounding the body, this commotion is transmitted to the central organs producing commotion there, and we experience what we call a *sensation.* The physical disturbance which excites the nerves is called a *stimulus.*

2 **Proper stimuli.** *End organs. Localization.* — Each set of sensory nerves, such as the nerves of sight or the nerves of smell, is especially sensitive to a certain kind of disturbance. The disturbance to which a given set of sensory nerves is especially sensitive is called the *proper stimulus* of that set of nerves.

A set of sensory nerves is rendered especially sensitive to its proper stimulus in two distinct ways, namely, (*a*) by being provided with *specialized end organs*, and (*b*) by being *located* so as to be exposed to the proper stimulus but protected to a great extent from other stimuli. Thus, the nerves of sight terminate in minute organs, the so-called rods and cones, which are situated in the retina of the eye. These organs are sensitive to the disturbances which can reach them through the transparent humors of the eye, and they are to a great extent protected from all other physical disturbances.

When a given set of sensory nerves is excited, *the sensation which corresponds to the set of nerves is always produced* no matter what the character of the stimulus may be. Thus, a sensation of light is always produced when the nerves of sight are excited, be the excitation caused by a severe mechanical shock, by an electric current passing through the eye, by abnormal conditions of blood circulation through the retina, or by the proper stimulus (the light which passes through the transparent humors of the eye. See Art. 3).

3. Light, the sensation; light, the proper stimulus. — The sensation which is experienced when the nerves of sight are excited is called *light*. That physical disturbance which constitutes the proper stimulus of the nerves of sight is also called *light*. The most familiar property of the physical disturbance which is called light is that it can pass through many substances, such as glass and water; that is to say, we can "see through" such substances, and they are said to be *transparent*.

The study of light as a sensation belongs to the subject of psychology; the study of the physical disturbance which constitutes the proper stimulus of the nerves of sight belongs to the subject of physics. This treatise is devoted to the study of light, the proper stimulus.

4. Nerves of hearing. *Sound, the sensation; sound, the proper stimulus.* — The nerves of hearing terminate in end organs which float in a watery fluid which is contained in a bone-walled cavity called the *inner ear*. These end organs are to a great extent protected by the massive bones of the head from all disturbances except vibratory movements of the air which reach them as follows: Figure 1 shows the essential features of the ear.* The

* This figure is not intended to show the actual structure but to illustrate the principles of action of the ear. No attempt has been made in this figure to show the cavity of the inner ear (the *cochlea*) in which the *basilar membrane* is situated, which contains the sense organs of musical pitch. The principles of action of the basilar membrane are explained in Chapter XV. A very full discussion of the structure and action of the ear may be found in Helmholtz's *Tonempfindungen*, pages 189–226 of the English translation by Alexander J. Ellis (Longmans & Co.).

cavity C which is filled with watery fluid constitutes the *inner ear*, and it is provided with two "windows," the *oval window* O and the *round window* R which are closed by very thin membranes. The *middle ear* M is an air-filled cavity which communicates with the mouth cavity through the *Eustachian tube* E, and three small bones in the form of a chain bridge across from the *ear drum* D to the membrane which covers the oval window. The vibratory motion of the outside air causes the ear drum D to vibrate. These vibrations are transmitted to the oval window by the chain of small bones, the vibration of the membrane of the oval window causes the fluid in the inner ear to surge back and forth through the complicated channels of the inner ear between the oval window and the round window, as indicated by the double-headed arrow in Fig. 1, and the end organs of the nerves of hearing are excited by this surging fluid.

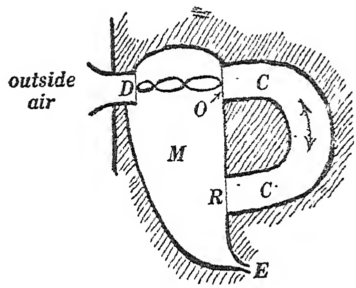


Fig. 1.

The sensation which is experienced when the nerves of hearing are excited is called *sound*. That physical disturbance which constitutes the proper stimulus of the nerves of hearing is also called *sound*. The study of sound, the sensation, belongs to the science of psychology. This treatise is devoted to the study of sound, the proper stimulus.

5. **Transmission of light and sound** —We have come by experience to associate *objects at a distance* with our sensations of light and sound and to draw from these sensations more or less complicated inferences concerning such distant objects. This process is called the *seeing* or *hearing* of the distant objects.

The attempt to explain the evident connection between a distant object and the sensations of light and sound which are associated with it has led philosophers to assume the existence of physical agencies which are transmitted to us from distant objects

and which affect our organs of sight and hearing. The various theories of light and sound which have thus arisen are explained briefly in the immediately following articles.

6. The corpuscular theory of light.* — The phenomena of shadows and the obstruction of vision of a distant object by intervening objects show that light travels sensibly in straight lines. In accordance with this fact, it was the accepted theory, until long after the time of Sir Isaac Newton, that light consisted of particles or corpuscles which were thrown off from luminous bodies at great velocity, traveling in straight lines until reflected or stopped by objects upon which they impinged. This was called the corpuscular theory of light.

7. The wave theory of light and sound.† — The most comprehensive understanding of the phenomena of light and sound has been reached on the hypothesis that light and sound are wave-like disturbances which pass out in all directions from luminous bodies and from sounding bodies, respectively. No attempt will be made to lead up to this hypothesis by preliminary discussion, but its justification, in the reader's mind, will become more and more complete as he has occasion to use it.

8. Transmitting media. *The luminiferous ether.* — The conception of light as a wave-like disturbance requires the assumption of a transmitting medium. The fact that light reaches us from the sun and stars across apparently empty space, and the fact that no known material substance is capable of transmitting waves with anything approaching the enormous velocity of light, necessitate the assumption of a special light-transmitting medium, *the ether*, which fills all space.

Sound, on the other hand, cannot travel through a vacuum;

* A very interesting discussion of the corpuscular theory of light is to be found in Preston's *Theory of Light*, pages 15–21. Sir Isaac Newton was the greatest exponent of the corpuscular theory.

† See "The Wave Theory of Light," a collection of original memoirs by Huygens, Young and Fresnel, translated and arranged by Henry Crew. The Scientific Memoirs Series, edited by J. S. Ames.

the transmission of sound is, in fact, accomplished by a wave-like disturbance of air, or water, or other material medium.

9. The elastic-solid theory of light and the electromagnetic theory of light. — Everyone is familiar with the behavior of water waves, how they travel out in all directions from the center of disturbance at a more or less definite velocity and how they produce commotion at any point on the surface of the water as they pass by. Without this familiarity with the behavior of water waves, it is probable that the wave theories of light and sound would never have been developed because sound waves are under ordinary conditions invisible, and light waves are even less directly evident to the senses than sound waves.

In earlier days, light waves were considered to be waves of distortion traveling through an elastic medium, similar in many respects to the waves of distortion which travel through a mass of jelly. Indeed, light waves are known to be of a kind called transverse waves, and such waves can be propagated only in a medium which has rigidity, that is, a solid medium. The older wave theory of light is therefore called the *elastic-solid* theory, and a curious difficulty arose in this older theory, namely, that the transmitting medium, the ether, was necessarily assumed to have the properties of a solid in order to transmit light waves and yet the planets were observed to travel through this solid ether without perceptible friction. This hypothetical ether possessed the rigidity of steel and at the same time it possessed a greater fluidity than the thinnest air!

The theory of electricity and magnetism as developed by Maxwell is based upon the assumption of a medium capable of transmitting electric and magnetic stresses. About 1870 Maxwell developed a theory of electromagnetic waves, which was based upon known phenomena of electricity and magnetism, and the velocity of such waves as calculated by Maxwell from electromagnetic data* came out equal to the velocity of light. The

* See Franklin and MacNutt, *Elements of Electricity and Magnetism*, page 268.

theory of electromagnetic waves was therefore proposed by Maxwell as a basis for the wave theory of light, the electromagnetic theory of light.

Most of the essential points in Maxwell's theory have been verified in recent years, especially by the experiments of Hertz on electromagnetic waves in 1887, and the electromagnetic theory of light is now universally accepted. The difference, from the philosophical point of view, between the elastic-solid theory of light and the electromagnetic theory of light has been pointed out most clearly by Heaviside.* "The electromagnetic theory is abstract and highly generalized and it rests mainly on facts; the elastic-solid theory is concrete and highly specialized and it rests mainly on hypotheses." The elastic-solid theory is indeed very particular in saying what light consists of, and for this reason it is useful in giving clear ideas to one who is not practised in thinking in terms of the abstract notions of electricity and magnetism. It is for this reason that the terminology of the elastic-solid theory is used in this text in the chapters on wave motion and on polarization and double refraction.

10. The measurement of the velocity of sound. † — It is a familiar fact that sound requires a perceptible time to reach the ear from a sounding body. The first attempt to measure accurately the velocity of sound was made by a committee of members of the French Academy of Sciences in 1738. The observers were placed at the Paris Observatory and at three distant stations visible from the observatory. Every ten minutes a cannon was fired at one of the stations, and the observers at the other stations noted the time intervals which elapsed between the flash and the sound of the cannon. The light flash was transmitted almost instantaneously so that the observed time intervals were taken to

* *Electromagnetic Theory*, Heaviside, Vol. I, page 325.

† The velocity of sound varies quite perceptibly with the loudness. See *Encyclopedia Britannica*, 9th edition, article *Acoustics*, sections 20 to 27. The velocity of sound varies greatly with the temperature of the air This matter is discussed in the Britannica article. See also Poynting and Thomson's *Text-Book of Physics*, Volume on *Sound*, pages 16–31.

be the time intervals required for the sound to travel over the measured distances between the stations.

In 1822 this experiment was repeated at Paris in a slightly modified form. Two stations were selected, cannon were fired at these stations alternately, and the time intervals between flash and sound were observed as before. By firing at the two stations alternately the influence of the wind was eliminated. The distance between the stations was 18,622 meters (about 10 miles). The observed time interval between flash and sound at one station was 54.84 seconds and at the other station it was 54.43 seconds. These data gave 340.9 meters per second as the velocity of sound; allowing for the temperature of the air at the time the observations were taken, this corresponds to 331.2 meters per second at 0° C. More recent determinations of the velocity of sound give 331 meters per second for the velocity in dry air at 0° C. The velocity of sound in air depends upon the temperature, and it also depends upon humidity.

The following table gives the velocity of sound in various substances in terms of the velocity of sound in air:

TABLE.

Velocities of Sound in Different Media.

Air	Unity
Iron	15.1
Glass	15.3
Water	4.3

11. The measurement of the velocity of light. * — A Danish astronomer, Roemer (1675), was the first to show that light has a finite velocity. He found that the observed time of revolution of the satellites of Jupiter varies with the position of the earth in its orbit. When the earth is moving towards Jupiter the observed time of revolution of a satellite is less than the true time of revolution, and when the earth is moving away from Jupiter the observed time is greater than the true time; the true time of revo-

* For full account of the researches upon the velocity of light, see Preston's *Theory of Light*, Chapter XIX.

lution being the mean of all the observed times during one revolution of the earth in its orbit. The cause of the variation of the observed time of revolution of Jupiter's satellites is as follows: Suppose a light signal is flashed at equal intervals of time. A stationary observer sees these flashes separated by the true time interval between them. If, during the interval between two flashes, the observer moves towards or away from the source of the flashes, the interval between observed flashes will be less or more than the true interval, and the amount of this difference will be equal to the time required for light to pass over the distance that the observer has moved.

A laboratory method for measuring the velocity of light was devised by Fizeau in 1849. This method was employed under more favorable conditions by Cornu in 1874. Another method was devised by Foucault in 1850. This method has been used by various observers, notably by Michelson in 1880.

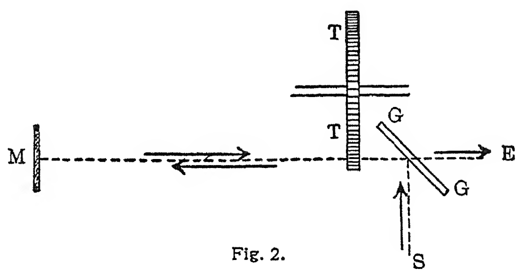


Fig. 2.

The method of Fizeau.—Light from a brilliant source S, Fig. 2, is reflected from an unsilvered glass GG, whence it passes between the cogs of a toothed wheel TT to a distant mirror M. From this mirror the light is reflected back so as to pass * between the same pair of teeth on the wheel, and a portion of this light passes on through the unsilvered glass GG to the observer's eye at E. If the wheel is now started rotating and slowly increased in speed, no light will reach the eye at E when the wheel has reached such a speed that, in the time t, during

* The actual arrangement of this apparatus involves the use of several lenses.

which the light is traveling to the distant mirror and back again, the opening through which the outgoing light passed will have been supplanted by the first following tooth. As the speed is further increased, light will again reach the eye at *E*, to be again cut off when the wheel reaches such a speed that, during the time *t*, the opening through which the outgoing light passed will have been supplanted by the second following tooth, and so on. In Cornu's experiments the distance from the wheel to the mirror *M* was 23 kilometers (about 15 miles), and the speed of the wheel was slowly increased until the light at *E* had disappeared and reappeared fifteen times. The wheel was then held steadily at this speed for a sufficient time for the speed to be

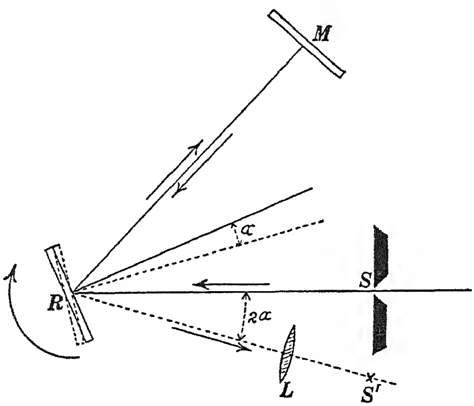


Fig. 3.

accurately observed, and from this speed the time *t* (required for the light to travel 46 kilometers) was determined as the time required for a given space to be supplanted by the fifteenth following tooth.

The method of Foucault.—Light from a brilliant source passes through a slit *S*, Fig. 3, and falls upon a rapidly rotating mirror *R* whence, once each revolution, the light is reflected towards a distant mirror *M* which reflects the light back to *R*. During the time *t* that the light is traveling from *R* to *M* and back again the mirror will have turned through an angle α and

the returning ray will be reflected along the dotted line making an angle 2α with the incident ray. This returning ray passes through a lens L which forms an image of the slit at S'. The angle 2α is determined by measuring the distances SR and SS'. From this angle and the observed speed of the mirror the time t is found. In the experiments of Michelson the distance RM was about 600 meters and the mirror was driven at about 250 revolutions per second. Newcomb in 1882 carried out Foucault's method. He found, from an extended series of observations, a velocity of 299,778,000 meters per second in air (299,860,000 meters per second in vacuum). This is perhaps the most reliable determination that has been made.

CHAPTER II.

THE WAVE THEORY.

12. Wave motion. — The group of ideas which relates to that kind of mechanical action which is called wave motion is perhaps more useful in general physics than any other group of mechanical ideas. Nearly every phenomenon of sound and light, and nearly all of the phenomena of oscillatory motion become easily intelligible in terms of the ideas of wave motion. In undertaking to establish the more important ideas of wave motion, however, we are confronted with a serious difficulty, namely, that ordinary water waves, the only kind of waves with which every one is familiar, are excessively complicated; invisible sound waves in the air and the even more intangible light waves in the ether, in their more important aspects at least, are very simple in comparison. The wave theory, however, originated in the applications to sound and light of the ideas which grew out of a familiarity with the behavior of water waves, and in attempting to establish the wave theory one is obliged to base it upon the familiar phenomena of wave motion as exemplified by water waves.

13. Wave media. — The material or substance through which a wave passes is called a *wave transmitting medium*. Thus, the air is a medium which transmits sound waves, and the ether is the medium which transmits light waves. During the passage of a wave, the medium always moves to some extent, but the velocity with which the medium actually moves is generally very much less than the velocity of progression of the wave.* Thus, when the end of a long rope is moved rapidly to and fro sidewise, waves travel along the rope and each point of the rope oscillates as the

* The mathematical theory of wave motion is developed on the assumption that the actual velocity v of the medium is very small in comparison with the velocity of wave progression V.

waves pass by. In some cases the medium is left permanently displaced after the passage of the wave and in other cases the medium returns to its initial position after the passage of the wave.

14. Wave pulses and wave trains. — When a stone is pitched into a pond a wave emanates from the place where the stone strikes. When a long stretched wire is struck sharply with a hammer, a single wave (a bend in the wire) travels along the wire in both directions from the point where the wire is struck. When a long steel rod is struck on the end with a hammer, a single wave (an endwise compression of the rod) travels along the rod. When an explosion takes place in the air, the firing of a gun for example, a single wave (a compression of the air) travels outwards from the explosion. Such isolated waves are called *wave pulses*. When a disturbance at a point in a medium is repeated in equal intervals of time the disturbance is said to be *periodic*. Such a disturbance sends out a succession of similar waves constituting what is called a *wave train*.

Clear ideas of the physical action which is involved in wave motion may be best obtained by considering the behavior of wave pulses. The applications of the wave theory to sound and light, however, depend very largely upon a consideration of wave trains.

15. Wave shape. — A term which is frequently used in the discussion of wave motion is the term *wave shape*, and the meaning of

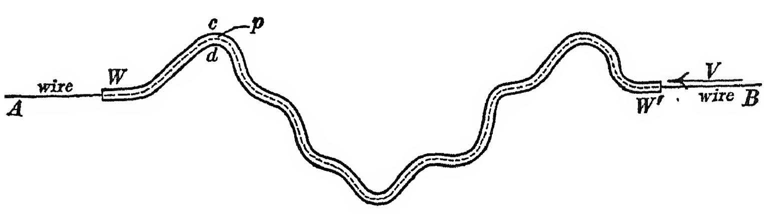


Fig. 4.

this term may be best explained by considering wave motion along a stretched wire or string. Let us consider first an entirely general case, as follows: Let AB, Fig. 4, be a wire under a tension of T dynes, and suppose that each centimeter of length of the

wire weighs m grams. Imagine the wire to be drawn at a velocity of V centimeters per second through an irregularly bent tube WW', and let it be assumed that the wire slides through the tube without friction. Then the wire will not exert any force against the sides of the tube if the velocity V satisfies the equation

$$V = \sqrt{\frac{T}{m}} \tag{1}$$

and at this velocity, therefore, the moving wire would retain its bent (stationary) shape if the tube could be removed. This tendency of a bend, once established in a moving flexible wire or cord, to persist is strikingly illustrated by the stationary bends which are often seen on a rapidly moving belt.

The absence of force action between the tube and wire in Fig. 4 may be shown as follows: Consider any point p of the tube. The portion of the tube in the immediate neighborhood of this point is a portion of a circle of radius r. Therefore, if the wire were stationary, its tension would produce a force against the side d of the tube, and this force would be equal to T/r dynes per centimeter of length of wire.* But when the wire is being drawn through the tube, the particles of the wire have to be constrained to move along the small circular arc at p, and an unbalanced force equal to mV^2/r dynes per centimeter of wire must act upon the wire pulling it towards the side d of the tube.† Therefore when $T/r = mV^2/r$, or when $V = \sqrt{T/m}$, the side force due to the tension of the wire is just sufficient to constrain the particles of the wire to the curved path at p, whatever the curvature at that point may be, and no force need act upon either side of the tube. In this discussion the wire is assumed to be perfectly flexible.

We may imagine everything in Fig. 4 (moving wire and tube) to be set moving to the right at a velocity equal and opposite to the velocity V at which the wire is supposed to be moving to

* See Franklin and MacNutt's *Elements of Mechanics*, pages 86 and 121.

† See Franklin and MacNutt's *Mechanics*, page 79.

the left. The wire would then be stationary and the tube would be moving to the right at velocity V. The wire would pass through the tube without exerting any force on the sides of the tube as before, so that the bend WW' would continue to move along the wire without changing its shape even if the tube were non-existent. Such a moving bend constitutes a wave, and the only motion of a given point of the wire during the passage of the wave would be its sidewise motion. *The term* **wave shape** *refers to the distribution of velocity of the medium (sidewise velocity of the wire in Fig. 4) in a wave.** This matter may be made clear by the following example. Imagine the straight tube WW', Fig. 5, to slide along the wire AB at velocity V, thus produc-

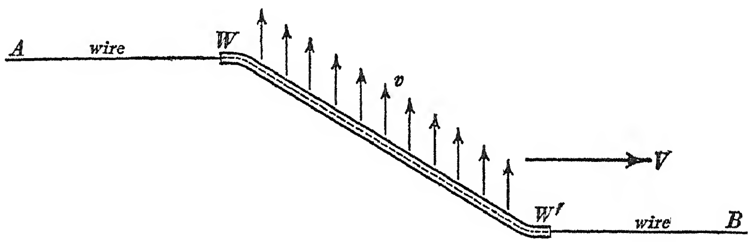


Fig. 5.

ing a wave. The wire at each point in the tube is moving sidewise at constant velocity v, as indicated by the small arrows, and the portion of the wire in the tube is uniformly stretched. The uniform stretch of the wire in the tube in Fig. 5 is evident when we consider first that the horizontal component of the tension of the wire in the tube must be equal to the tension T of the portions of the wire beyond the tube so that the tension of the wire in the tube must be greater than T, and second that the tube is straight so that the tension of the wire in the tube must be uniform.

* In some respects it would be simpler to think of the shape of the wave in Fig. 4 as represented by the shape of the moving bend, but the two things in a wave which it is important to consider are velocity and distortion (or stretch) because the kinetic energy in the wave depends upon the one and the potential energy in the wave depends upon the other.

In discussing a wave on a wire, it is convenient to draw a reference axis OO, Fig. 6, in the direction of progression of the wave and to represent the actual velocity of the medium at each point in the wave by an ordinate y, as shown; or to represent the actual stretch or compression of the medium at each point by the ordinate y. Thus, the wave shown in Fig. 5 would be represented by the rectangle ab in Fig. 6, inasmuch as the sidewise velocity of the wire and the stretch of the wire are everywhere

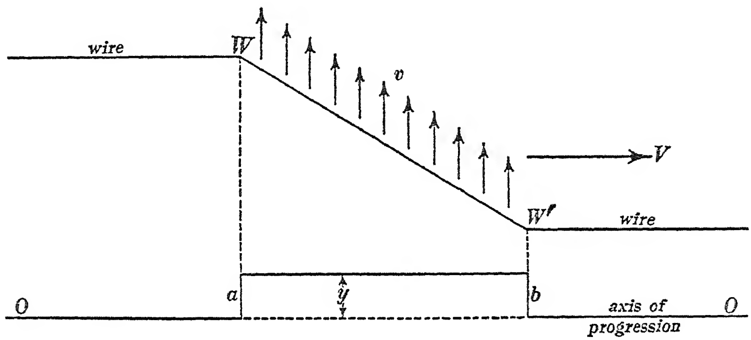


Fig. 6.

the same in value between W and W'. The wave represented in Figs. 5 and 6 is therefore called a *rectangular wave* or a *rectangular wave pulse.* The discussion of wave pulses which is given in the following articles refers to rectangular wave pulses because rectangular wave pulses are the simplest to describe.

16. Pure waves and impure waves. — When the kinetic energy in a wave due to the velocity of the medium is at each point equal to the potential energy due to the distortion of the medium, the wave is called a *pure wave,* otherwise the wave is called an *impure wave.* A remarkable property of a pure wave is that it travels along without changing its shape. Thus, for example, a bend travels along a flexible string with unchanging shape. An impure wave, on the other hand, spreads out more and more as it travels through the medium. This matter is discussed in Art. 21.

17. Wave pulses in a canal. — The simplest kind of wave motion in a canal is that in which the only perceptible motion of the water in the wave is a *uniform horizontal flow*.* Imagine a canal filled with still water, and imagine a gate to be moved slowly along like a piston at velocity v as indicated in Fig. 7. As the water is set in motion by the gate, it heaps up to a definite level, and a wave of starting† W travels along the canal at a definite velocity V (V very much larger than v). If the gate is suddenly stopped, the wave of starting W continues to move along as before, the water next to the gate in being stopped drops to its normal level, and a wave of arrest W' moves along the canal as shown in Fig. 8. The uniformly moving‡ and uniformly elevated body of water A, Fig. 8, constitutes what is called a wave. The water in front of the wave is continually being set in motion at velocity v and raised to a higher level, the water in the back part of the wave is continually being brought to rest and lowered to the normal level of the still water in the canal. Thus the

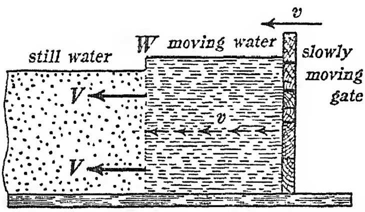


Fig. 7.

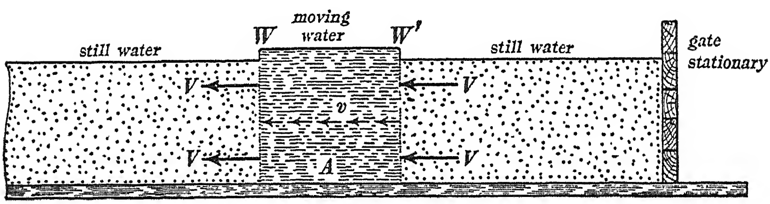


Fig. 8.

* A good discussion of ordinary waves, on the surface of deep water is to be found in *Encyclopedia Britannica*, 9th edition, article *Wave*, sections 5 to 8. A fascinating discussion of waves produced by ships may be found in the third volume of Sir William Thomson's (Lord Kelvin's) *Popular Lectures and Addresses*.

† Franklin and MacNutt's *Elements of Mechanics*, page 255.

‡ The actual motion of the water in the wave is at velocity v; this must not be confused with the velocity of progression V of the wave.

state of motion which constitutes the wave A travels along without changing its character, friction being neglected.

Figure 8 represents a wave of elevation in which the direction of motion of the water v is the same as the direction of progression of the wave V. Figure 9 represents a wave of depres-

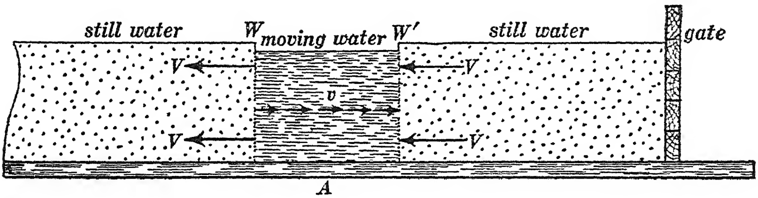


Fig. 9.

sion in which the direction of motion of the water is opposite to the direction of progression of the wave. This wave of depression is produced by moving the gate in a direction opposite to that shown in Fig. 7.

Figure 10 represents a wave of elevation followed by a wave

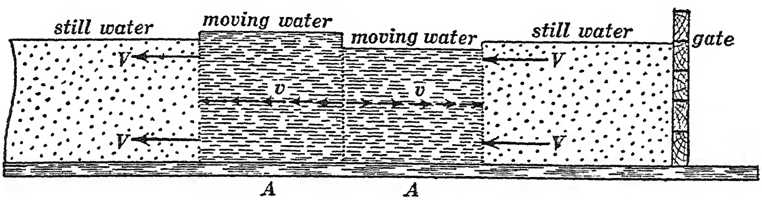


Fig. 10.

of depression. Such a wave is produced by a to and fro movement of the gate. The wave shown in Fig. 8 leaves the water in the canal permanently displaced by an amount equal to the original movement of the gate. The same is true of the wave shown in Fig. 9. The wave which is shown in Fig. 10 leaves the water in the canal in its initial position.

18. Transverse waves and longitudinal waves. — The waves which are described in Art. 15 as traveling along a stretched wire are called *transverse waves* because the motion v of the wire in

the wave is at right angles to the motion of progression of the wave V. Any wave in which the motion of the medium is at right angles to the direction of progression of the wave is called a transverse wave.

The waves which are described in Art. 17 as traveling along a canal are called *longitudinal waves* because the motion v of the water in the wave is parallel to the direction of progression V of the wave. Any wave in which the motion of the medium is parallel to the direction of progression of the wave is called a longitudinal wave.

19. The principle of superposition.—One of the most interesting properties of water waves is that a number of distinct sets of waves can travel over a given portion of water simultaneously without becoming confused. Thus, one can often distinguish the ocean swell, the smaller waves formed by the local breeze, and the peculiar waves formed by a boat, simultaneously on a given portion of water surface. It is a familiar fact that *a number of objects remain distinctly visible to a number of observers when the light in passing from the various objects to the various observers has to cross the same region at the same time.* It is also true that *one sound does not sensibly alter the character of another sound which accompanies it* although a combination of sounds is more or less distracting to the attention. *A number of waves or system of waves can travel through a given region simultaneously, each wave or system of waves behaving exactly as if it were traversing the region*

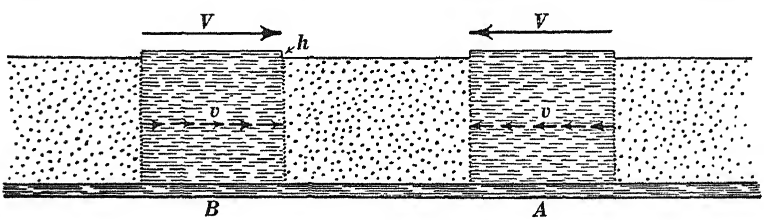


Fig. 11a.

alone. The actual velocity of a particle of the medium at a given instant is the vector sum of the velocities which correspond to the separate waves.

Examples.—Two interesting examples of the principle of superposition in the case of canal waves are shown in Figs. 11 and 12. Two waves of elevation approach each other as shown

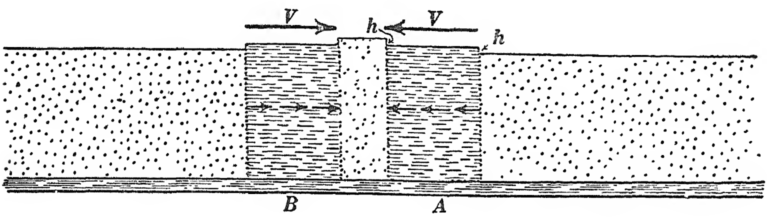


Fig. 11b.

in Fig. 11*a*. When these waves begin to overlap, the overlapping portions produce a stationary body of water of doubled elevation

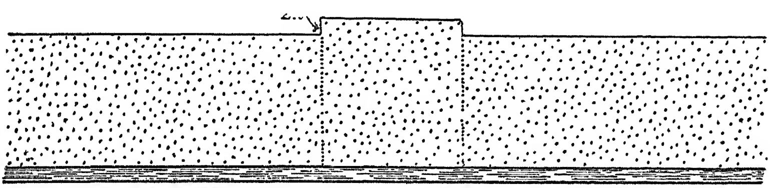


Fig. 11c.

(the opposite velocities when added give zero velocity and the elevations when added give a doubled elevation) as shown in

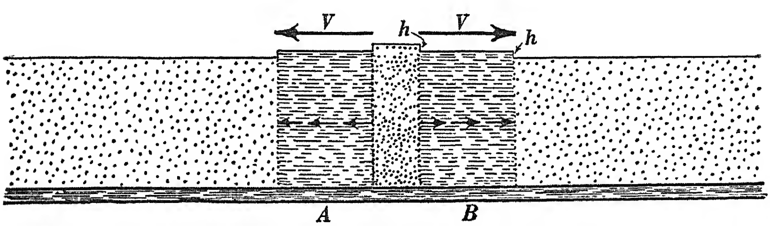


Fig. 11d.

Fig. 11*b*. Figure 11*c* shows the state of affairs at the instant when the two similar waves *A* and *B* completely overlap each other, Fig. 11*d* shows the state of affairs at a later instant, and Fig. 11*e* shows the state of affairs after the two waves have become entirely separated.

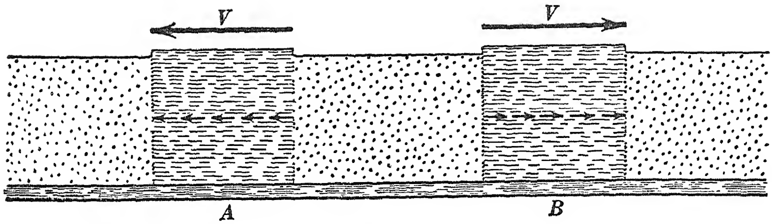


Fig. 11e.

Figure 12*a* shows a wave of elevation and a wave of depression approaching each other. In this case the velocity of flow of the water is in the same direction in both waves, and when

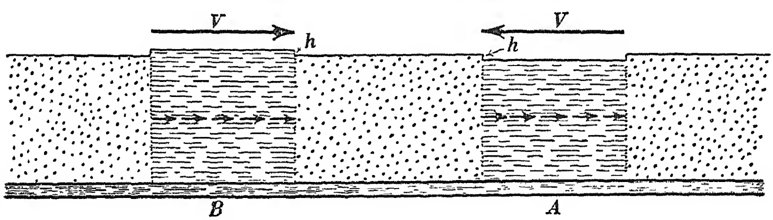


Fig. 12a.

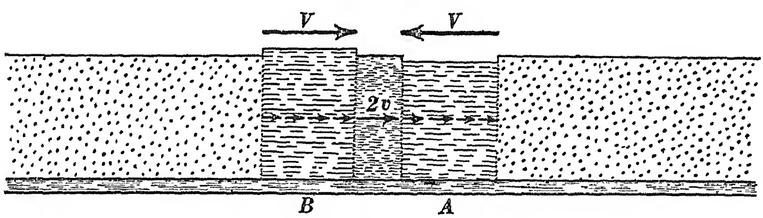


Fig. 12b.

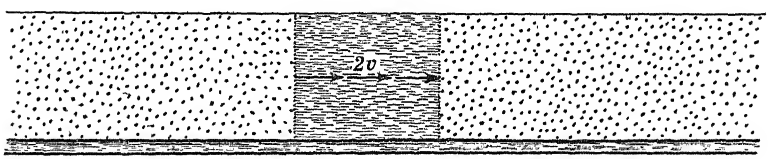


Fig. 12c.

these waves begin to overlap the overlapping portions constitute a body of water of normal depth but having a doubled velocity of flow as shown in Fig. 12*b*. Figures 12*b*, *c* and *d* represent successive stages of the passage of the two waves through each

other, and Fig. 12*e* shows the two separate waves after they have passed through or over each other.

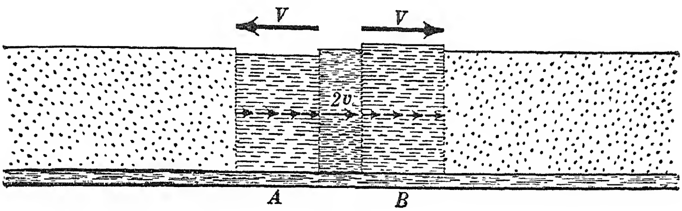


Fig. 12d.

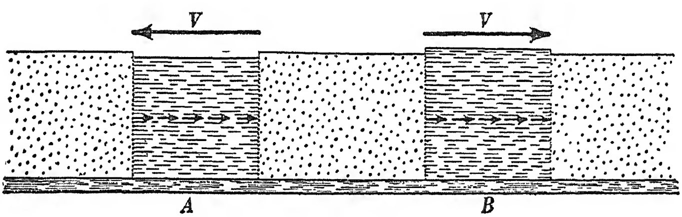


Fig. 12e.

It is especially interesting to note that the uniformly elevated but stationary body of water in Fig. 11*c*, and that the uniformly moving body of water in Fig. 12*c* which is neither elevated nor depressed, both break up into oppositely moving waves as shown in Fig. 11*d* and in Fig. 12*d*, respectively.

20. Reflection of canal waves. — A matter of considerable importance in the theory of interference of light and in the theory of oscillation of organ pipes is the so-called reversal of phase of

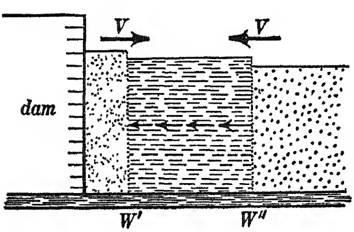


Fig. 13a.

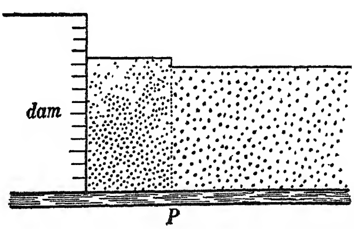


Fig. 13b.

a wave by reflection. Some understanding of this matter may be obtained by considering the reflection of canal waves as follows:

Imagine a wave like that which is shown in Fig. 8 to approach a rigid dam. When the wave reaches the dam the moving water in the wave, in being brought to rest at the face of the dam, is heaped up to a doubled elevation, as shown in Fig. 13*a*; the wave of arrest W' in Fig. 13*a* travels to the right; and at a certain instant the state of affairs is as represented in Fig. 13*b*,

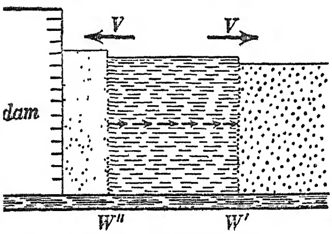


Fig. 13c.

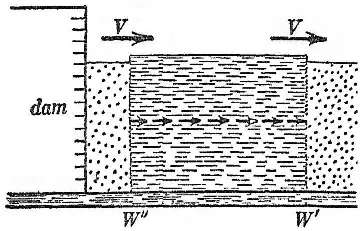


Fig. 13d.

the entire energy of the wave being represented at this instant by the doubled elevation of a portion of water one half as long as the original wave. The body of stationary water in Fig. 13*b* begins to flow at the point P, and the potential energy of elevation is partly transformed into kinetic energy of flow as shown in Fig. 13*c*; and when the wave of starting W'' in Fig. 13*c* reaches the dam, the water next to the dam is brought to rest and lowered to the normal level of the water in the canal as

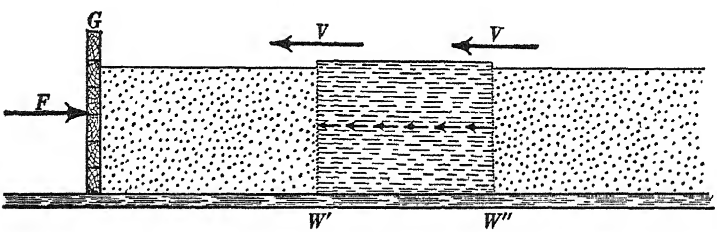


Fig. 14a.

shown in Fig. 13*d*. Figure 13*d* represents the reflected wave which is exactly like the original wave except that its velocity of flow has been reversed.

Imagine a gate G in Fig. 14*a* which is free to move along a canal like a piston, and upon which acts a force F barely suffi-

cient to balance the normal push of the still water in the canal. Imagine the gate, furthermore, to be without inertia and without friction so that the least increase of push of the water would cause the gate to move to the left, or the least decrease of push of the water would cause the gate to move to the right. When the advancing wave which is shown in Fig. 14*a* reaches the gate,

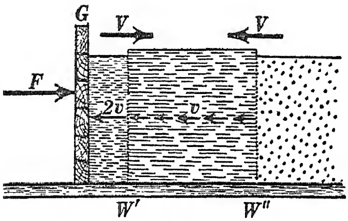


Fig. 14b.

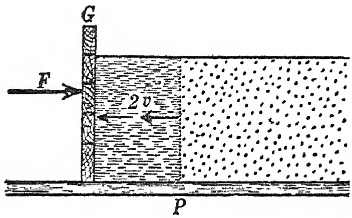


Fig. 14c.

the elevated water in the wave sets the gate in motion, and the water next the gate drops to the normal level of the still water in the canal and has a doubled velocity imparted to it, as shown in Fig. 14*b*. A moment later the entire energy of the original wave

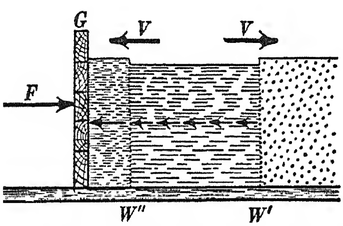


Fig. 14d.

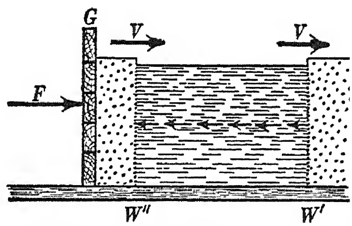


Fig. 14e.

is represented by the energy of flow (at doubled velocity) of a portion of water one half as long as the original wave as shown in Fig. 14*c*; then the water level at the point P is suddenly lowered and the velocity of flow is reduced to the value v, as shown in Fig. 14*d*. Figure 14*e* shows the complete reflected wave which is exactly like the original wave except that the original wave is a wave of elevation whereas the reflected wave is a wave of depression.

A wave always consists of two elements, namely, motion and distortion traveling along together and mutually sustaining each other, and it is convenient to speak of the motion of the medium in the wave as the *velocity phase* of the wave, and to speak of the distortion (elevation or depression in the case of water waves) as the *distortional phase* of the wave. When a wave is reflected from a rigid boundary, that is, a boundary where no motion is possible, the velocity phase of the wave is reversed and the distortion phase is not reversed. When, however, a wave is reflected from a free boundary, that is, a boundary where no distortion is possible, the distortional phase of the wave is reversed and the velocity phase is not reversed.

21. Wave diffusion.—When a wave like *A*, Fig. 8, travels along a canal, the shape of the wave would remain unaltered if the water were frictionless. As a matter of fact, however, the velocity of flow of the water in the wave is continually reduced by the friction of the water against the sides and bottom of the canal, whereas there is no action tending to reduce the elevation of the water in the wave. The result is that the wave becomes impure (kinetic energy of flow not equal to potential energy of elevation), and that *portion* of the elevation which is in excess of what is required to constitute a pure wave with what remains of the velocity of flow behaves exactly like the elevated body of still water in Fig. 11*c*, that is, this excess of elevation breaks up into two pure waves *A* and *B*, as shown in Fig. 11*d*; one of these, *A* or *B*, merges with the original wave (according as the original wave is moving to the right or to the left), and the other shoots backwards. The upper part of Fig. 15 represents on an exaggerated scale the elevated portion of water which constitutes a rectangular wave pulse in a canal. The velocity of flow v in this wave is continually reduced by friction as the wave travels along, and the *excess of elevation* which is being thus continually left in the wave causes a long-drawn-out wave to shoot backwards, and after a time the wave takes on the form shown in the

lower part of the figure. The energy in the head of the wave is greatly reduced, partly because of the direct losses due to friction and partly because of the shooting of a portion of the energy backwards into the tail of the wave. This spreading out of a

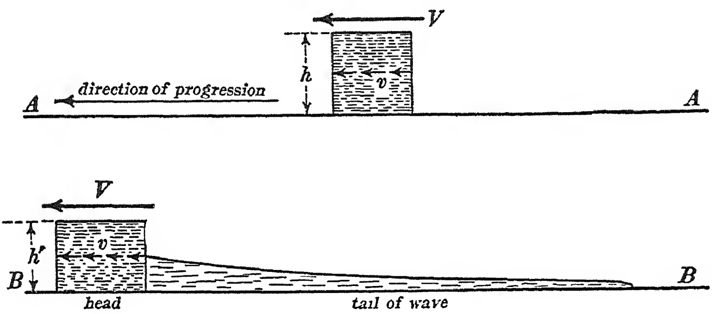


Fig. 15

wave is a phenomenon like diffusion and it is called *wave distortion* or *wave diffusion.*

If the canal is brimful of water so that the elevation of the water in the wave causes an overflow or spill, the tendency is for a wave to remain pure and therefore to be propagated without diffusion (without the development of a tail), because the elevation is reduced by spill and the velocity of flow is reduced by friction.

The effects of wave diffusion are often noticeable in the case of sound waves as follows: The clear transmission of articulate speech depends upon the passage of complicated wave shapes of sound from the speaker to the hearer. * When the sound passes through thick foliage the velocity of the air in the waves is reduced by friction and there is no action which tends to reduce the compression of the air in the waves. The result is that every

* See, for example, the discussion of Fig. 21. A very interesting experimental study of the wave shapes which correspond to articulate speech is published by the Carnegie Institution of Washington under the title, *Researches in Experimental Phonetics and the Study of Speech Curves,* by E. W. Scripture, Washington, November, 1906. See especially the plates facing pages 40, 44 and 50 in Professor Scripture's paper.

part of the wave tends to diffuse over the adjoining parts causing the obliteration of all the fine details of wave shape and making the sound very indistinct to the hearer. When one attempts to speak through a very long speaking tube the friction of the moving air against the sides of the tube decreases the velocity of the air in the waves and there is no tendency for the compression in the waves to be decreased. The result is that wave diffusion takes place and the speech becomes indistinct.

22. Oscillation of the water in a short canal. — If a wave is started along a portion of a canal the ends of which are terminated by rigid dams as shown in Fig. 13, or by freely moving gates, as shown in Fig. 14, then the wave will be repeatedly reflected from the ends of the canal, and the to and fro motion of the wave along the canal will constitute an oscillatory movement of the water in the canal.

The time required for one complete cycle of movements of the water in the canal is related to the velocity of the wave and to the length of the canal as follows: (*a*) If both ends of the canal are rigid dams, then the wave starting from end *A* is reflected with reversal of velocity phase at end *B* and again reflected with reversal of velocity phase at end *A*, so that after two reflections the wave is exactly in its initial condition. Therefore one complete cycle of movements of the water in the canal takes place during the time required for the wave to travel from one end of the canal to the other and back again. This is also true in case both ends of the canal are formed by freely moving gates as shown in Fig. 14.

(*b*) If one end of the canal is formed by a rigid dam and the other by a freely moving gate, then a complete cycle of movements of the water in the canal takes place in the time required for the wave to travel over four times the length of the canal. Starting from the dam-end of the canal, the distortional phase of the wave is reversed by reflection at the gate-end, then the velocity phase is reversed by reflection at the dam-end, then the distortional phase is again reversed by reflection at the gate-end,

and then the velocity phase is again reversed by reflection at the dam-end, thus bringing the wave into its initial condition after two reflections from each end.

Figure 16 shows a string AB which is pulled to one side, and the figure also shows a canal CD in which the water in one end

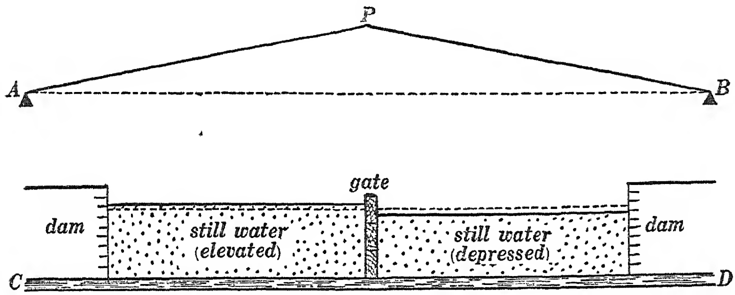


Fig. 16.

is elevated and the water in the other end is depressed. When the stretched string AB, Fig. 16, is released, it oscillates, and $A'B'$, Fig. 17, shows the state of affairs at a given instant.

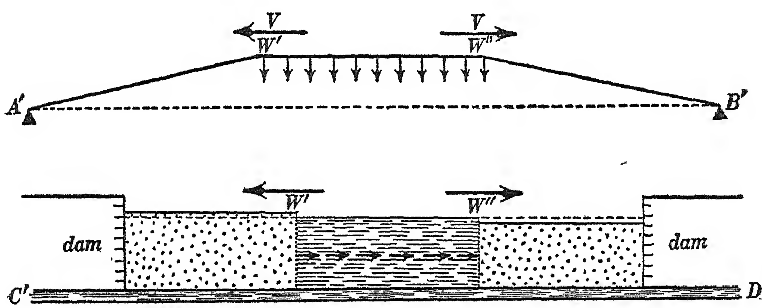


Fig. 17.

When the gate is lifted from the canal CD, Fig. 16, the water in the canal oscillates, and $C'D'$, Fig. 17, shows the state of affairs at a given instant.*

*The student should make a series of drawings showing, say, eight successive stages of one complete oscillation of the string AB, and a series of drawings showing eight successive stages of one complete oscillation of the water in the canal CD. The student should also plot a curve like that which would be traced upon a uniformly moving strip of paper by a pencil-point attached to any given point on the

23. Waves from periodic disturbances; wave-trains. — A periodic disturbance is one which is repeated in every detail in equal intervals of time. The time interval τ during which one repetition of the disturbance takes place is called the *period* of the disturbance, and the number of repetitions per second is called the *frequency*. A periodic disturbance sends out a succession of similar waves constituting what is called a *wave-train*. The distance λ between similar parts of two adjacent waves of a train is called the *wave-length* of the train; it is the distance traveled by the waves during the period τ. Let V be the velocity of the waves; then

$$\lambda = V\tau \tag{2}$$

Examples. — One end of a long rubber tube is held in the hand and the other end is fixed to a wall, and when the hand is moved up and down periodically (one movement being of any character whatever, simple or complicated) a wave-train will be transmitted along the tube. The motion of the tube in this case is complicated by the fact that the waves that go out from the

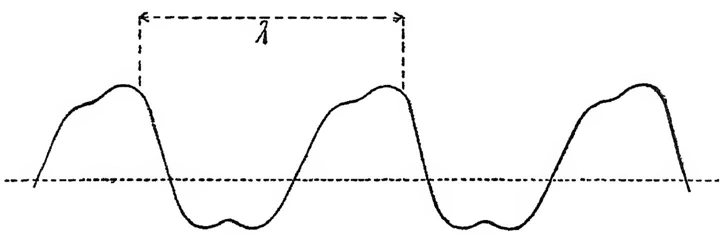


Fig. 18.

hand are reflected from the fixed end of the tube. Immediately after beginning to move the hand, however, and before the waves have reached the distant end of the tube, the wave-train may be seen very distinctly, and an instantaneous photograph of the rubber tube would show a series of similar bends like Fig. 18, for

string *AB*, Fig. 16, or by a pencil-point carried by a float at any given point in the canal *CD*, Fig. 16; the direction of motion of the strip of paper being at right angles to the direction of motion of the pencil-point in each case.

example. The dotted line in Fig. 18 represents the undistorted position of the rubber tube.

Graphical representation of a wave-train. — For many purposes it is best to think of the shape of a wave as having reference to the velocity of the medium at each point of the wave, as explained in Art. 15. For present purposes, however, it is more convenient to represent the shape of a wave by a curve like Fig. 18, in which *the ordinate of the curve at each point represents the distance of the medium at that point from its equilibrium position*, the *displacement* of the medium, as it is called.

Simple wave-trains and compound wave-trains. — When a wave-train passes through a medium, each particle of the medium oscillates. When each particle of the medium performs simple harmonic motion during the passage of a wave-train, the wave-train is called a *simple wave-train.* A simple wave-train is represented graphically by a curve of sines * as shown in Fig. 19.

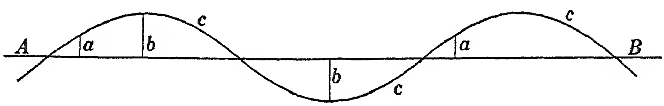


Fig 19.

When, during the passage of a wave-train the particles of the medium perform periodic movements which are not simply harmonic, the wave-train is called a *compound wave-train.* A compound wave-train is represented graphically by a periodic curve (a curve of which the successive sections are exactly alike), which is not a curve of sines. Simple and compound wave-trains are discussed in Chapter XII.

The character of the motion of a medium during the passage of a simple train of longitudinal waves may be understood from Fig. 20, which represents a simple wave-train travelling along a canal. Where the velocity of flow is greatest to the right, there the elevation of the water surface is greatest, and where the

* This is true whether the ordinates of the curve represent the velocity of the medium at each point or the displacement of the medium at each point.

velocity of flow is greatest to the left, there the depression of the water surface is greatest. The velocity of flow is represented by the shading and also by the horizontal arrows at the bottom.

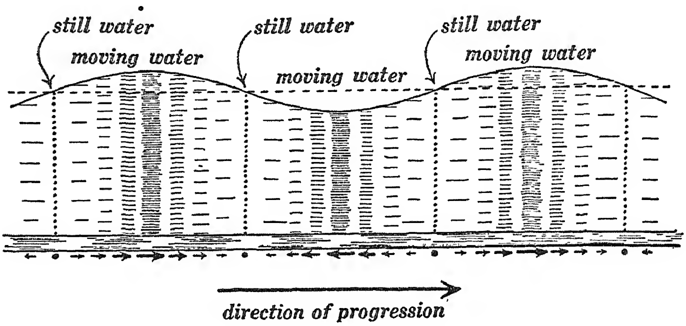


Fig. 20

When a simple wave-train (of longitudinal waves) travels along the air in a pipe or through the open air, the motion of the air is exactly like the motion of the water as represented in Fig. 20, but in this case *compression* of the air corresponds to elevation of the water surface and *rarefaction* of the air corresponds to depression of the water surface.

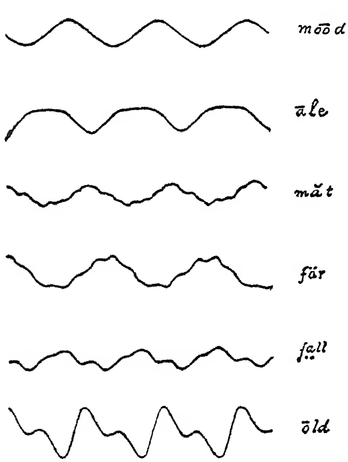


Fig. 21.

Examples.—The curves of Fig. 21 * represent the wave-trains which issue from the mouth when the indicated vowel sounds are produced by a baritone voice. The first curve represents approximately a simple wave-train, the other curves represent compound wave-trains. It must not be un-

* By L. B. Spinney. A very small mirror was mounted upon flexible hinges in a small rectangular opening in a wall. The vowel sound striking this wall caused the mirror to oscillate and a beam of light reflected from the mirror was focused upon a rapidly moving photographic plate. Figure 21 is a reproduction full size of these photographic tracings.

derstood from these examples that a given vowel sound is always associated with a wave-train of the same shape. This is by no means the case. See Chapter XV.

Amplitude and phase. — The maximum displacement *b*, Fig. 19, in a simple wave-train is called the amplitude of the train. Two points in a simple wave-train distant a whole wave-length from each other, always have equal displacements of the same sign and they are said to be *in the same phase;* two points in a simple wave-train distant half a wave-length from each other always have equal displacements of opposite sign and they are said to be *opposite in phase.* Thus, the points *aa* in Fig. 19 are in the same phase and the points *bb* are opposite in phase. The terms *crest* and *hollow* as applied to water waves are sometimes used to designate those points of a wave-train where the displacement has the greatest positive value and the greatest negative value, respectively.

Transmission of energy. — When a disturbance is produced in an elastic medium all or nearly all of the energy of the disturbance is carried away by the waves which are produced. On the other hand, waves give up their energy to obstacles which absorb them. Thus, water waves give up nearly the whole of their energy when they strike a shelving shore where they are not reflected; light waves give up their energy to a black body which absorbs them. The rate at which energy which is transmitted by light waves and by sound waves is the physical measure of the intensity of the light or sound. Thus, a beam of light of one square centimeter section would be said to have unit intensity if it transmitted one unit of energy per second. The energy transmitted by a simple wave-train is proportional to the square of its velocity phase or to the square of its distortional phase.*

* The velocity phase of a simple wave-train is proportional to its amplitude (maximum displacement) and inversely proportional to its wave-length. This is evident when we consider that the velocity of the medium at each point is represented by the steepness of the displacement curve, Fig. 18, and that this steepness is everywhere doubled if the ordinates are everywhere doubled or if the abscissas are everywhere halved.

24. Stationary wave-trains. — When one end of a stretched rubber tube is moved rapidly up and down, the tube quickly settles to a steady state of oscillation in which a series of points *n* along the tube remain stationary, while the intervening portions of the tube surge up and down, as indicated in Fig. 22. The heavy line in Fig. 22 shows the position of the tube at a given instant; a snap-shot of the tube, as it were. This oscillatory motion, which is entirely devoid of progressive character, is called a *stationary wave-train.* The stationary points *n* are

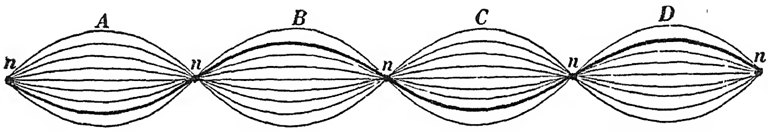


Fig. 22.

called *nodes*, the intervening portions of the vibrating rubber tube or string are called *vibrating segments*, and the middle point of a vibrating segment * is called an *antinode.* It is important to keep in mind the distinction between an advancing wave-train (which is usually called, simply, a wave-train) and a stationary wave-train. In an advancing wave-train no portion of the medium remains stationary, in fact, every particle of the medium moves in precisely the same way and to exactly the same extent, but not simultaneously, each succeeding particle being a little later in its movements. Thus, every particle of water in Fig. 20 oscillates to and fro, and the particles of water which at the given instant are stationary are merely at the middle points of their oscillations. In a stationary wave-train, on the other hand, the medium does not move at all at the nodes, the amplitude of motion increases from node to antinode, and all the particles move simultaneously, that is, all the particles in a vibrating segment move to and fro, or up and down, together.

The discussion of the oscillatory motion of the watei in a short

* A vibrating segment is sometimes called a *loop*.

canal and the discussion of the oscillatory motion of a string as carried out in Art. 22 involve the use of the idea of progressive wave motion although the oscillation itself has no progressive character, and *the vibratory motion which is represented in Fig. 22 may be resolved into two oppositely moving (progressing) wave-trains.* Consider two similar wave-trains *AA* and *BB*, Figs. 23, 24, 25 and 26, moving in opposite directions as indicated by

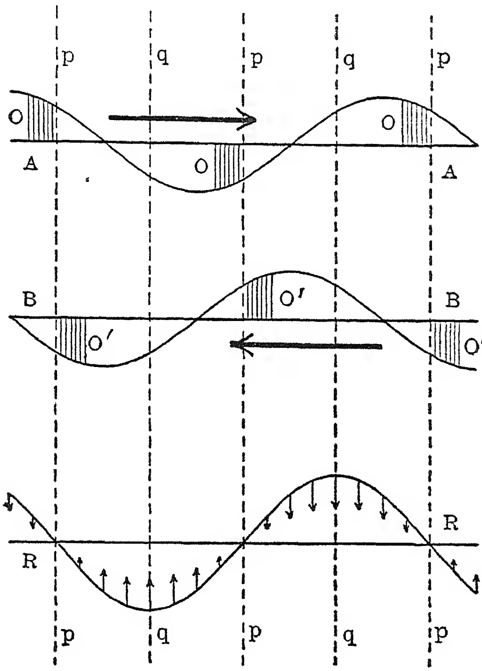


Fig. 23.

the heavy arrows. These wave-trains are supposed to be traversing the same region at the same time, and *AA* is drawn above *BB* merely to avoid confusion. According to the principle of superposition, the actual displacement of each particle of the medium is equal to the sum of the displacements of that particle due to each wave-train, and the actual velocity of each particle is equal to the sum of the velocities of that particular due to each

wave-train. Now, the ordinates O of the wave-train AA, Fig. 23, which reach the points ppp as the wave-train AA moves to the right are at each instant equal and opposite to the ordinates O' of the wave-train BB which reach the points ppp as the wave-train BB moves to the left. Therefore the points ppp of the medium remain stationary. The regions between the

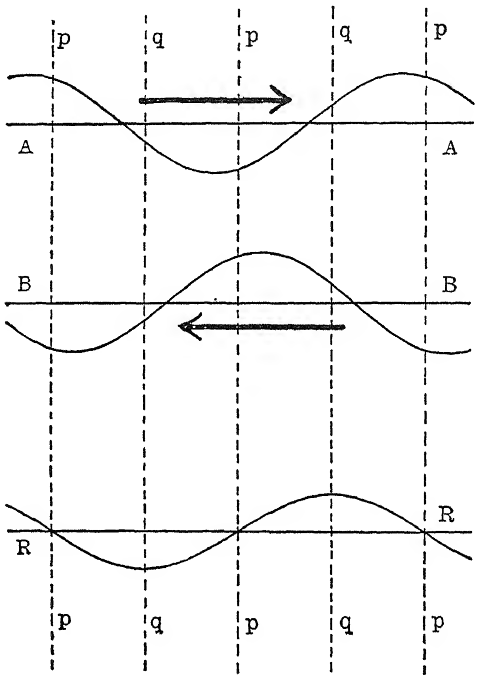


Fig. 24.

points pp, on the other hand, move up and down (to right and left in case of a longitudinal wave) as the two wave-trains AA and BB travel through or over each other. The resultant of the two wave-trains AA and BB is therefore a stationary wave-train with nodal points at ppp.

Figure 23 shows the positions of the two wave-trains AA and BB and their resultant RR at a given instant. The small vertical arrows show the velocities of the various parts of the

medium. Figure 24 shows the positions of the two oppositely moving wave-trains *AA* and *BB* and their resultant *RR* at a later instant when *AA* has moved one sixteenth of a wave-length to the right and *BB* has moved one sixteenth of a wave-length to the left. Figure 25 shows the position of *AA* and *BB* and their resultant *RR* (a straight line) at a still later instant when *AA* has moved one eighth of a wave-length to the right and *BB* has moved one eighth of a wave-length to the left.

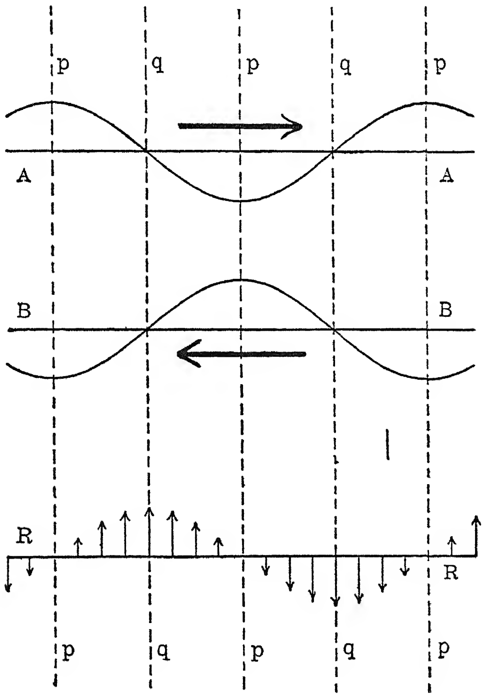


Fig. 25.

The small vertical arrows show the velocities of the various parts of the medium as before. Figure 26 shows the positions of *AA* and *BB* and their resultant *RR* at a still later instant when *AA* has moved three sixteenths of a wave-length to the right and *BB* has moved three sixteenths of a wave-length to the left.

It is to be particularly noted that the component wave-trains

AA and *BB* are always *opposite to each other in phase* at the nodes *pp*, and that they are always *in the same phase* at the antinodes *qq*.

The medium at the antinodes of a stationary wave-train has at times considerable velocity but is never distorted. The medium at the nodes of a stationary wave-train never moves but is at times much distorted. This is shown for transverse waves in Fig. 27.

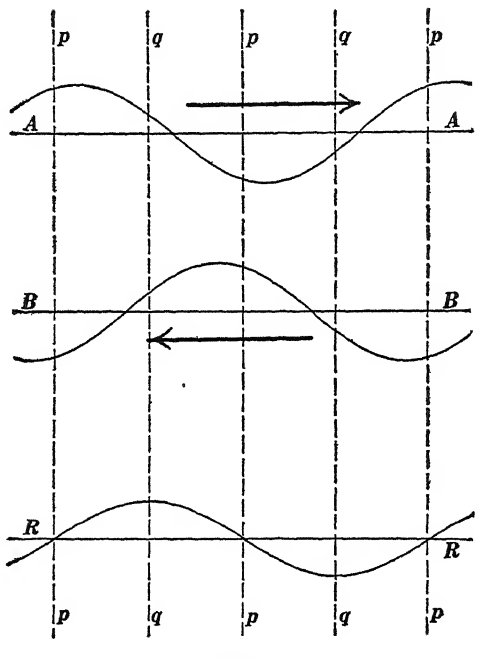


Fig. 26.

The shaded areas represent portions of the medium, *AB* represents the state of affairs when the velocities in the vibrating segments are greatest as indicated by the vertical arrows and when the displacements are everywhere zero, and *CD* represents the state of affairs one fourth of a period later where the velocities are everywhere zero and the displacements are at their greatest. It may be seen from *AB* that the velocity is greatest at the antinode *q*, and it may be seen from *CD* that the medium at

the antinode is not distorted, whereas the medium at the node p is distorted as indicated by the shaded area at p.

In a stationary train of transverse waves the medium at a node is distorted as shown in Fig. 27. In a stationary train of longitudinal waves the medium of the two sides of a node moves

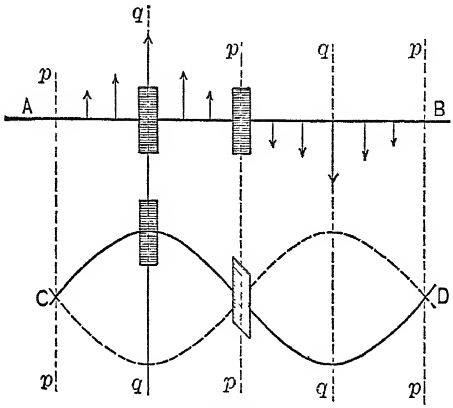


Fig. 27.

towards the node simultaneously and away from the node simultaneously, and the medium at the node is alternately condensed and rarefied.

Note. — Stationary wave-trains may result from the superposition of wave-trains of any shape provided only that the advancing train is exactly similar to the receding train turned end for end and upside down.

25. Stationary wave-trains by reflection. — When a wave-train reaches the boundary of a medium it is reflected. The reflected train is similar to the incident train, and, if the incident train strikes the boundary normally, then a stationary wave-train is produced by the superposition of the two.

If the boundary is *free*, that is, if there is perfect freedom of motion at the boundary, then the surface layer of the medium cannot be distorted because there is no possibility of this layer of the medium being squeezed, as it were, by the material behind it. In such a case there must be an antinode of the stationary

wave-train at the reflecting boundary, inasmuch as an antinode of a stationary wave-train is a place where the medium is never distorted.

If the boundary is *rigid*, as, for example, where a medium like air is bounded by a solid wall, then the layers of the medium next the wall cannot move. In this case there must be a node of the stationary wave-train at the reflecting boundary, inasmuch as a node of a stationary wave-train is a place where the medium never moves.

The components (advancing wave-trains) of a stationary wave-train are opposite to each other in phase at a node and in the same phase with each other at an antinode (velocity phase is here referred to) as explained in Art. 24. Therefore, from the above statements concerning free and rigid boundaries, it follows that the reflected wave-train is in phase with the incident wave-train at a free boundary and opposite in phase at a rigid boundary. That is to say, a wave-train is reflected from a free boundary without reversal of phase (velocity phase), and a wave-train is reflected from a rigid boundary with reversal of phase (velocity phase).

Stationary wave trains may be easily produced on a stretched cord or wire or on a stretched rubber tube as explained in Art. 24. One end of the rubber tube is held in the hand. A series of periodic movements of the hand generates a wave-train on the tube, this wave-train is reflected from the other end of the tube, and the tube is broken up into segments with intervening nodes as the reflected train and advancing train come into superposition. When the end of the rubber tube is fixed (that is, when it is tied to a rigid support), the waves are reflected with reversal of velocity phase and the fixed end of the tube is a nodal point of the stationary wave-train. When the end of the rubber tube is free (that is, when the rubber tube hangs vertically from the hand), the waves are reflected without reversal of velocity phase and the loose end of the tube is an antinode of the stationary wave-train.

A stationary wave-train may also be produced in the air in a

long glass tube The lips applied to one end of the tube as to a bugle are thrown into periodic motion producing a wave-train which is reflected from the open end of the tube without reversal of velocity phase, and a stationary wave-train is produced in the tube when the reflected train and advancing train come into superposition. In this case the open end of the tube is at the center (nearly) of a vibrating segment. If lycopodium powder is strewn inside of the tube it is swept into the nodal points by the violent to and fro motion of the air giving a striking indication of the existence of the stationary wave-train.

26. Wave front. — One of the most important ideas in connection with the wave theory of light is the idea of the *wave front.* Every one is familiar with the fact that waves on the surface of a pond always resolve themselves into clearly defined ridges at a distance from the disturbance, however complicated the disturbance may be. Thus, when a handful of pebbles is thrown into a pond the wave motion in the immediate neighborhood of the disturbance is excessively complicated, but the waves become a clearly defined series of ridges at a considerable distance from the disturbance.

Consider a region *AB*, Fig. 28, on the surface of a pond, *C* being a point at which a pebble is dropped into the pond. The

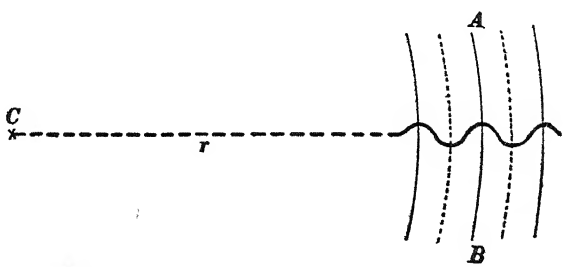


Fig. 28.

above-mentioned fact that the waves from *C* resolve themselves into clearly defined ridges at a distance from *C* means that *all points of the water surface which lie in a certain line AB rise and fall together or, in other words, the line AB on the water surface*

moves up and down as a whole as the waves pass by. Such a line is called a *wave front.*

Sound waves in the air and light waves in the ether always resolve themselves at a great distance from the disturbance into a clearly defined **layer** *or* **series of layers** (*if they could but be seen*), *and an indefinitely thin portion of such a layer moves up and down or to and fro as a whole, and is called a wave front.*

The direction of progression of a water wave is at right angles to its front. The direction of progression of a sound wave or light wave is at right angles to its front.*

A wave which has a plane wave front is called a *plane wave.* A wave which has a spherical wave front is called a *spherical wave.*

27. Huygens' principle. — Let AB, Fig. 29, be the instantaneous position of a wave which has come from a disturbance at C. The disturbance which is produced later at the point p when the wave reaches that point is of course to be thought of as having come originally from C; *it may be, however, considered to have come from the disturbance which constitutes the wave* AB. In this case, each point of the wave AB is to be considered as a center of disturbance from which a spherical wave emanates. The waves which thus emanate from each point of a *primary wave* are called *secondary waves* or *wavelets.* The actual disturbance produced at p is the resultant of the effects of all these wavelets.

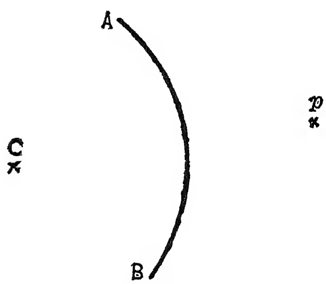


Fig. 29.

* When the medium through which the wave passes has different properties in different directions, the direction of progression may not be at right angles to the front. Thus, in a substance like wood which has a grained structure, a sound wave does not in general progress in a direction at right angles to its front, in a crystal like Iceland spar a light wave does not in general progress in a direction at right angles to the front.

28. Huygens' construction for wave front. — Let AB, Fig. 30, be the front of a wave which is advancing towards $A'B'$, and let it be required to find the wave front after a time has elapsed during which the wave has traveled a distance r. Describe circles (spheres) of radius r about each point of the wave front AB.

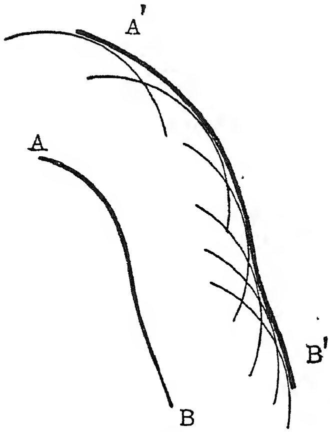


Fig. 30.

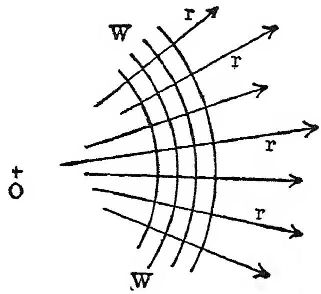


Fig. 31.

The envelope $A'B'$ of these circles (spheres) is the required wave front. These spheres are the secondary wavelets described in the previous article.†

29. The ray of light. — Consider a train of waves WW, Fig. 31, passing out from a center of disturbance O. These waves progress at each point in a direction at right angles to the wave front, and the lines rrr along which the wave disturbance travels are called *rays*. A bundle of rays drawn from the various points of a small portion of a wave front is called a *pencil of rays*.

The conception of the ray carries with it the idea that a wave disturbance is propagated in straight lines, that a wave disturbance will not, for example, bend round an obstacle into the region behind the obstacle. Water waves do, however, bend

† A full discussion of Huygens' principle and of Huygens' construction for a wave front, together with Fresnel's improvement thereon, is to be found in Chapter III of Drude's *Theory of Optics*, translated by Mann and Millikan (New York, 1902).

round an obstacle and so also do sound waves inasmuch as it is a common experience that a sound can be heard around a corner. The familiar phenomena of shadows * and the fact one cannot see around a corner seem to indicate that light travels in straight lines. As a matter of fact, however, light bends round a corner in the same way that sound does but to an extent that is, under ordinary conditions, scarcely noticeable. This matter is taken up again very briefly in the chapter on Diffraction.

30. Homocentric pencil of rays. — When a portion of a wave front is a sector of a spherical surface (the entire wave front may not be a sphere) the rays from the portion intersect at the center of the sphere, the pencil of rays is said to be *stigmatic* or *homocentric*, and the center of the sphere is called the *focal point* of the pencil. The circle *WW*, Fig. 32, represents a spherical wave

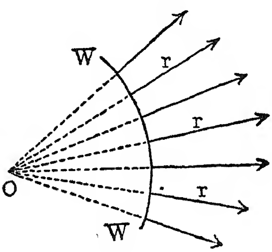


Fig. 32.

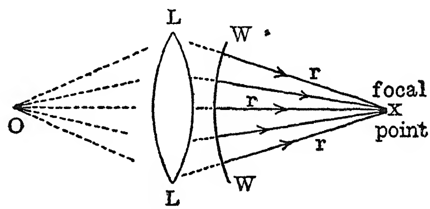


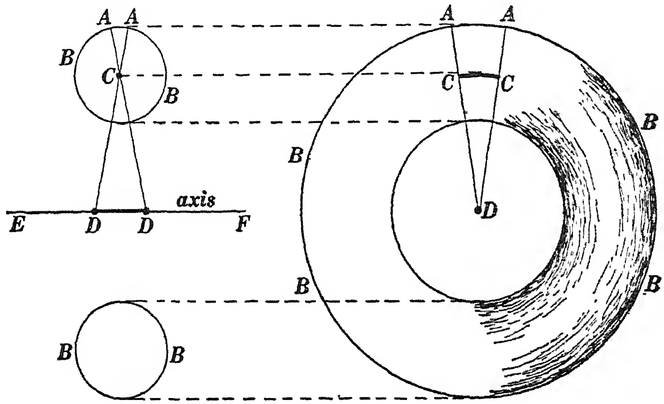
Fig. 33.

emanating from a center of disturbance *O*. In this case the wave is shown as traveling towards its convex side and the pencil of rays *rrr* is said to be *divergent*. In Fig. 33, *WW* represents a spherical wave which has come from a center of disturbance *O* and has passed through a lens *LL*. In this case the wave *WW* is shown as traveling towards its concave side and the pencil of rays *rrr* is said to be *convergent*.

31. Astigmatic pencil of rays. — Imagine a circle *BB*, Fig. 34*a*, to be rotated about the axis *EF* so that the circle may describe a ring-shaped surface. Consider a small portion *AA* of

* The phenomena of shadows are briefly discussed on pages 3 to 5 of Edser's *Light for Students*, Macmillan and Co., 1902.

the surface of this ring. If normals (lines) are drawn from every point of the portion AA these lines will intersect first along a short line at C (perpendicular to the plane of the figure) and second along the line DD. The portion AA of the ring surface is sharply curved in one plane (the plane of the paper in Fig. 34*a*) and less sharply curved in another plane at right angles to the first (the plane perpendicular to the paper in Fig. 34*a*).



sectional view Fig. 34a. *side view* Fig. 34b.

When a small portion of a wave front is shaped like the portion AA of the ring surface in Fig. 34, the pencil of rays which corresponds to the portion of the wave front passes first through the line C (perpendicular to the plane of the paper in Fig. 34*a*) and then through the line DD. Such a pencil of rays is called an *astigmatic pencil*, and the lines C and DD are called the *focal lines* of the pencil.

Example. — Light emanates from a point O, Fig. 35, and passes obliquely through a lens LL. After passing through the lens the wave fronts are shaped like a portion of the surface of a ring (approximately), and the rays are focused first along the line C (perpendicular to the plane of the paper in Fig. 35) and then along the line DD. If the light from the lens LL, Fig.

35, is allowed to fall on a screen *SS* a short bright line will appear on the screen if the screen passes through *C* or through *DD*. If the screen is between *C* and *DD* as shown in Fig. 35, a blurred spot will show on the screen. The light from *O* passing

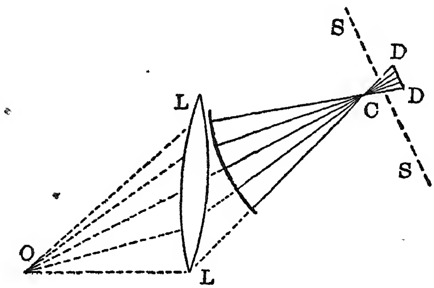


Fig. 35.

obliquely through *LL* cannot be sharply focused at a point on a screen. The nearest approach to a sharp point is obtained when the screen is midway between *C* and *DD*, in which case the blurred spot is a small circle which is called the *circle of least confusion*.

CHAPTER III.

REFLECTION AND REFRACTION.

32. Regular and diffused reflection and refraction. — When light falls upon the surface of a body it is in part turned back or reflected. When light enters the eye from a polished surface (a mirror), we see the objects from which the light originally came. On the other hand, we see only the reflecting body if its surface is rough. Reflection from a polished surface is called *regular reflection.* This kind of reflection is discussed in the following articles. Reflection from a rough surface is called *diffuse reflection.*

When light passes from one medium into another the direction of progression of the light is in general altered. This phenomenon is called *refraction.* The familiar broken appearance of a straight oar at the surface of clear still water is due to refraction. This effect is explained in Art. 43. Refraction at a polished surface is called *regular refraction.* This kind of refraction is discussed in the following articles. Refraction at a rough surface is called *diffuse refraction.*

33. Index of refraction. — The velocity of light is different in different media. Thus, light travels about three quarters as fast in water as in air. The ratio: $\dfrac{\text{velocity of light in air}}{\text{velocity of light in a given substance}}$ is called the index of refraction * of the substance and it is usually represented by the Greek letter μ. The index of refraction of a substance may be determined by the direct measurement of the velocity of light in air and in the substance; by the method of Foucault, for example; but the most convenient method for

* The ratio: $\dfrac{\text{velocity of light in a vacuum}}{\text{velocity of light in a given substance}}$ is called the absolute index of refraction of the substance.

determining the index of refraction of a substance is that which is described in Art. 41.

The velocity of light in a given substance is different for light of different colors (different wave-lengths), and therefore the index of refraction of a substance varies with the wave-length of the light. This matter is discussed in the chapter on Dispersion.

34. Application of Huygens' principle to reflection and refraction. — When a light wave approaches the polished surface of a transparent substance, such as glass, a portion of the wave is reflected and a portion enters the glass and is refracted. The determination of the location of the reflected and refracted wave-fronts at any given instant depends upon the application of Huygens' principle (Art. 27) slightly modified as follows: Any *given point on the surface of the glass* is a center of disturbance at the instant that the advancing wave reaches that point, and a wavelet emanates from the given point at this instant, one half of the wavelet passes into the glass at a definite velocity and the other half passes back into the air at a velocity μ times as great, μ being the index of refraction of the glass.

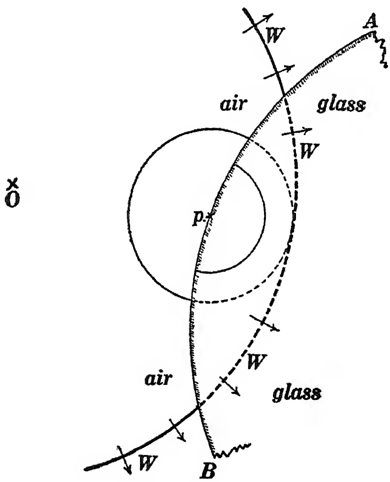


Fig. 36.

A wave emanates from a point O, Fig. 36, and approaches a reflecting and refracting surface AB. At a certain instant, which we will designate by the letter X, the wave reaches a given point p on the surface, and at a later instant Y the wave *would have reached the position WW if it had not encountered the glass.* It is required to find the position of the reflected wave and the position of the refracted wave at the instant Y. Describe about

the point p a circle (sphere) tangent to the dotted line WW. The radius r of this circle is the distance that the light travels in air during the time from X to Y, and r/μ is the distance that the light travels in glass during the same interval of time. Therefore, that portion of the wavelet from p which is in air has a radius r, and that portion of the wavelet from p which is in glass has a radius r/μ. The two parts of the wavelet from p are shown by the small full-line semicircles in Fig. 36.

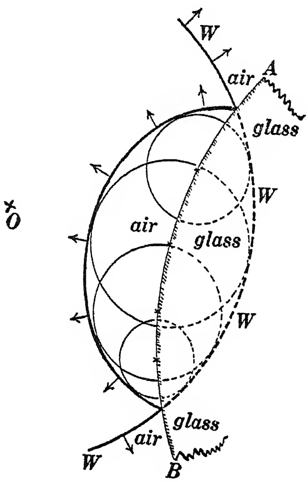


Fig. 37.

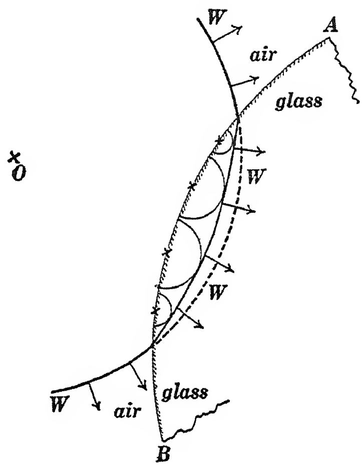


Fig. 38.

Figure 37 shows Huygens' construction for the reflected wave. About each point on the surface of the glass a sphere is described of such radius as to be tangent to WW. The envelope of these wavelet spheres is the reflected wave.

Figure 38 shows Huygens' construction for the refracted wave. About each point on the surface of the glass a sphere is described of which the radius is $1/\mu$ as great as it would have to be to touch WW. The envelope of these wavelet spheres is the refracted wave.

35. Reflection of a plane wave from a plane surface. — Consider a plane wave CD, Fig. 39, approaching a plane polished surface

AB. At *a given instant* this wave would have reached the position *WW* if it had not encountered the reflecting surface; it is required to find the position of the reflected wave at the given

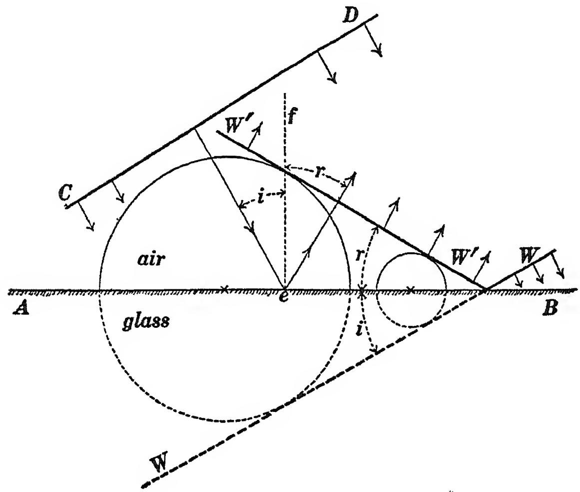


Fig. 39.

instant. About each point of the surface *AB* describe spheres tangent to *WW*. The envelope *W′ W′* of these wavelet spheres is the reflected wave. The angle *i* between the incident wave-front and the reflecting surface (or between the incident ray and the normal *ef* to the reflecting surface) is called the *angle of incidence*. The angle *r* between the reflected wave-front and the reflecting surface (or between the reflected ray and the normal to the reflecting surface) is called the *angle of reflection*. These two angles are evidently equal to each other.

36. Reflection of a spherical wave from a plane surface. — Let *O*, Fig. 40, be a luminous point from which spherical waves emanate, let *AB* be a plane reflecting surface, and let *WW* be the position which would be reached at a given instant by a spherical wave from *O* were it not for the reflecting surface *AB*. It is required to find the position of the reflected wave at the given instant. About each point of the surface *AB* describe a

sphere tangent to WW. The envelope $W'W'$ of these wavelet spheres is the reflected wave.

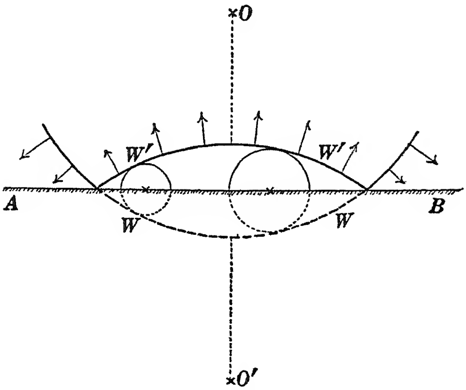


Fig. 40.

As is evident from the symmetry of the figure, the reflected wave $W'W'$ is a sphere exactly like WW, and the center of curvature of the reflected wave is at the point O'. Therefore, when light from a luminous point O is reflected from a plane surface the reflected light appears to come from a fictitious luminous point O'. This point O' is called the image of O. The line OO' is perpendicular to the plane of the reflecting surface and bisected thereby.

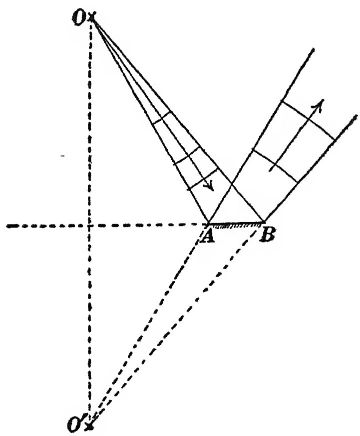


Fig. 41.

Figure 41 shows the reflection of light from a small plane mirror. The relative positions of O and O' in Fig. 41 are the same as if the mirror were large.

37. Image of an object in a plane mirror. — An ordinary object is merely a group of luminous points in so far as its light-giving properties are concerned. Consider such an object O, Fig. 42.

The light from the various points of this object, after reflection from a plane surface AB, or from a portion of such a surface, appears to come from a similar group of points O'. This group of fictitious luminous points O' is called the *image* of the object O.

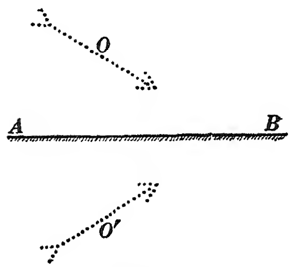


Fig. 42.

38. Reflection from curved surfaces.—The reflection of plane or spherical waves from a spherical surface is rather complicated because of the fact that the reflected wave is, in general, not plane or spherical, that is to say, light which emanates from a point does not appear to come from a clearly defined point after reflection from a spherical surface, or in other words, a spherical mirror does not form clearly defined images.* When, however, the diameter of a spherical mirror is small in comparison with the radius of curvature of the mirror and when light is reflected in a

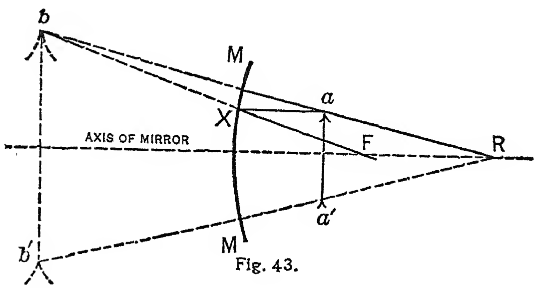


Fig. 43.

direction nearly at right angles to the surface of such a mirror, fairly distinct images are formed. Thus MM, Fig. 43, represents a concave spherical mirror, aa' represents an object placed in front of the mirror and bb' represents the image of the object in the mirror.†

* The pupil of the eye is very small and therefore *only a very small portion* of any reflecting surface reflects light from a given point to the eye. It is this fact which enables us to see fairly distinct images in spherical and even in irregularly curved mirrors.

† A mirror like MM in Fig. 43 is called a mirror of small aperture. The discussion of the optical properties of such mirrors is of but little practical importance.

Reflection from a paraboloid. — Figure 44 represents a parabolic mirror. A beam of parallel rays falling upon such a mirror is concentrated at the focus of the paraboloid, or, light emanating from the focus of the paraboloid is reflected by the paraboloid as a beam of parallel rays. The parabolic reflector is much used for giving a beam of parallel rays of light from a light source placed at its focus, as in locomotive headlights and in searchlights.

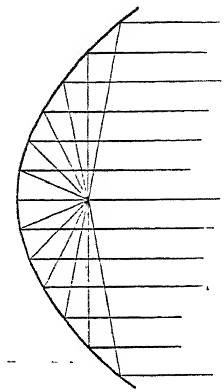


Fig. 44.

Reflection from an ellipsoid. — Figure 45 represents an ellipsoid of revolution (prolate ellipsoid). Light emanating from one focal point of the ellipsoid is concentrated at the other focal point. Vaulted ceilings sometimes approximate so closely to the ellipsoidal form that a faint sound emanating from one point is concentrated by reflection at some other point.

39. Reflection of a plane wave from a spherical surface. — Let *MM*, Fig. 46, represent a polished spherical surface with its center of curvature at *R*. A plane wave comes from the right and the

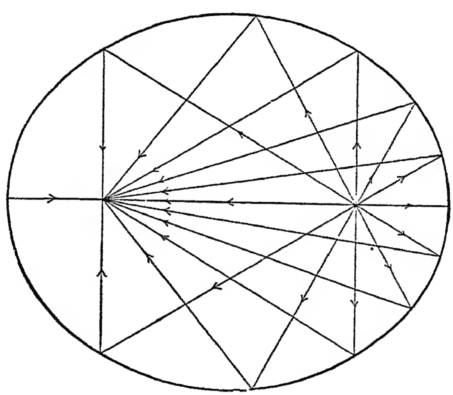


Fig. 45.

line *WW* represents the position this wave would have reached at a given instant if it were not for the mirror *MM*. It is

required to find the position of the reflected wave at the given instant. About each point of the reflecting surface describe spheres tangent to *WW*. The envelope *W′ W′* of these wavelet spheres is the required reflected wave.

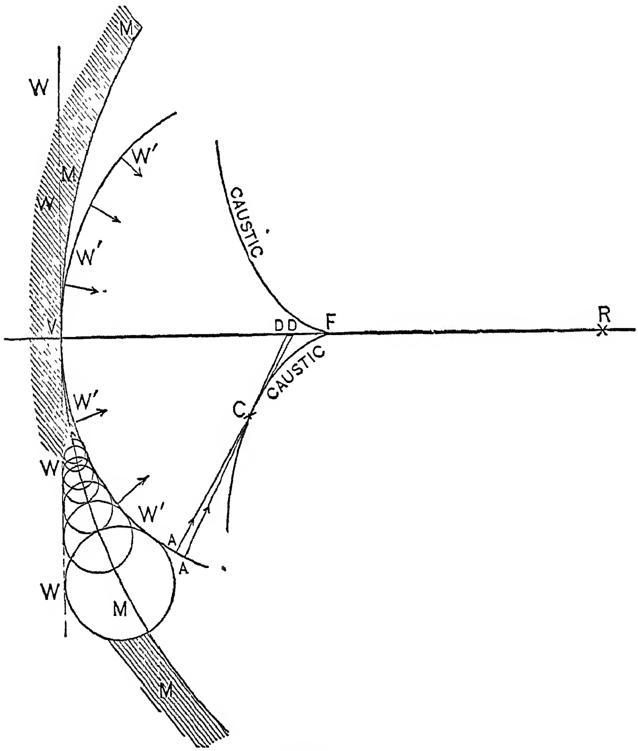


Fig. 46.

The reflected wave in Fig. 46 is not spherical. The portion of the wave near *V*, however, approximates very closely to a spherical shape with its center of curvature at *F*. The portion *AA* of the reflected wave has its center of curvature at the point *C* lying on a curve which is called the *caustic curve* of the mirror, that is to say, the caustic curve is the locus of the centers of curvature of the reflected wave.*

* Compare the portion *AA*, *C* and *DD* of Fig. 46 with the portion *AA*, *C* and *DD* of Fig. 34. The pencil of rays *AA*, Fig. 46, is an astigmatic pencil.

Figure 46 shows only a portion of the reflected wave. Figure 47*a* shows the reflected wave more completely. Indeed, three successive positions, 111, 222, 333 of the reflected wave are

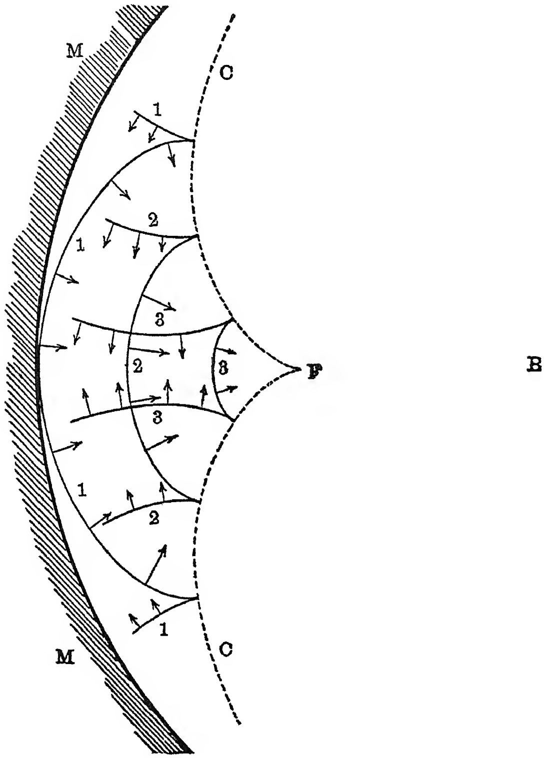


Fig. 47a.

shown in Fig. 47*a*. Figure 47*b* is a more complete construction (Huygens' construction) for the reflected wave than that which is shown in Fig. 46; *MM* is the spherical mirror, *WW* is the position which would be reached at a given instant by a plane wave coming from the right if it were not for the mirror, and the fine curved lines in the figure are circles (spheres) which are tangent to *WW* and whose centers lie on *MM*. The reflected wave has two sharp points or cusps which travel along the caustic curve *C* as indicated in Fig. 47*a*.

The caustic curve is a region of intense illumination inasmuch as the disturbance which is represented by each portion *AA* of the reflected wave is concentrated at the center of curvature *C*

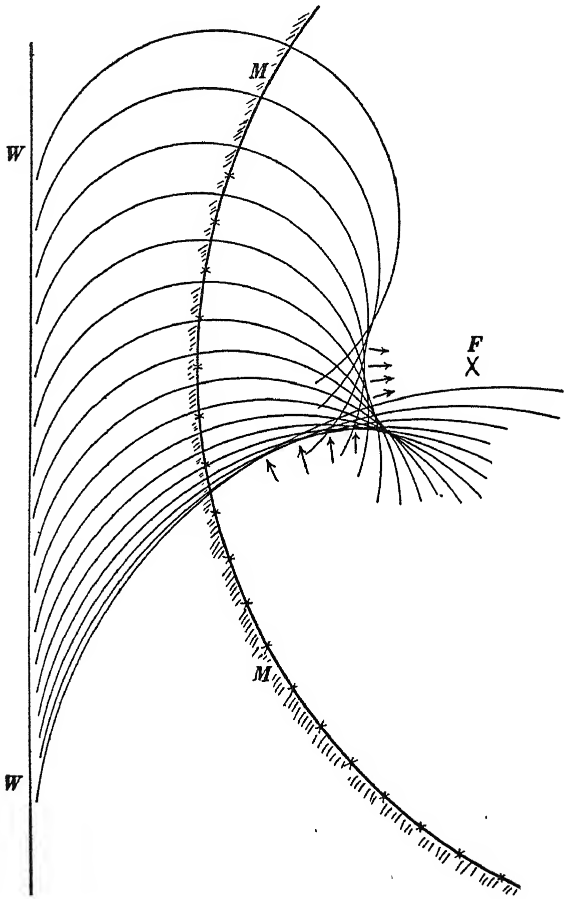


Fig. 47b.

of that portion as indicated in Fig. 46. The caustic curve is therefore visible as a sharp bright line when the light which is reflected from the curved surface is thrown obliquely on a flat sheet of paper. Caustic curves are very distinctly visible when

light from a bright lamp shines obliquely into a glass which is partly filled with milk or into a cup which is partly filled with coffee. In this case a brilliant line of illumination is formed along the surface of the liquid.

40. Refraction of a plane wave at a plane surface. — Consider a plane wave *CD*, Fig. 48, approaching the plane surface *AB*

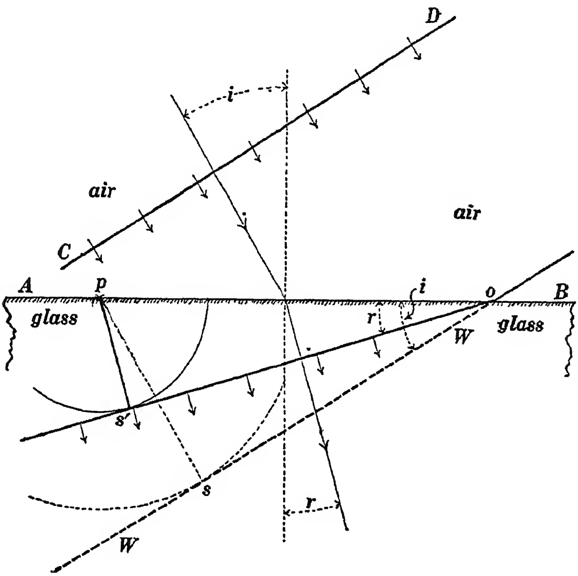


Fig. 48.

of a piece of glass. At *a given instant* this wave would have reached the position *WW* if it were not for the piece of glass. It is required to find the position of the refracted wave at the given instant. About each point on the surface *AB* describe a sphere of which the radius *ps'* is equal to the distance *ps* divided by the index of refraction of the glass. The envelope of these wavelet spheres (of which only one is shown in the figure) is the required refracted wave.

The angle *i* between the incident wave front and the refracting surface (or between the incident ray and the normal to the refracting surface) is called the *angle of incidence*. The angle *r*

between the refracted wave front and the refracting surface (or between the refracted ray and the normal to the refracting surface) is called the *angle of refraction.* From the right triangle *pos*, Fig. 48, we have

$$\overline{po} \sin i = \overline{ps} \tag{i}$$

and from the right triangle *pos'* in Fig. 48 we have

$$\overline{po} \sin r = \overline{ps'} = \frac{\overline{ps}}{\mu} \tag{ii}$$

Whence, dividing equation (i) by equation (ii), member by member, we have

$$\frac{\sin i}{\sin r} = \mu \tag{3}$$

This relation between the index of refraction μ and the angles of incidence and refraction i and r was discovered by Snell and is sometimes called *Snell's law.*

41. Refraction of a plane wave by a prism. — The prism is a portion of a transparent substance, such as glass, bounded by two

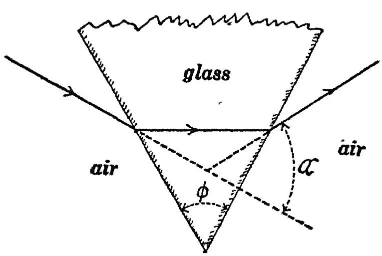


Fig. 49.

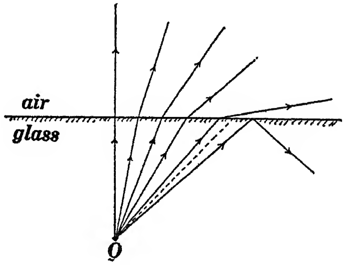


Fig. 50.

plane polished faces inclined to each other. A ray of light in passing through a prism is refracted as shown in Fig. 49. The total angle of deflection a depends upon the index of refraction of the glass, upon the angle ϕ and upon the direction of the ray in the prism. When the ray in the prism is equally inclined to

the two faces of the prism, as shown in Fig. 49, the deflection α is a minimum, and in this case

$$\mu = \frac{\sin \frac{1}{2}(\phi + \alpha)}{\sin \frac{1}{2}\phi} \tag{4}$$*

The index of refraction of a sample of glass is usually determined by making a prism of the glass, measuring the angle ϕ of the prism and observing the minimum angle of deviation α of light by the prism, whence the index of refraction may be calculated from equation (4).

42. Total reflection. — Consider a luminous point O, Fig. 50, in glass from which light emanates and passes across a plane surface into air. A ray which strikes the surface normally remains unchanged in direction in the air. The more inclined the ray, however, the greater the change of direction due to refraction as shown in the figure, and for a certain critical inclination which is represented in Fig. 50 by the dotted line, the light which emerges into the air is parallel to the refracting surface. Beyond this critical inclination the light is *totally reflected*, no portion of it being able to pass out into the air.

Total reflection is the cause of the brilliant silvery appearance of the surface of water in a tumbler when viewed obliquely from below.

The total reflecting prism. — In many optical instruments a prism of glass like ABC, Fig. 51*a*, with a right angle at C is used instead of a mirror for reflecting a beam of light. The beam of light passes through the prism as indicated by the arrows, being totally reflected by the face AB.

The well-known experiment of *the illuminated water jet* is based upon the phenomenon of total reflection. In this experiment water escapes from an orifice o in a tank, as shown in Fig. 51*b*. In the opposite wall of the tank is a lens which directs the light from a lamp L into the orifice o. The rays of light, striking the boundary of the jet very obliquely, are totally reflected

* See Preston's *Theory of Light*, page 123, for a derivation of this equation.

and thus prevented from escaping from the jet. Where the jet strikes an obstacle, or where it breaks up into drops, however, the bounding surface of the jet is sufficiently changed in direction to allow the light to escape, thus producing brilliant illumination.

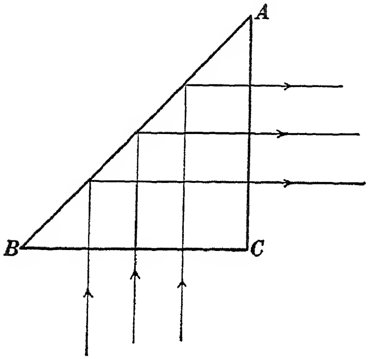


Fig. 51a.

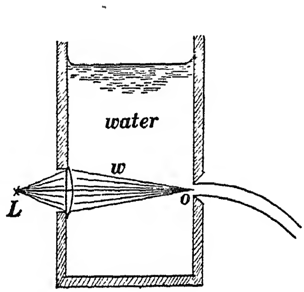


Fig. 51b.

The field of a microscope is sometimes illuminated by conveying a light along a solid glass rod one end of which is beneath the microscope slide while the other end is polished flat and is near the lamp flame. The light which enters the rod from the lamp is kept in the rod by total reflection until it reaches the extreme end where it emerges into the air.

43. Refraction of a spherical wave at a plane surface. — Figure 52 represents the refraction of a spherical wave when it enters glass from air, and Fig. 53 represents the refraction of a spherical wave when it enters air from glass; *AB* is the surface which separates the glass and air, *O* is a luminous point from which spherical waves emanate, *WW* represents the position that would be reached at *a given instant* by one of these spherical waves without refraction, and *W′ W′* represents the position of the refracted wave at the given instant. The refracted wave is not spherical in either case, and the pencil of rays *aa, C, DD* in each figure is an astigmatic pencil.*

* Compare the portions *aa, C, DD* of Figs. 52 and 53 with the portions *AA, C, DD* of Fig. 34.

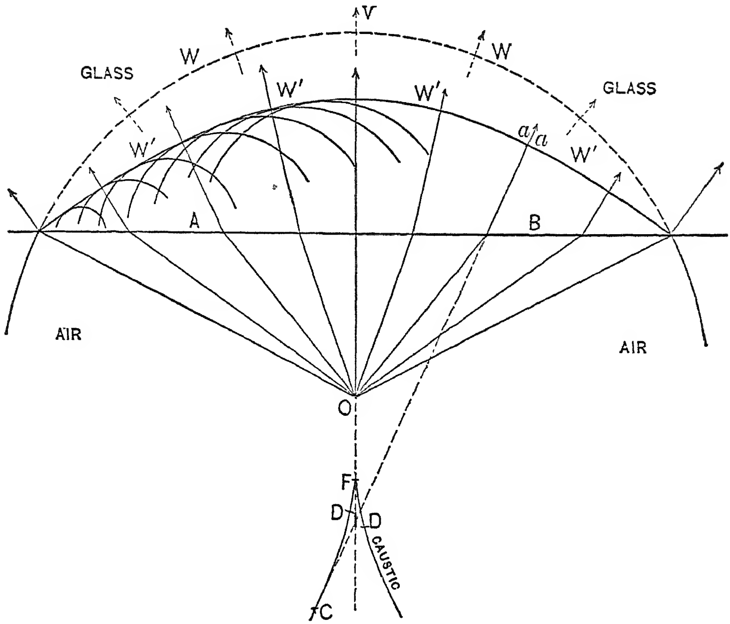

V
W
W
GLASS
GLASS
W′
W′
W′
W′
a
a
A
B
AIR
AIR
O
F
D
D
CAUSTIC
C

Fig. 52.

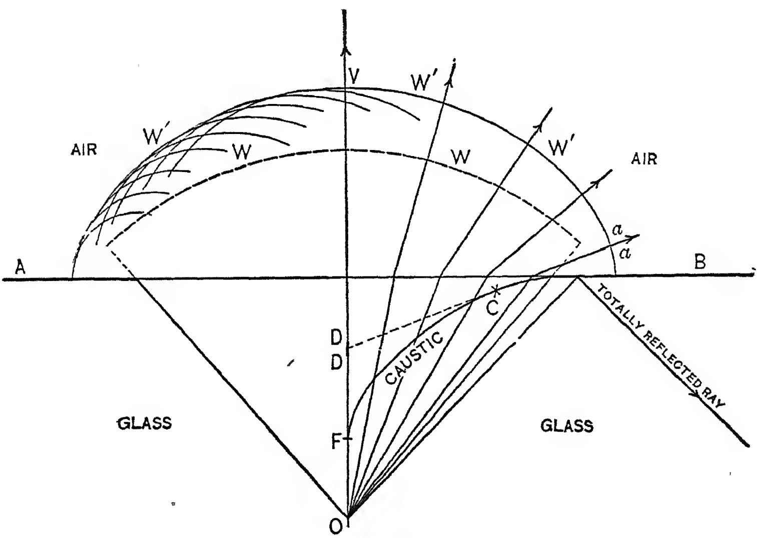

V
W′
W′
W′
AIR
AIR
W
W
a
a
A
B
TOTALLY REFLECTED RAY
C
D
D
CAUSTIC
F
GLASS
GLASS
O

Fig. 53.

A very striking experiment is the following: A fine bit of chalk is placed at the bottom of a basin of water (at O, Fig. 53). With the eyes at aa the bit of chalk is seen at DD *if the line joining the eyes is horizontal*, and it is seen at C *if the line joining the eyes is vertical.* The familiar broken appearance of a straight oar in still, clear water is due to the fact that any point (like O, Fig. 53) of the submerged portion of the oar seems to be raised (to DD, Fig. 53).

44. Spherical aberration. Aplanatism. — A spherical or plane wave is in many cases *not spherical or plane* after reflection or refraction, and the departure of the reflected or refracted wave from a true spherical shape is called *spherical aberration.* When a reflecting or refracting surface does not produce spherical aberration the surface is said to be *aplanatic.* Thus, a plane reflecting surface is always aplanatic because a plane or spherical wave always remains plane or spherical after reflection from a plane surface. A plane surface is aplanatic for the refraction of plane waves as explained in Art. 40, but a plane surface is not aplanatic for the refraction of spherical waves, as explained in Art. 43.

When a plane or spherical wave is refracted at a spherical surface the refracted wave is in general subject to spherical aberration. That is, a spherical surface is in general not aplanatic.* There is, however, one particular case in which a spherical wave remains spherical after refraction at a spherical surface, that is, one particular case in which a spherical refracting surface is aplanatic.

Discussion of particular case of aplanatism of a spherical refracting surface. — Given two fixed reference points P and P', Fig. 54. Imagine the point p to move so that the distance pP' is always μ times as great as the distance pP. It can be shown that under these conditions the point p describes the surface of a sphere CC. Let this sphere be made of glass of which the index of refraction is μ, and let P be a luminous point inside of the glass sphere. Then, spherical waves which emanate

* In order that a spherical wave may be spherical after refraction the refracting surface must have a form which is called the *Cartesian oval* from its discoverer. The Cartesian oval is, however, of no practical importance, because a spherical or plane surface is the only kind of surface which can be mechanically produced with ease and accuracy. An interesting paper on the Cartesian oval may be found in the Collected Papers of J. C. Maxwell, Vol. I, page 1, Cambridge, 1890.

from P and pass to the left through the dotted portion of the spherical surface remain spherical after passing into the air, and the center of the new spherical waves is the point P'. *This fact was discovered by Amici and it is utilized as one of the most important features of the modern compound microscope,* as explained in Chapter VI.

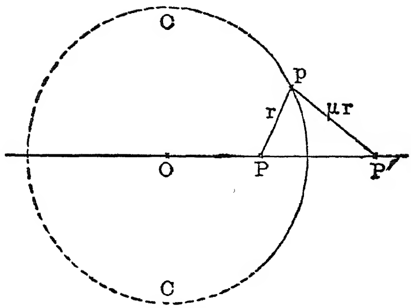


Fig. 54.

Let the dotted circle WW, Fig. 55, of radius r be the position which the spherical wave from P would reach at *a given instant* if the whole region were glass. Describe a circle $W'W'$ of radius μr with its center at P'. Consider the point p, Fig. 55, distant b from P and distant μb from P'. At the given instant, the wavelet from the point p has had time to travel a distance $(r-b)$ in glass, or

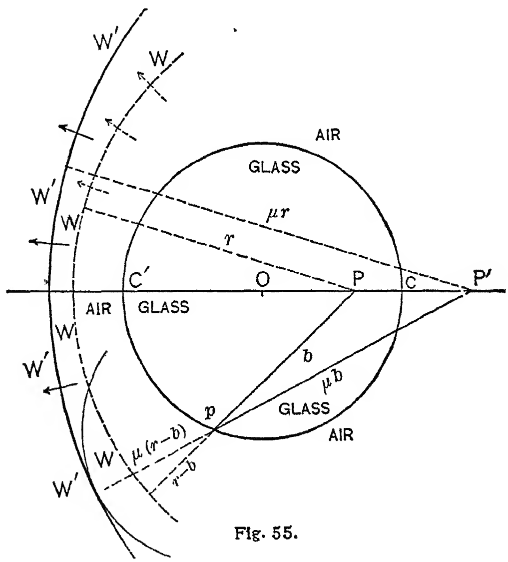


Fig. 55.

$\mu(r-b)$ in air, so that the radius of this wavelet is $\mu(r-b)$. Therefore, since $\mu(r-b)+\mu b=\mu r$, the wavelet under consideration is tangent to $W'W'$, and consequently $W'W'$ is the envelope of all such wavelets, and it is therefore the refracted wave.

CHAPTER IV.

LENSES AND LENS SYSTEMS.

45. The lens is a portion of glass bounded by polished spherical surfaces, as indicated in Figs. 56, 57 and 58, in which C_1 and C_2 are the centers of curvature of the respective surfaces of the

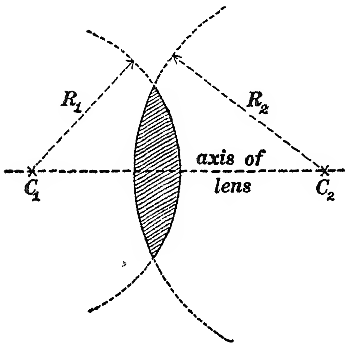


Fig. 56.

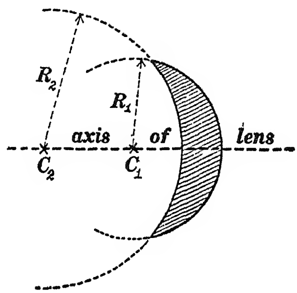


Fig. 57.

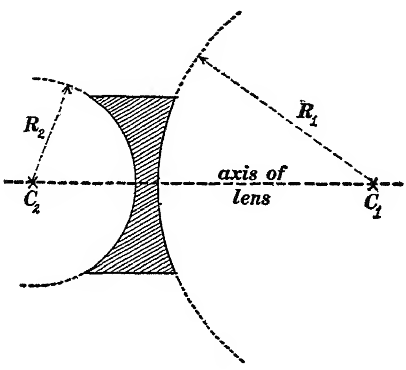


Fig. 58.

lenses, and R_1 and R_2 are the radii of curvature. The line joining the centers of curvature is called *the axis of the lens.*

The surface of a lens is shaped by grinding the glass and a

matrix together with every possible variety of sliding motion. The lens and matrix are thus brought automatically to almost perfect spherical shape. The polishing is accomplished by using finer and finer grinding material in succession, usually powdered emery, and ending with rouge. The matrix is usually of iron, and in the later stages the matrix is lined with a layer of stiff pitch with cross grooves cut in its surface.*

A plane or spherical wave is, in general, *not spherical* after passing through a lens. The departure of the wave from a true spherical shape is especially great when the lens is thick (thick, that is to say, in comparison with its diameter), or when the light passes obliquely through the lens. This is shown in a very striking way by attempting to focus the light of the sun by a very thick lens, or by attempting to focus the light of the sun by allowing it to pass obliquely through a thin lens. *This entire chapter applies to thin lenses, and the light in every case is assumed to pass through the lens in a direction not greatly inclined to the axis of the lens.* In the illustrations, however, the lenses are shown of considerable thickness and the rays are frequently shown passing quite obliquely through a lens for the sake of clearness, but it must be remembered that the lenses are supposed to be very thin and the light never greatly inclined to the axis of the lens.

Figure 59 shows the action of a lens which is thick at the center and thin at the edge, and Fig. 60 shows the action of a lens which is thin at the center and thick at the edge. Light travels slower in glass than in air so that the portions of the plane waves *AA* in Figs. 59 and 60 which pass through the thicker parts of the lens fall behind the portions of the waves which pass through the thinner parts of the lens. Therefore, the

* For a simple account of the processes of lens grinding see Lockyer's *Stargazing*, pages 117–138; see also "How to make a Refracting Telescope" in the *Scientific American Supplement*, Nos. 581, 582 and 583 (February 19 and 26 and March 5, 1887), pages 9283–9285, 9296–9299 and 9312–9316. A very interesting account of the manufacture of a large reflecting telescope is given by Henry Draper in the *Smithsonian Contributions to Knowledge*, No. 180, Vol. XIV, 1865.

waves $A'A'$ in Fig. 59 have *concave* fronts and they are concentrated at the point F; whereas the waves $A'A'$ in Fig. 60 have *convex* fronts as if they had come from the point F. A lens

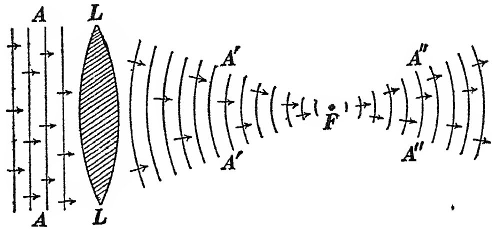


Fig. 59.

which is thicker at the center than at the edge is called a *converging lens.* A lens which is thicker at the edge than at the center is called a *diverging lens.*

For the sake of clearness it is best in lens diagrams to show the rays without showing the wave fronts. Thus Figs. 61 and 62 are duplicates of Figs. 59 and 60 but showing the rays only.

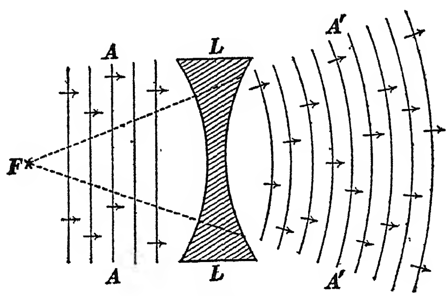


Fig. 60.

46. Focal points. Focal planes. Focal lengths.* — The point F in Figs. 59 and 61 at which a converging lens concentrates a

* A lens of short focal length is sometimes spoken of as a lens of "high power." Thus, a lens of which the focal length is one meter is said to have unit "power," a lens of which the focal length is 3 meters is said to have a "power" of ⅓ of a unit, a lens of which the focal length is ⅓ of a meter is said to have a "power" of 3 units, and so on. The unit of "power" of a lens here used is called a *diopter*, that is to say, the "power" of a lens in diopters is equal to the reciprocal of its focal length expressed in meters. When a number of thin lenses are placed *close together* their combined "power" is equal to the sum of their respective "powers."

parallel beam of rays is called the *focal point* of the lens. The point F in Figs. 60 and 62 from which a parallel beam of rays seems to come after passing through a diverging lens is called the *focal point* of the lens. The plane AB, Figs. 61 and 62, which is at right angles to the axis of a lens and which contains

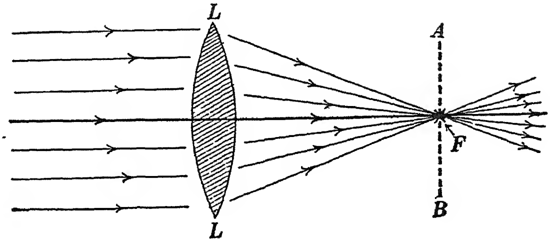


Fig. 61.

the focal point of the lens is called the *focal plane* of the lens. A lens has two focal points (and two focal planes), one on each side of the lens.

The distance from the center of a lens to one of its focal points is called the *focal length* of the lens. It is usual to consider the

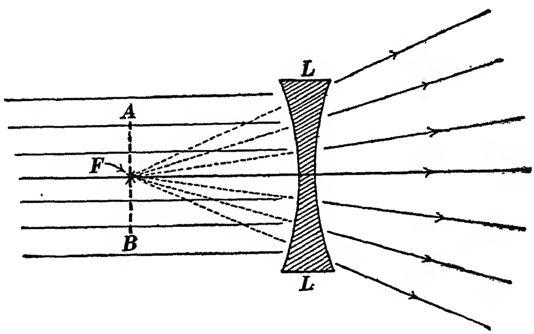


Fig. 62.

focal length of a converging lens as positive and the focal length of a diverging lens as negative.

Figure 63 shows a beam of parallel rays passing through a converging lens in a direction slightly inclined to the axis of the lens, and Fig. 64 shows a beam of parallel rays passing through

a diverging lens in a direction slightly inclined to the axis of the lens. In Fig. 63 the beam after passing through the lens is concentrated at the point F' in the focal plane of the lens, and in

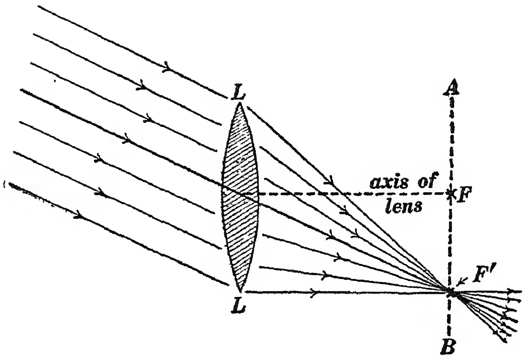


Fig. 63.

Fig. 64 the light after passing through the lens appears to come from the point F' in the focal plane of the lens.

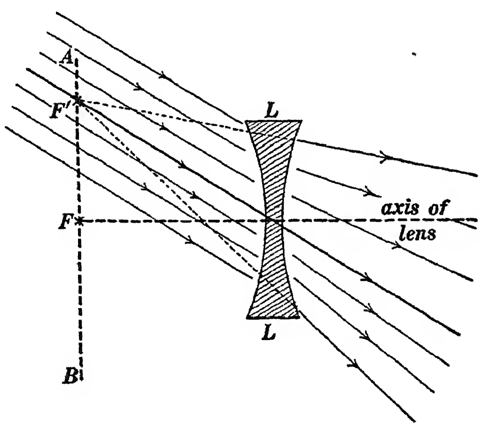


Fig. 64.

47. Conjugate points and conjugate planes. — Figure 65 shows two points a and b so situated with reference to a converging lens that the light which emanates from either one of the points is concentrated at the other. Two such points are called *conjugate points* or *conjugate foci.*

Figure 66 shows two points a and b so situated with respect to a diverging lens that the light which emanates from a appears to come from b after passing through the lens, or light which

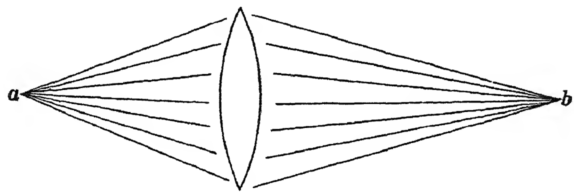


Fig. 65.

is converging towards b is actually focused at a after passing through the lens. Two such points are called *conjugate points* or *conjugate foci*.

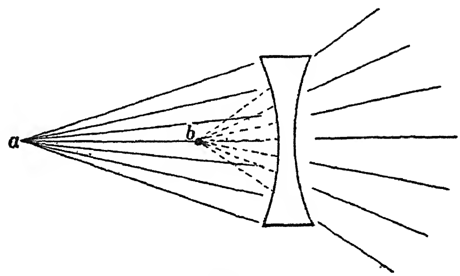


Fig. 66.

When a point lies in the axis of a lens its conjugate also lies in the axis as shown in Figs. 65 and 66. When a point is moved

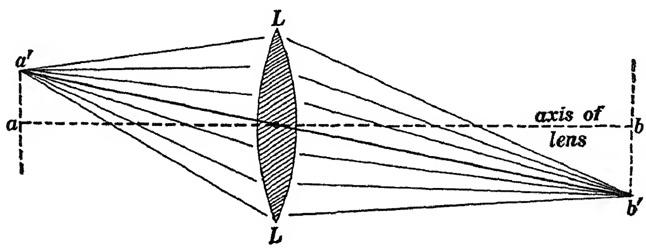


Fig. 67.

away from the axis of a lens in a plane perpendicular to the axis, its conjugate moves away from the axis in a plane perpendicular to the axis. This is shown for a converging lens in Fig. 67.

The planes aa' and bb' in Fig. 67 are called *conjugate planes*.

When a luminous point moves farther and farther away from a lens (along the axis) its conjugate approaches one of the focal points of the lens.

48. Real and virtual points. — When light passes through a lens and is actually concentrated at a point, the point is said to be a *real focus* or a *real point*. When, after passing through a lens, the light merely appears to have come from a certain point, the point is said to be a *virtual focus* or a *virtual point*. The focal point of a converging lens is real; the focal point of a diverging lens is virtual, as shown in Figs. 61 and 62. *When the conjugate of a given point is virtual it is on the same side of the lens as the given point.* It is customary to consider the distance from the center of a lens to a real focus or real point as positive, and to consider the distance from the center of a lens to a virtual focus or virtual point as negative. A complete statement of the conditions under which virtual foci occur with converging lenses and with diverging lenses is given in the next article.

49. Proposition. — Let f be the focal length of a lens and let a and b be the respective distances of a pair of conjugate foci from the center of the lens. Then

$$\frac{1}{f}=\frac{1}{a}+\frac{1}{b} \tag{5}$$

In using this equation it is important to remember that f is positive for a converging lens and negative for a diverging lens, and that the distance from the center of a lens to a conjugate point is positive when the conjugate point is real and negative when the conjugate point is virtual.

Proof of equation (5). — The following proof applies primarily to a converging lens but the same argument very slightly modified applies to a diverging lens. Let us assume that the lens comes to a sharp edge where its thickness is zero. On this as-

sumption let d be the diameter of the lens and let h be its thickness at the center. While the middle portion of a wave is traveling through thickness h of glass the edge portion of the wave, that is, the portion which grazes the edge of the lens, travels μ times as far in air. Therefore *the effect of the lens on a wave is to cause the middle portion of the wave to fall behind the edge portions by the amount* $\mu h - h$, or $(\mu - 1)h$.

Consider a plane wave approaching a converging lens as shown in Fig. 59. After passing through the lens, the wave becomes a sector of a sphere of which the chord is equal to the diameter d of the lens and the versed sine is equal to $(\mu - 1)h$, the amount that the central portion of the wave is retarded by the lens. Therefore, the radius of curvature f of the wave after passing through the lens is given by the equation

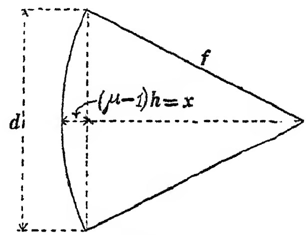


Fig 68.

$$f = \frac{d^2}{8(\mu - 1)h} \qquad (i)^*$$

The radius of curvature f of the wave after it has passed through the lens is the distance from the lens to the point at which the wave is concentrated, that is to say, the radius of curvature f is the focal length of the lens.

Consider a wave WW, Fig. 69, which has reached the lens from the point A, the distance of which from the lens is a.

* This equation expresses the relation between the radius f of a circle or sphere, the length of chord d, and the versed sine $(\mu - 1)h$ as shown in Fig. 68. From this figure we have :

$$f^2 = \frac{d^2}{4} + (f - x)^2 = \frac{d^2}{4} + f^2 - 2fx + x^2$$

in which x is written for $(\mu - 1)h$. But x is very small in comparison with f and therefore x^2 may be discarded, so that we have equation (i) directly.

It is easy to find from equation (i) an expression for f in terms of μ, R_1 and R_2, where R_1 and R_2 are the radii of curvature of the respective surfaces of the lens.

Consider a portion of this wave of which the diameter is d (the same as the diameter of the lens). Let k be the distance that the middle portion of the wave WW in Fig. 69 is ahead of the edge portions. Then we have

$$a = \frac{d^2}{8k} \tag{ii}$$

In passing through the lens, the central part of the wave WW in Fig 69 falls behind the edge portions by an amount which is equal to $(\mu - 1)h$, as explained above. Therefore the central

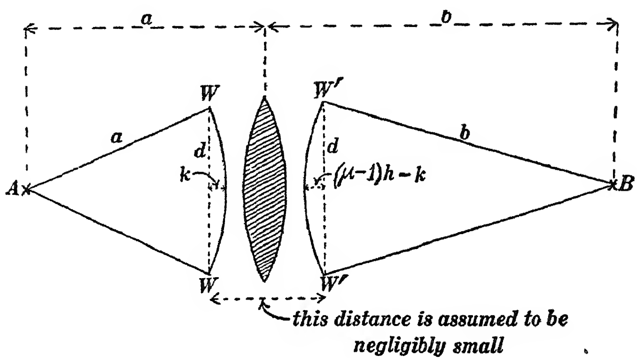


Fig. 69.

portion of the wave $W'W'$ is behind the edge portions by the amount $(\mu - 1)h - k$, so that the radius of curvature b of the wave $W'W'$ is given by the equation

$$b = \frac{d^2}{8[(\mu - 1)h - k]} \tag{iii}$$

Substituting the values of f, a and b from equations (i), (ii) and (iii) in equation (5), we find that equation (5) is satisfied.

50. Proposition. — *A pair of conjugate points which do not lie on the axis of a lens, lie on a straight line passing through the center of the lens.* The truth (approximate) of this proposition may be made evident as follows: Consider the ray R, Fig. 70, which emanates from a luminous point, passes through the center of the lens C, and emerges from the lens as the ray R'. *Any straight*

line which passes through the center of the lens which is shown in Fig. 70 cuts the surfaces of the lens at points where the surfaces are parallel to each other. Therefore the lens acts like a flat plate

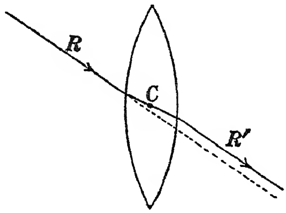


Fig. 70.

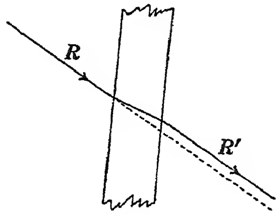


Fig. 71.

of glass on the ray RR', as shown in Fig. 71, and consequently R' is parallel to R, but displaced slightly sidewise. The lens is supposed, however, to be indefinitely thin so that the sidewise displacement of the ray in Fig. 70 is negligible and therefore RR' is sensibly one straight line. The ray R comes from a luminous point a and the ray R' passes through b (the conjugate of a). Therefore the points a, C and b are sensibly on one straight line.

This proposition is of great importance in the geometrical construction for determining the position of the conjugate to a given point as discussed in the next article.

51. Geometrical construction for determining the location of the conjugate to a given point. (*a*) *For the case of a converging*

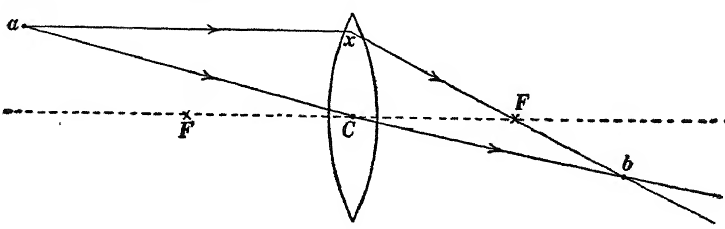


Fig. 72.

lens. — All the rays which emanate from the point a in Fig. 72 are concentrated by the lens at the point b. That particular ray ax which is parallel to the axis of the lens passes through the

opposite focal point after traversing the lens because all rays parallel to the axis of the lens are concentrated at the opposite focal point. Furthermore, the ray aC which passes through the center of the lens continues unchanged in direction after traversing the lens, as explained in the previous article. Therefore the conjugate of a lies at the intersection of the lines xF and aC.

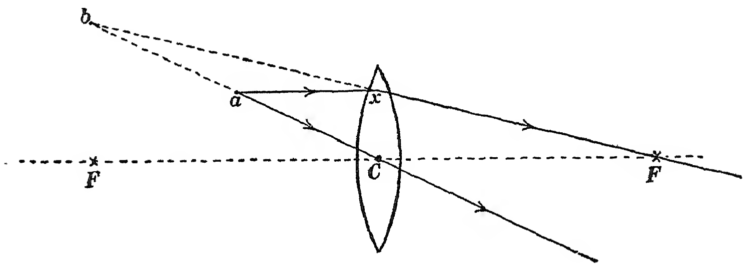


Fig. 73.

Figure 73 shows the geometrical construction for the virtual conjugate of a point a which is nearer to the lens than its focal point F. In this case, also, the ray ax passes through the opposite focal point after traversing the lens, and the ray aC continues in an unchanged direction after traversing the lens. Therefore, since all the rays from a appear to come from the conjugate of a after traversing the lens, it is evident that the conjugate point b must lie at the point of intersection of the lines xF and aC.

(*b*) *For the case of a diverging lens.* — Figure 74 shows a diverging lens with one of its focal points F. All the rays which emanate from a luminous point a appear to come from the conjugate point b after traversing the lens. That particular ray ax which is parallel to the axis of the lens appears to come from the focal point F as shown in Fig. 74, because all rays parallel to the axis appear to come from the focal point after traversing the lens. Furthermore the ray aC continues unchanged in direction after traversing the lens, as explained in the previous article. Therefore the conjugate of a must lie at the intersection of the lines xF and aC.

It is desirable in making lens diagrams to indicate only those particular rays (lines) which serve to determine the location of a conjugate point as here explained, and in many cases it is desirable to omit all rays (lines) except those which pass from a given point through the center of the lens to the conjugate of the point.

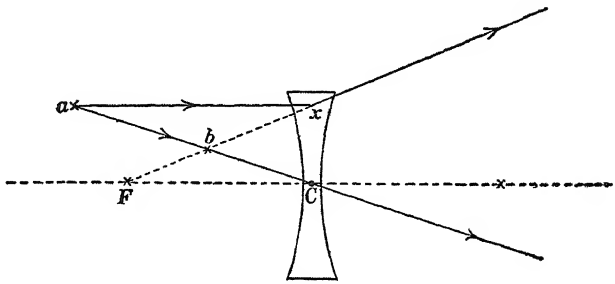


Fig. 74

52. Formation of images by lenses. — The group of points which are conjugate to the respective points of an object constitute an *image* of the object. When these conjugate points are real (see Art. 48) the image is said to be *real*. When these conjugate points are virtual the image is said to be *virtual*. Real

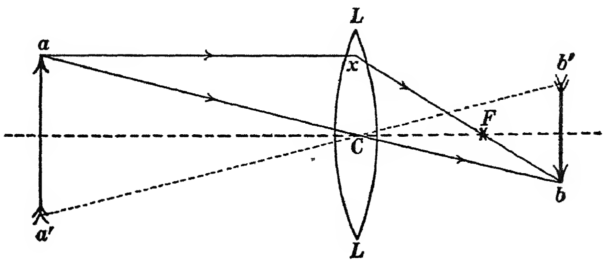


Fig. 75.

images are always on the side of the lens opposite to the object, virtual images are always on the same side of the lens as the object.

Figure 75 shows a converging lens, an object *aa'*, and the real image *bb'* of the object. One of the focal points of the lens is shown at *F*. All of the rays which emanate from the

point a are concentrated at the point b by the lens, but those rays only are shown which serve to determine the location of the point b, as explained in Art. 51. All the rays which emanate from the point a' are concentrated by the lens at the point b', but that ray only is shown which passes from the point a' through the center of the lens.

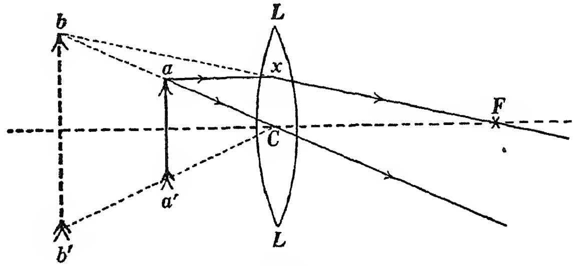


Fig. 76.

Figure 76 shows a converging lens, an object aa' and a virtual image bb' of the object. One of the focal points of the lens is shown at F. All of the rays which emanate from a appear to come from b after traversing the lens, but those rays only are shown which serve to determine the location of b as explained in Art. 51. All of the rays which emanate from a' appear to come from the point b' after traversing the lens, but only that ray is shown which passes from a' through the center of the lens.

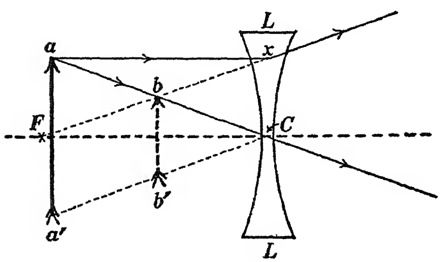


Fig. 77

Figure 77 shows a diverging lens, an object aa' and the virtual image bb' of the object.

All rays which emanate from a appear to come from b after traversing the lens, but those rays only are shown which serve to locate the position of b. All the rays from a' appear to come from b' after traversing the lens, but that ray only is shown which passes from a' through the center of the lens.

An important case of image formation by a diverging lens is shown in Fig. 78. Light from a distant object passes through a converging lens (not shown in the figure) *and would form a real image of the object at* O *if it were not for the interposition of a diverging lens* LL as shown in the figure. Consider that pencil of rays RR which is converging towards the point a. That particular ray rx which is parallel to the axis of the lens appears to come from the focal point F after traversing the lens,

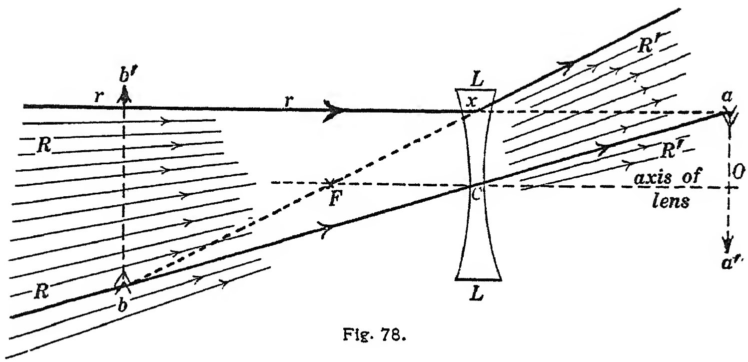


Fig. 78.

and that particular ray bC which passes through the center of the lens continues unchanged in direction after traversing the lens. Therefore the conjugate of a is at the point of intersection of the lines xF and aC, that is to say, the light which is converging towards the point a appears to come from the point b after traversing the lens. The image O may be thought of as a *virtual object* in so far as the diverging lens LL is concerned, and the lens LL forms an inverted virtual image of O at bb'. The action of the diverging lens which is represented in Fig. 78 is utilized in the opera glass.

A general idea of the relation between the position of an object O and the position of its image I [a relation which is expressed algebraically by equation (5)] may be obtained with the help of the geometrical construction of Figs. 72, 73 and 74. Thus Fig. 79 applies to the case of a converging lens.* Consider three

* The student should make a drawing somewhat like Fig. 79 for the case of a diverging lens.

objects O, O' and O'' all of the same size, draw the line $aa'a''x$ parallel to the axis of the lens, and draw the line xF. Then the lines aC, $a'C$ and $a''C$ cut the line xF at the points b, b' and b'' which are the conjugates of a, a' and a'' respectively.

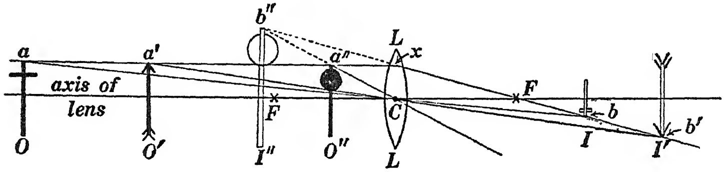


Fig. 79.

An important fact which is at once evident from the construction of Fig. 79 [and which may also be established with the help of equation (5)] is that *when* a *is large as compared with* b *then* b *is very nearly equal to* f. This fact is exemplified in all cheap photographic cameras in which the "focus" is not adjustable; the focal length f of the camera lens is short in comparison with the distance a of any object which is to be photographed, and the distance b from the lens to the photographic plate is sensibly invariable and equal to f.

53. Linear magnification. — Lines drawn from the center of a lens through the extremities of an object aa' pass through the extremities of the image bb', as shown in Figs. 75 to 78. The distance of the object from the center of the lens is represented by a and the distance of the image from the center of the lens is represented by b, and therefore *the diameter of the object is to the diameter of the image as* a *is to* b.

54. Lens systems.* Compensation of imperfections. — A number of lenses used together constitute a *lens system.* When the centers of curvature of the spherical surfaces of the various lenses all lie on a straight line, the system is called a *centered system.* Centered lens systems are almost universally used in practice instead of simple lenses in order that the very appreciable imper-

* Certain fundamental properties of lens systems are described in Appendix A. The terms *principal planes*, *inverse principal planes*, and *nodal points* are there explained.

fections of simple lenses may be obviated. This obviation of the imperfections of simple lenses by combining a number of them into a system is accomplished by so designing the system that the imperfections of one lens are balanced or compensated by opposite imperfections of another lens. An outline of the theory of compensated lens systems is given in Chapter VI.

CHAPTER V.

SIMPLE OPTICAL INSTRUMENTS.

55. The photographic camera * consists of a light-tight box in one side of which a lens is mounted. Inside of the box and opposite to the lens is a sensitive plate upon which a real inverted image of external objects is projected by the lens. Various types of highly perfected photographic lens systems are described in Chapter VI.

56. The magic lantern is an arrangement for projecting the image of a brilliantly illuminated object or picture upon a screen. The light from a lamp *L*, Fig. 80*a*, passes through lenses *CC*,

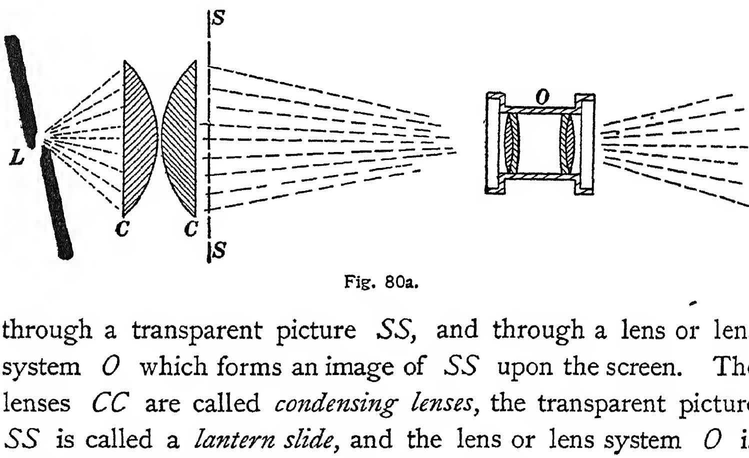


Fig. 80a.

through a transparent picture *SS*, and through a lens or lens system *O* which forms an image of *SS* upon the screen. The lenses *CC* are called *condensing lenses*, the transparent picture *SS* is called a *lantern slide*, and the lens or lens system *O* is called the *objective* or *object lens* of the lantern.

Figure 80*a* shows the usual form of lantern, for projecting transparent pictures. A lantern for projecting opaque pictures is shown in Fig. 80*b*. Two lamps *LL* illuminate the picture *PP*, the illumination being increased to the utmost by the polished

* A very good book on the physical and chemical aspects of photography is *Photography for Students of Physics and Chemistry* by Louis Derr, The Macmillan Co., New York, 1906.

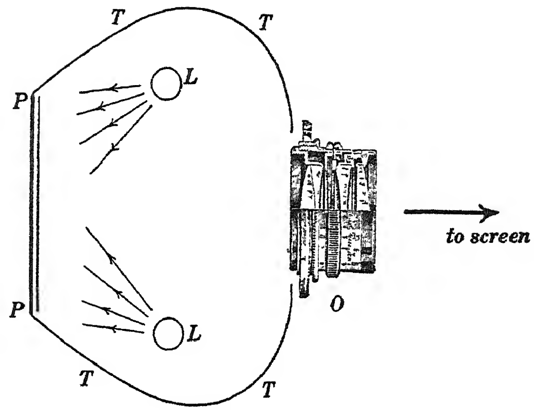


Fig. 80b.

metal reflectors *TT*, and the light from the picture passes through an object lens *O* which gives upon the screen an enlarged real image of the picture *PP*. In this form of lantern it is necessary to use a very high grade wide-aperture lens system for the object lens *O*, whereas in the form of lantern shown in Fig. 80*a* a much cheaper grade of object lens can be used with entirely satisfactory results.

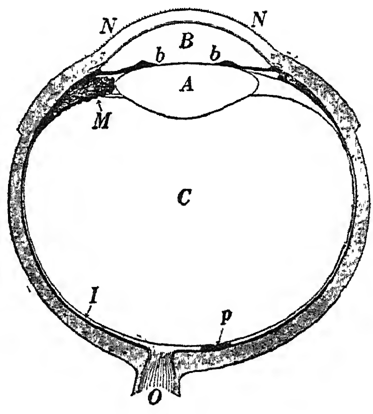


Fig. 81.

57. The eye. *— Figure 81 shows a horizontal section of the human eye. The tough outer coating of the eye-ball is sharply curved and transparent in front forming the *cornea NN*. Between the cornea and the *crystalline lens A* is a clear watery fluid, the *aqueous humor B*.

*A good discussion of the structure of the eye and its functions; of the ophthalmoscope; of binocular vision and the stereoscope; and of the persistence of vision and the stroboscope is given in Edser's *Light for Students*, pages 159–196, Macmillan and Company. See also Müller-Pouillet's *Lehrbuch der Physik*, Vol. II, part I (on light by Otto Lummer), pages 580–643. The great work on this subject is Helmholtz's *Handbuch der physiologischen Optik*, Leipzig, 1896.

Behind the crystalline lens and filling the remainder of the eyeball is a clear semi-fluid substance, the *vitreous humor* C. The front surface of the cornea and the two surfaces of the crystalline lens are sensibly spherical and they constitute a lens system which projects an image of external objects upon a sensitive membrane, the *retina* I. The retina consists of a great number of minute end-organs of nerve fibers which enter the eye in a bundle at O, constituting what is called the *optic nerve*. The most sensitive part of the retina is a slightly depressed place at p which is called from its color the *yellow spot*. In the yellow spot the terminal organs of the optic nerve are packed very close together, and in order to see an object distinctly its image must fall upon this spot. Light which enters the eye passes through an aperture bb in a muscular membrane which is called the *iris*. The aperture bb is called the *pupil* of the eye.

Accommodation. — In the photographic camera the distance of the lens from the sensitive plate is adjustable so that distant objects or near objects may be sharply focused upon the sensitive plate at will. In the eye, however, the distance from the crystalline lens to the retina is invariable and the lens of the eye is focused by the action of a muscle M which surrounds the crystalline lens. When this muscle is under tension the lens is drawn outwards all around its edge and thereby somewhat flattened. When this muscle is relaxed the crystalline lens resumes its normal shape becoming thicker at the center. This adjustment of the focus of the lens of the eye to give distinct vision is called *accommodation*. Ordinarily the eye has power of accommodation for objects at any distance greater than 15 centimeters from the eye. The distance of most distinct vision for the normal eye is about 25 centimeters. The accommodation of the eye is sensibly invariable for all distances exceeding eighty or ninety meters (see last paragraph of Art. 52).

Imperfections of the eye. — Some persons can accommodate the eye to distant objects only with great effort or not at all; some

persons can accommodate the eye to near objects with great difficulty or not at all; some persons can accommodate the eye so as to see vertical lines or horizontal lines sharply but not so as to see both vertical and horizontal lines distinctly at the same time. A person who cannot accommodate his eyes to distant objects but who can accommodate his eyes to near objects is said to be *near sighted.* A person who can accommodate his eyes to distant objects but who cannot accommodate his eyes to near objects is said to be *far sighted.* Near-sightedness is relieved by the use of spectacles with diverging lenses, far-sightedness is relieved by the use of spectacles with converging lenses. Inability to see vertical lines and horizontal lines simultaneously is due to inaccurate "centering"* of the eye lenses or to more or less deviation from true spherical shape of the various refracting surfaces of the eye, and it is called *astigmatism.* Astigmatism is corrected by the use of spectacles having cylindrical surfaces.

58. Apparent size of objects. Visual angle. — An object appears large when its image covers a large portion of the retina.

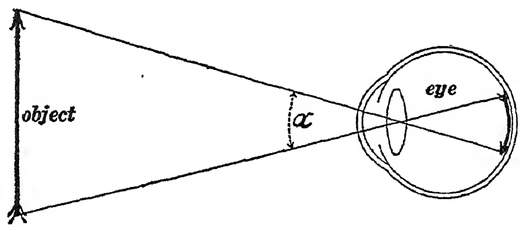


Fig. 82.

Lines drawn from the extremities of the object through the center† of the eye lenses, as shown in Fig. 82, pass through the extremities of the image on the retina. The angle α between these lines determines, therefore, the size of the image on the retina; this

* See Art. 54.

† Strictly, those lines pass through the extremities of the image which are drawn through the *posterior nodal point* of the lens system of the eye parallel, respectively, to lines drawn through the extremities of the object to the *anterior nodal point* of the lens system of the eye. See Appendix A.

angle is called the *visual angle* of the object and it is taken as the measure of the apparent size of the object.

It must be remembered that the angle α in Fig. 82 is always quite small because an image of an object must fall upon a very small portion of the retina (the yellow spot) if one is to see the whole object distinctly. In the diagrams of the eye, however, Figs. 82 and 83, the angle α is shown rather large for the sake of clearness.

59. The simple microscope. Definition of magnifying power. — The *simple microscope* or *magnifying glass* is a converging lens

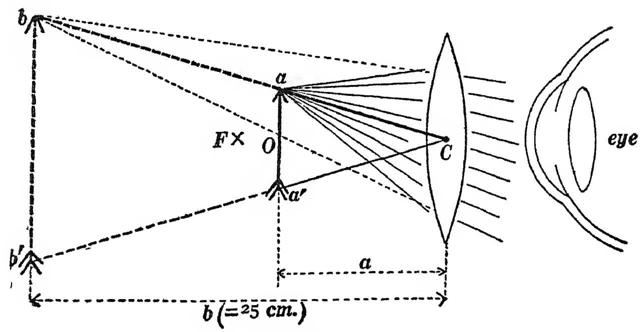


Fig. 83.

which is held as near to the eye as possible, the object to be examined being moved up until it is seen most distinctly. The eye is then looking at an enlarged virtual image of the object and this virtual image is at the distance of most distinct vision from the eye, say, 25 centimeters, as shown in Fig. 83. The light from the point a of the object appears to have come from the point b after passing through the magnifying glass.

The magnifying power of a microscope is defined as the ratio of the apparent size (visual angle) of an object as seen with the microscope to its apparent size (visual angle) as seen with the naked eye at a distance of 25 centimeters.

Proposition. — The magnifying power of a magnifying glass is

$$m = \frac{25}{f} + 1 \tag{6}$$

in which f is the focal length of the magnifying glass in centimeters. The eye is assumed to be accommodated for a distance of 25 centimeters, that is to say, the plane bb' in Fig. 83 is assumed to be 25 centimeters from the eye.

Proof of equation (6). — Consider the angle α between the lines bC and $b'C$ in Fig. 83. This is the angle which is subtended by the object O at a distance $OC(=a)$. Let us consider the distance between the magnifying glass and the eye to be negligible, then the angle α between the lines bC and $b'C$ in Fig. 83 is the same as the angle between lines drawn from b and b' to the center of the lenses of the eye, but lines drawn from b and b' to the center of the lenses of the eye determine the visual angle of the image bb', or, in other words, the *visual angle of the object as seen through the magnifying glass.* Therefore the angle subtended by the object O at the distance a is the visual angle of the object as seen through the magnifying glass, whereas the visual angle of the object O as seen with the naked eye at the distance of 25 centimeters would be less than the angle α in the ratio of 25 to a.* Therefore, the magnifying power of the simple microscope shown in Fig. 83 is equal to $25/a$. From equation (5), however, we have, since bb' is a virtual image:

$$\frac{1}{f} = \frac{1}{a} - \frac{1}{b}$$

whence, substituting the value of b, namely, 25 centimeters as indicated in Fig. 83, we find

$$a = \frac{25f}{25+f}$$

whence $25/a$ is found to be equal to $25/f + 1$.

Note. — When the eye is accommodated for parallel rays the distance b in Fig. 83 is infinity. Under these conditions, a is

* It must be remembered that the angles under consideration are all small so that to increase the distance of O from a given point in the ratio of a to 25 would decrease the angle subtended by O in the ratio of 25 to a.

equal to f and the magnifying power of the magnifying glass is $25/f$.

A high-power magnifying glass must have a very short focal length according to equation (6), and it must be held very near to the object to be examined inasmuch as a, Fig. 83, is sensibly equal to f. Furthermore, a short focal length lens is necessarily small in diameter and the field of view in such a magnifying glass is extremely limited. Therefore high-power magnifying glasses are inconvenient and unsatisfactory. For magnifying powers exceeding 20 or 30 diameters the compound microscope is always used.

60. The compound microscope* consists of a lens A, Fig. 84, which forms an enlarged real image I of an object O, and a magnifying glass B for viewing this image. The lens A is called the *object-glass* of the microscope and the lens B is called the *eye-piece*. In modern high-grade microscopes the object-glass and eye-piece both consist of compensated lens systems as explained in Chapter VI. No attempt is made in Fig. 84 to show the actual paths of the rays of light through the microscope; the dotted lines are drawn from the extremities of the object

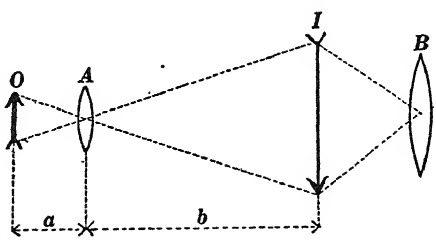


Fig. 84.

* See "The Microscope and Its Revelations," by W. Carpenter (revised by W. H. Dallinger), London, 1901. An interesting outline of the historical development of the microscope is given in Hastings' *Light*, pages 83–110, Charles Scribner's Sons, 1901. Very good practical directions for the use and care of the microscope are published by the Bausch and Lomb Optical Company, of Rochester, New York. This publication is in the form of a small pamphlet and it consists of extracts from a larger work "The Manipulation of the Microscope," by Edward Bausch. A very complete discussion of the microscope by Czapski is given in Winkelmann's *Handbuch der Physik*, Vol. VI, pages 328–373.

through the center of the object-glass, and these lines therefore determine the extremities of the image. The image I in Fig. 84 is viewed through the magnifying glass (eye-piece) B exactly as if it were an ordinary object like O, Fig. 83.

Proposition.—The magnifying power of a compound microscope is

$$m = \frac{b}{a}\left(\frac{25}{f} + 1\right) \tag{7}$$

in which a and b are the respective distances of object and image from the center of the object glass as shown in Fig. 84, and f is the focal length of the eye-piece.

Proof of equation (7).—The image I in Fig. 84 is b/a times as large as the object, and the eye-piece (an ordinary magnifying glass) makes this image appear $(25/f + 1)$ times as large as it would appear to the naked eye at a distance of 25 centimeters, or $b/a(25/f + 1)$ times as large as the object itself would appear at a distance of 25 centimeters from the naked eye.

The compound microscope has reached almost the limit of magnifying power that is possible, namely, about 900 diameters with ordinary light and about 1,800 diameters with ultra-violet light. See discussion of resolving power in Chapter VI.

61. The telescope* consists of a large long-focus lens O, Fig. 85, which forms an image i of a distant object, and a magnifying glass E for viewing this image. The lens O is called the *object-glass* of the telescope and the lens E is called the *eye-piece*. In modern high-grade telescopes the object-glass and the eye-piece both consist of compensated lens systems as explained in Chapter VI. Before the introduction of the modern compensated lens systems the best telescopes were about 150 feet long so as to permit of the use of a very thin (simple) lens as an object-glass. See Arts. 45 and 65.

*A very interesting discussion of the telescope is given in Lockyer's *Stargazing*, pages 138–172, and in Hastings' *Light*, pages 53–82. A very complete discussion of the telescope by Czapski, is given in Winkelmann's *Handbuch der Physik*, Vol. VI, pages 386–432.

The dotted lines in Fig. 85 are not intended to represent the paths of the rays through the telescope; the dotted lines are drawn from the extremities of the object through the center of

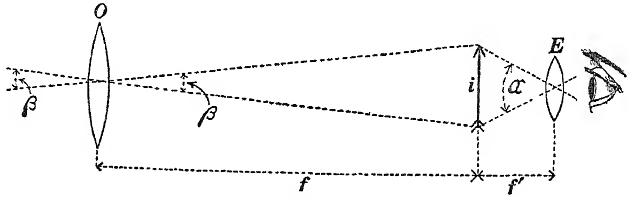


Fig. 85.

the object-glass to the extremities of the image i; and also from the extremities of the image i through the center of the eye-piece E; thus showing the visual angle β of the distant object as it would be seen by the naked eye, and the visual angle α of the distant object as seen through the telescope. The image i in Fig. 85 is viewed through the magnifying glass (eye-piece) E exactly as if it were an ordinary object like O, Fig. 83.

The magnifying power of a telescope is defined as the ratio of the visual angle of a distant object as seen with the telescope to the visual angle of the object as seen with the naked eye. The visual angle of the object as seen with the naked eye is the angle β in Fig. 85 because the distance from the object-glass O to the distant object is sensibly the same as the distance from the eye to the distant object; the visual angle of the object as seen through the telescope is the angle α in Fig. 85 between lines drawn from the extremities of the image through the center of the eye lens E (see Art. 59); and therefore the magnifying power m of the telescope is equal to α/β. If the object is at a very great distance from the observer then the image i is at the focal point of O and the distance Oi is the focal length f of O. If the eye is accommodated for parallel rays then the image i is at the focal point of E and the distance Ei is the focal length f' of E. Therefore, since both angles α and β are supposed to be very small, we have

$$m = \frac{\alpha}{\beta} = \frac{f}{f'} \tag{8}$$

in which m is the magnifying power of the telescope in Fig. 85, f is the focal length of the object-glass, and f' is the focal length of the eye-piece.

62. The erecting telescope or spy-glass. — The simple telescope, the elements of which are shown in Fig. 85, shows objects inverted. The spy-glass is a telescope so modified as to make distant objects appear erect. Figure 86 shows the essential parts

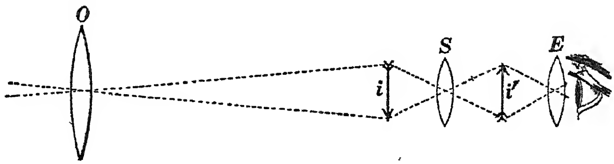


Fig. 86.

of a spy-glass. The object-glass O forms at i an inverted image of a distant object, and the lens S forms at i' an inverted image of i or an erect image of the distant object. This erect image i' is viewed by the magnifying glass E. The introduction of the erecting lens S lengthens the telescope very materially and the spy-glass is consequently quite long. In practice each of the lenses O, S and E, Fig. 86, consists of a compensated lens system.

63. The opera glass is a telescope of which the eye-piece is a diverging lens, as shown in Fig. 87. The object-glass O would

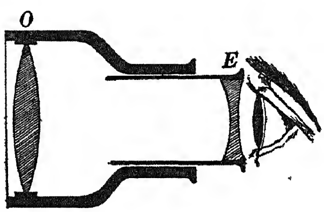


Fig. 87a.

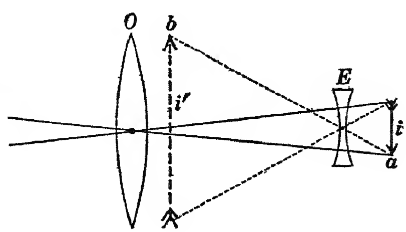


Fig. 87b.

form at i an inverted real image of the distant object if it were not for the interposition of the diverging lens E, the eye-piece.

Light from a point in the object converges towards the point a after passing through the object-glass O, and the effect of the eye-piece is to cause the pencil of rays which is converging towards a to appear to come from the point b, that is to say, the lens E forms an inverted enlarged virtual image at i' of the inverted image i. Therefore the image i' is erect, and the eye in looking through the lens E sees the distant object in an erect position. The action of the lens E is fully explained in connection with Fig. 78.

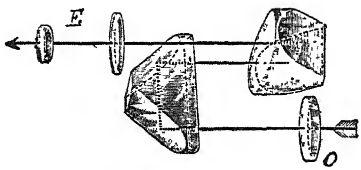


Fig. 88a.

In practice, the object-glass O and the eye-piece E in Figs. 87*a* and 87*b* both consist of compensated lens systems as explained in Chapter VI.

One great advantage of the opera-glass is its shortness. Figure 88*a* shows two glass prisms arranged to reflect the light back and forth as it travels from the object-glass O to the eye-piece E of a telescope, thus greatly shortening the telescope, and at the same time bringing the image which is formed by the object-glass into an erect position. This arrangement was devised by J. Porro in 1852. Figure 88*b* shows the essential features of a shortened binocular telescope or field glass using Porro's prisms.

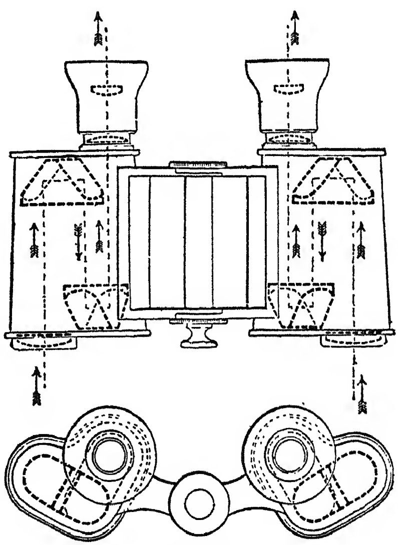


Fig. 88b.

64. The use of the telescope for sighting. — Sighting cannot be accurately done where the two sights consist of pins or point

like projections as on a gun, because one or both of the sights must be blurred on account of the impossibility of sharply focusing the eye on both sights and on the distant object simultaneously. The most accurate sighting that can be done with the naked eye in this way is with an error of one or two minutes of angle (one or two inches in a distance of 100 yards). Accurate

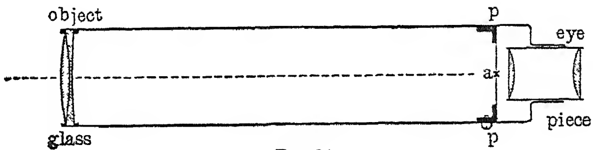


Fig. 89.

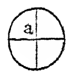


Fig. 90.

sighting is always done by means of the telescope. Two very fine wires are stretched across the focal plane *pp* of the object-glass of the telescope, as shown in Fig. 89. Figure 90 shows a view of these cross wires as they appear to an observer looking through the eye-piece. These cross wires are in the same plane as the image (formed by the object-glass) of the distant object, these wires are therefore sharply in focus, and the telescope may be adjusted so as to bring the image of any given point of the object accurately into coincidence with the point *a*, Figs. 89 and 90, where the two cross wires intersect. When this adjustment is made, the line drawn from the point *a* through the center * of the object-glass passes through the given point of the object.

* This statement is strictly true when the point *a* lies on the axis of the object-glass, which is always approximately the case. When the point *a* does not lie on the axis of the object-glass, then the line drawn from *a* through the *posterior nodal point* of the objective lens system is parallel to the line drawn from the *anterior nodal point* of the lens system to the given point in the object. See Appendix A.

CHAPTER VI.

LENS IMPERFECTIONS AND THEIR COMPENSATION.*

65. Limitations of the simple theory of lenses. — The simple theory of lenses which is given in Chapter IV applies to a thin lens with light passing through it parallel to its axis, and the simple theory ignores the variation of focal length of a lens with the wave-length (color) of the light. Also the simple theory of Chapter IV is based on the idea of the *ray of light* as follows:

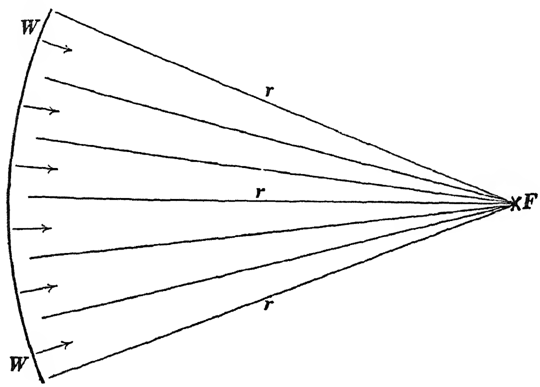


Fig. 91.

Consider a wave WW, Fig. 91, which is in the form of a sector of a spherical surface; *the simple theory of Chapter IV assumes that the entire disturbance which constitutes the wave* WW *is concentrated at the mathematical point* F *which is the center of curvature of* WW, or, in other words *the simple theory of Chapter*

* A good discussion of this subject is to be found in Lummer's *Photographic Optics*, translated by S. P. Thompson, Macmillan & Co., 1900. See also Drude's *Theory of Optics*, pages 31–92, translated by Mann and Millikan, Longmans, Green & Co., 1902. A very complete discussion of this subject by Czapski, Eppstein, and von Rohr is given in the second edition of Winkelmann's *Handbuch der Physik*, Vol. VI, on Optics, pages 1–470. See also Müller-Pouillet's *Lehrbuch der Physik*, ninth edition, Volume II, Part I (Optics), pages 443–880, by Otto Lummer.

IV assumes that light travels along the "rays" rr in Fig. 91 which come to a mathematical focus at the point F. As a matter of fact, however, a perfect lens would focus light as a small *spot* not as a *point;* one reason is that the disturbance which constitutes the wave *IWW*, Fig. 91, occupies a region of sensible thickness (the wave front *IWW* is a *surface* but the actual wave is a *layer*) and *the focus F is always a region of the same order of magnitude as the thickness of the wave.* Another reason is briefly explained in Art. 66*b*.

The various imperfections of simple lenses are briefly described in this chapter, and the methods of approximately eliminating these imperfections by combining two or more simple lenses into a *lens system* are briefly discussed. In general this elimination of imperfection is accomplished by balancing the opposite imperfections of two or more lenses.

In some cases the lenses of a system are cemented together by a thin layer of Canada balsam (a clear fluid resin), and in some cases the lenses of a system are separated by air spaces. A lens system which consists of two simple lenses is called a *doublet*, and a lens system which consists of three simple lenses is called a *triplet*. In some cases ten or more simple lenses are combined into a system.

66. Numerical Aperture. — The free diameter * of a lens divided by its focal length is called the *numerical aperture* of the lens. The numerical aperture of a lens is usually expressed by specifying the free diameter of the lens as a fraction of its focal length f, thus if the free diameter of a lens is one fourth of its focal length its aperture is expressed as $f/4$.

The free diameter of a lens is frequently reduced by the use

* The free diameter of a simple thin lens is the diameter of the lens. The "free diameter" of a lens system is not so easily defined. The definition of numerical aperture as here given is sufficient however for present purposes. For a full discussion of aperture see Drude's *Theory of Optics*, translated by Mann and Millikan, pages 73–92; see also Dallmeyer's *Telephotography*, pages 91–101. A very full discussion of this subject is given in Winkelmann's *Handbuch der Physik*, Vol. VI, pages 211–260, 298 and 347.

of a metal plate with a hole in it. Such a metal plate is called a *diaphragm* or a *stop*.

Examples. — The object-glass of the great telescope of the Lick Observatory is 3 feet in diameter and its focal length is 50 feet, that is, its diameter is 3/50 of its focal length. The best objectives for instantaneous photography have a numerical aperture of about 1/4 (an actual free diameter equal to 1/4 of the focal length, or $f/4$). High grade high power microscope objectives have a numerical aperture of 1.4, that is to say, the "free diameter" of a high power microscope objective is 1.4 times as great as its focal length.

Large numerical aperture is important for two reasons, namely, (*a*) the greater the numerical aperture the brighter the image which is formed, and (*b*) the greater the numerical aperture the greater the resolving power of the lens.

(*a*) *The brightness of the image which a lens forms of a given distant object is proportional to the square of the numerical aperture of the lens.* Consider two lenses A and B of the same focal length, the diameter of lens A being twice as great as the diameter of lens B (numerical aperture twice as great). These two lenses form images of the same size of a given distant object, and, inasmuch as the larger lens gathers four times as much light, its image is four times as bright. Consider two lenses A and B of the same diameter, lens A having a focal length twice as great as lens B (numerical aperture half as great). Under these conditions the lenses gather the same amount of light, but lens A forms an image twice as large in diameter or four times as large in area as that which is formed by lens B, so that the image formed by lens A is one quarter as bright as the image formed by lens B.

(*b*) *Resolving power.* — The power of a lens to reproduce fineness of detail of an object in an image is called the resolving power of the lens. If a lens concentrated the light from the various points of an object at actual points in the image, then infinite fineness of detail would exist in the image and the lens

would have unlimited resolving power. As a matter of fact, however, the light from a point of an object is focused as a *spot* in the image partly on account of imperfections of the lens and partly on account of the essential nature of light itself. The imperfections of a lens may, however, be practically eliminated by compensation, as described in the following articles, and therefore *the limit of resolving power of the best modern optical instruments depends upon the nature of light itself.*

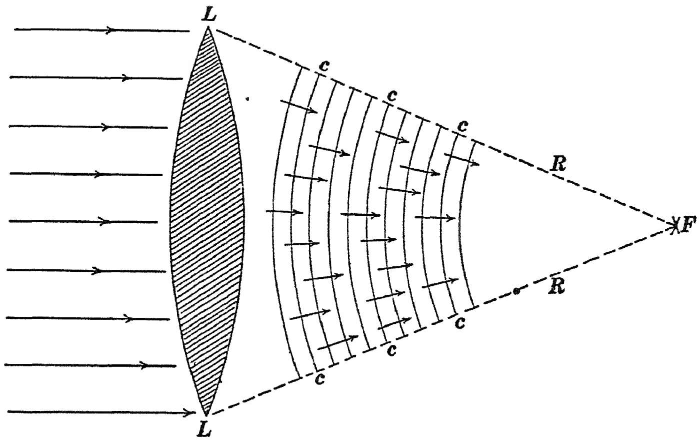


Fig. 92.

The limit of resolving power of a completely compensated lens depends upon the numerical aperture of the lens, the greater the numerical aperture the greater the resolving power. This dependence of resolving power upon numerical aperture may be explained in a general way as follows: * Imagine LL, Fig. 92, and $L'L'$, Fig. 93, to be lenses which make the wave fronts accurately spherical with their centers of curvature at the points F and F'. The entire disturbance which is represented by the converging waves in Figs. 92 and 93 is not concentrated at the points F

* A brief discussion of this matter is given in the chapter on Diffraction of Light (particularly on pages 235–240) of Drude's *Theory of Optics*, translated by Mann and Millikan.

The best discussion of this matter is, perhaps, that given by Michelson. See pages 27–30, *Light Waves and their Uses*, University of Chicago Press, 1903.

and F' as explained in Art. 65. Furthermore, a very considerable portion of the disturbance which constitutes the waves in Figs. 92 and 93 leaks off, as it were, from the edges of the waves at *ccc* into the regions *RR*, and the result is a focal *spot** at *F* and *F'*. *The leakage, if we may so call it, is a much larger part of the whole disturbance in Fig. 93 than it is in Fig. 92, and*

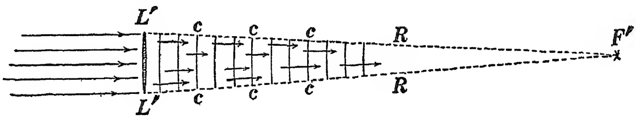


Fig. 93.

therefore the focal spot in Fig. 92 (lenses supposed to be perfect as above stated) is smaller than the focal spot in Fig. 93. The action here described as the "leaking off" of the disturbance from the free edge of a wave is called diffraction and it is discussed very briefly in Chapter VIII.

The largest numerical aperture lens in use is the high power microscope objective of which the "free diameter" is 1.4 times its focal length. With a *perfect* microscope of this numerical

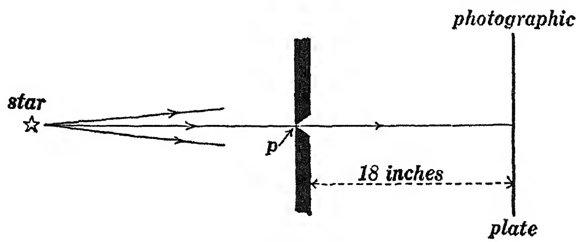


Fig. 94.

aperture the *spot* in the image corresponding to a *point* in the object would be barely visible as a spot with a total magnifying power of 900 diameters.† Therefore to increase the magnifying

* In fact, the focus is a central spot surrounded by bright and dark rings as shown in Fig. 95.

† This is the limit of magnifying power of a microscope (see Drude's *Theory of Optics*, pages 235–240) when ordinary light is used. By using ultra-violet "light," which cannot be seen but which can be photographed, the limit of effective magnifying power is about 2,000 diameters. The lenses of the ultra-violet microscope are made of quartz inasmuch as ordinary glass does not transmit ultra-violet light.

power even of a perfect microscope beyond 900 diameters is ineffective because such increase of magnifying power cannot bring out additional detail. Thus a mosaic picture shows more and more detail as one approaches it from a distance until the separate stones become visible as spots after which no additional detail is possible.

The blurred character of the image produced by a high-grade lens is usually not visible to the naked eye unless the numerical aperture of the lens is very small. Figure 94 shows the light from a fine point-source, like a star, shining through a pin-hole p upon a photographic plate, and Fig. 95 shows the *spot* which is produced on the plate. This spot is exactly like the focal spot of a perfect lens greatly magnified. The diameter of the pinhole in Fig. 94 was 0.006 inch and the diameter of the spot in Fig. 95 if there had been no spreading action would have been less than 0.01 inch whereas the central white spot in Fig. 95 is actually about 0.12 inch in diameter.

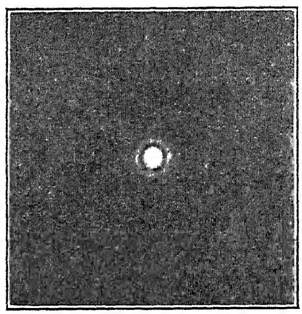


Fig. 95.

67. Field angle. — The angle between lines drawn from the center * of a lens to the extreme edges of the largest distinct image which the lens can produce is called the *field angle* of the lens. Thus the angle F is the field angle of the lens in Fig. 96. A simple lens usually gives an image so badly blurred that one can scarcely speak of the largest distinct image which the lens can produce and such a lens becomes extremely

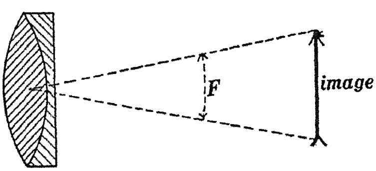


Fig. 96.

* The optical center of a simple thin lens is the geometrical center of the lens. The optical center of a thick lens or of a lens system is not so easily defined. See Appendix A.

unsatisfactory for field angles greater than a few degrees. The field angle of high-grade telescope and microscope objectives is ordinarily very small, seldom exceeding one or two degrees. Photographic objectives, on the other hand, are made which give excellent definition with a field angle as great as 135 degrees. Such lenses are called *wide angle lenses.*

68. Spherical aberration. — A plane or spherical wave is in general not plane or spherical after passing through a lens. This effect is called *spherical aberration.* There is, of course, an infinite variety of ways in which a wave front (or any surface) may be *not* spherical or plane, and spherical aberration is in fact an extremely complicated thing.

A very narrow pencil of rays passing through a lens parallel to the axis of the lens is not subject to spherical aberration because, in the first place, the transmitted waves are symmetrical about the axis (equal curvature in every direction), and, in the second place, the transmitted waves are so small that they cannot be distinguished from sectors of a spherical surface. A broad beam of parallel rays, however, is subject to spherical aberration to a very considerable degree as described in Art. 69.

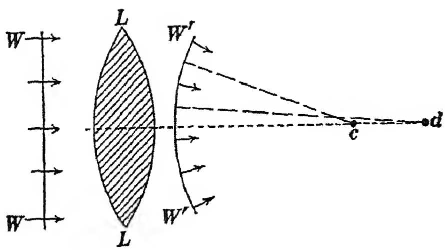


Fig. 97.

A narrow pencil of rays which passes obliquely through a lens becomes a well-defined astigmatic pencil, as described in Art. 70, and a broad oblique pencil or beam of rays is very greatly confused by a simple lens, as described in Art. 71.

There are, therefore, three kinds of spherical aberration, namely, (*a*) spherical aberration of a broad pencil or beam of rays parallel

to the axis of the lens, (*b*) spherical aberration of a narrow oblique pencil of rays, and (*c*) spherical aberration of a broad oblique pencil or beam of rays. The first is called *axial spherical aberration*, the second is called *astigmatism*, and the third is called *oblique spherical aberration*, or *coma*.

69. **Axial spherical aberration.** — Figure 97 represents a plane wave WW passing through a simple lens LL. After passing through the lens, the wave $W'W'$ is not spherical, the edge portions of the wave are concentrated at the point c and the middle portion of the wave is concentrated at the point d. Some idea of the action of a simple converging lens upon a broad beam of parallel rays passing through the lens parallel to its axis may be obtained from Fig. 98, which is a photograph of the beam after

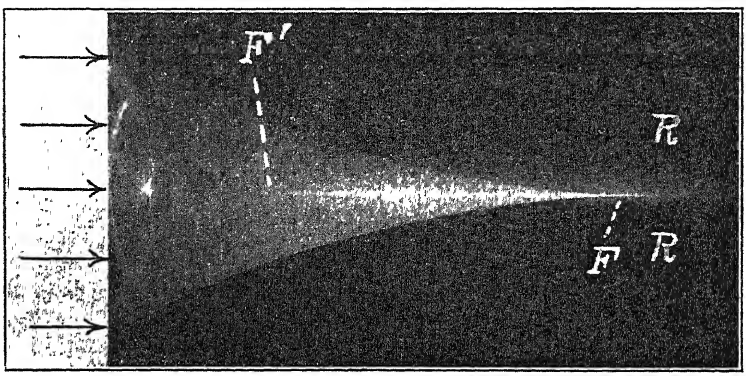


Fig. 98.

it has passed through the lens as indicated by the sketch lines in the figure, the beam being rendered luminous by a cloud of smoke. This photograph is entirely untouched except for the sketch lines and letters. Figure 99 shows how the light which passes through a narrow zone ZZ of the lens is focused at a point on the axis, and the fine line of light $F'F$ in Fig. 98 is due to the brilliant illumination of the smoke by the light which is focused at points along the axis by the various zones of the lens; F is the focus of the middle zone and F' is the focus of the edge zone of the

lens (in Fig. 98). The narrow pencil of rays P in Fig. 100 becomes an astigmatic pencil after passing through the lens, one focal line of the astigmatic pencil being at C (perpendicular to

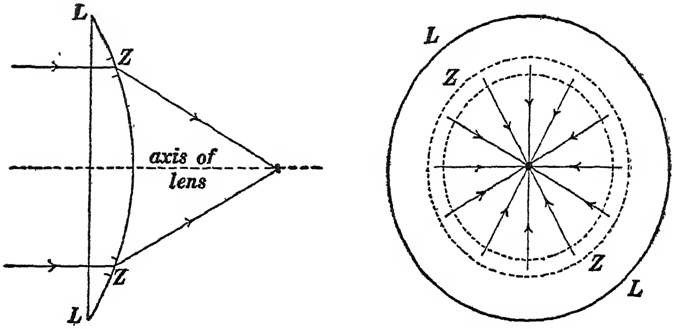


Fig. 99.

the plane of the paper) and the other focal line being at DD. It is evident from Fig. 100 that the smoke in the region RR is illuminated; in fact, the sharp, curved outline of the beam in Fig. 98 is due to the intense illumination of the smoke at all such points as C in Fig. 100. The sharp curved outline in Fig. 98 is indeed a *caustic curve* (see Art. 39).

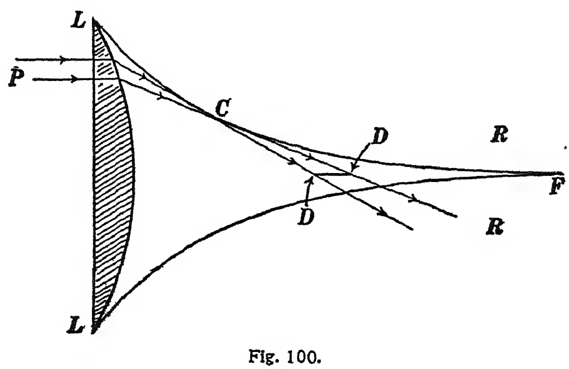
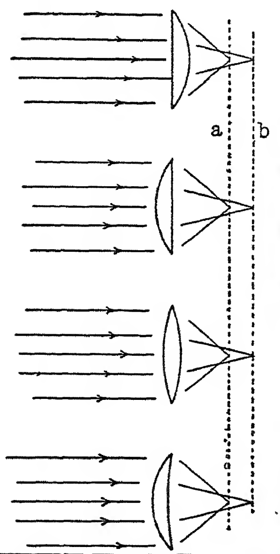


Fig. 100.

Some idea of the dependence of axial spherical aberration upon the shape of a lens and upon the kind of glass (value of index of refraction) may be obtained from the following table.

TABLE OF SPHERICAL ABERRATIONS.

Focal Length of Lens, 100 cm. Diameter of Lens, 10 cm.



Form of Lens.	$\mu=1.5$	$\mu=2.0$
1	$ab=4.5$ cm.	$ab=2.0$ cm.
2	$ab=1.17$ cm.	$ab=0.5$ cm.
3	$ab=1.67$ cm.	$ab=1.0$ cm.
4	$ab=1.07$ cm.	$ab=0.44$ cm.

*Compensation of spherical aberration.** *Aplanatism.*—The possibility of correcting a lens system for axial spherical aberration was discovered about 1760, and the first step towards the correcting of a lens system for oblique spherical aberration or coma was made by Fraunhofer about 1810. Fraunhofer's discovery referred to the condition which must be satisfied to partially eliminate oblique spherical aberration, and the condition which must be satisfied to eliminate oblique spherical aberration more completely from a very wide aperture lens was discovered in 1873 by Abbe, and it is known as Abbe's sine condition.†

* An example of the complete calculation of a telescope objective, according to Gauss, is given by Otto Lummer in Müller-Pouillet's *Lehrbuch der Physik*, Vol. II, part I, pages 573–579. The method for eliminating spherical aberration is brought out in this example.

† The theory of the aberrations of a lens was worked out very completely by von Seidel in 1855. A good discussion of von Seidel's theory may be found in Lummer's

Many of the earlier lens manufacturers had learned by trial to satisfy Abbe's sine condition, but since the discoveries of Abbe and Rudolph,* the lens designer recognizes more distinctly the necessity and understands more clearly the possibility of eliminating astigmatism and coma, but the lens designer must still work very largely by trial because of the infinite number of possible combinations of differently shaped lenses of different kinds of glass at different distances apart.

A lens system which is free from spherical aberration is said to be *aplanatic*. This term was originally applied to a lens system which had been corrected for axial spherical aberration, but it is now more properly applied to a lens system which has been corrected for axial spherical aberration and for oblique spherical aberration, or coma.

It is evident from the above table that the spherical aberration of a lens of given focal length (distance ab in the above table) depends upon the relative curvatures of the two surfaces of the lens and upon the index of refraction μ of the glass of which the lens is made. It is possible, therefore, to make two lenses of different focal lengths but having the same spherical aberration, or to make a diverging lens and a converging lens of *unequal focal lengths* but having *equal and opposite spherical aberrations*. When two such lenses are combined, the spherical aberration of the one annuls the spherical aberration of the other, but the diverging action of the one does not annul the converging action of the other. Such a combination is therefore free from spherical aberration and the combination is said to be aplanatic. A lens system can be made accurately aplanatic for *one pair of conjugate points, only*. These conjugate points are called the *aplanatic points* of the system.

Photographic Optics, translated by S. P. Thompson, pages 6–13, and pages 103–115; and a good discussion of Abbe's sine condition may be found on pages 116–121. See also Drude's *Theory of Optics* translated by Mann and Millikan, pages 58–63. An extremely simple discussion of the five aberrations of von Seidel on the basis of Hamilton's Principle is given by Lord Rayleigh in the *Philosophical Magazine* for June, 1908, pages 677–687.

* See Art. 70.

70. Astigmatism. — A narrow pencil of parallel rays, or indeed, any narrow homocentric pencil of rays, becomes an astigmatic pencil when it passes obliquely through a simple lens. Thus, Fig. 101 shows the focal lines *C* and *DD* of the astigmatic

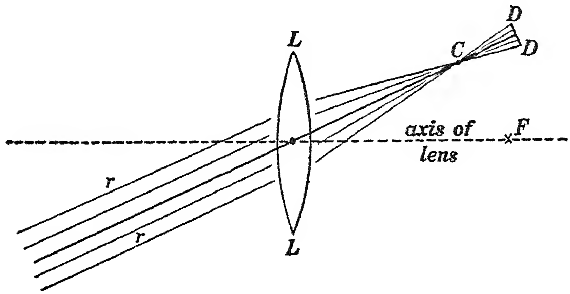


Fig. 101.

pencil of rays which is produced by the passage of the narrow pencil *rr* obliquely through the lens, as shown. The focal point of the lens is at *F*. This conversion of a homocentric pencil of rays into an astigmatic pencil by oblique passage through a lens is called *astigmatism*, and a lens system which is free or approximately free from astigmatism is called an *anastigmatic*, or simply a *stigmatic* system. The possibility of correcting a lens system for astigmatism was discovered by Rudolph about 1890.*

A very simple demonstration of the astigmatism of a simple lens may be made by looking obliquely through a magnifying glass at the cross-rulings on a sheet of cross-section paper. Under these conditions one or the other set of cross-rulings may be sharply focused according to the distance of the lens from the paper.

A more striking demonstration of astigmatism may be made by projecting a cross-ruled lantern slide upon the screen with the lantern objective turned so that the light passes through it

* A discussion of the elimination of astigmatism by the combination of simple lenses is beyond the scope of this text. A somewhat superficial but instructive discussion of this matter is given on pages 62–67 of Lummer's *Photographic Optics*, translated by S. P. Thompson, and a more elaborate discussion is given in Winkelmann's *Handbuch der Physik*, Vol. VI, pages 137–143, by Czapski.

obliquely as shown in Fig. 102, in which *AB* represents the lantern slide, and *LL* represents the object lens of the lantern. Light from a point *p* of the lantern slide is focused by *LL* along two lines, *C* and *DD*, the line *C* being perpendicular to the plane of the paper and the line *DD* being in the plane of the paper. If the focal line *C* is in the plane of the screen, then the horizontal lines of the lantern slide (lines which are per-

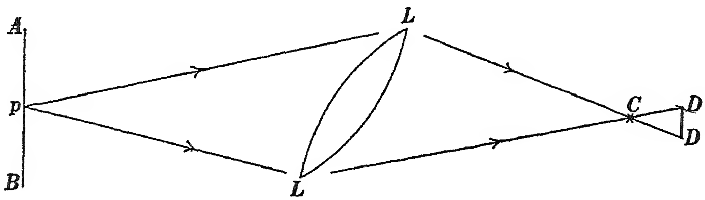


Fig. 102.

pendicular to the plane of the paper) are sharply focused on the screen. If the focal line *DD* is in the plane of the screen, then the vertical lines on the lantern slide are sharply focused on the screen. In this experiment, the central zone only of the objective lens should be used.

Astigmatism of the eye is due to unequal curvatures of the refracting surfaces of the eye in different directions so that a

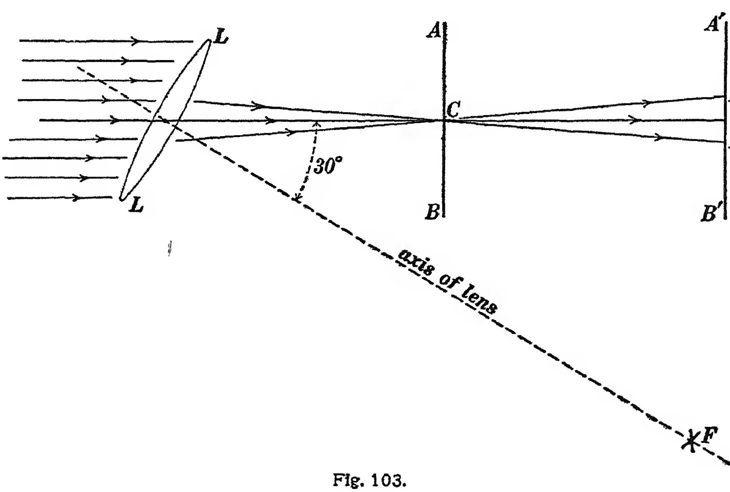


Fig. 103.

homocentric pencil of rays which enters the eye *axially* becomes an astigmatic pencil. This kind of astigmatism must not be confused with the astigmatism of a lens which converts an *oblique* homocentric pencil into an astigmatic pencil.

71. Oblique spherical aberration, or coma. — Figure 103 is a sketch, exactly one quarter size, showing a beam of parallel rays passing obliquely through a simple converging lens *LL*. Figure 104*a* is a photograph of the spot of light on the plate *AB* of

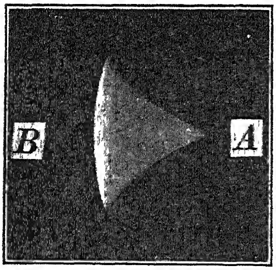


Fig. 104a.

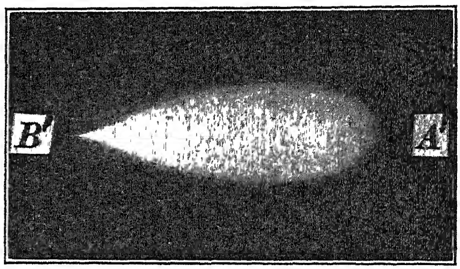


Fig. 104b.

Fig. 103, and Fig. 104*b* is a photograph of the spot of light on the plate *A'B'*, Fig. 103, the full opening of the lens *LL* being used. When the central zone only of the lens *LL* is used, the astigmatic pencil is focused along a horizontal line at *C* and along a vertical line at *DD* in Fig. 103, as shown by the two sketches in Fig. 105. In these sketches the outline of the coma is also shown for the sake of comparison.

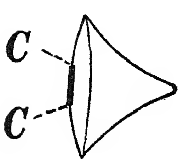


Fig. 105a.

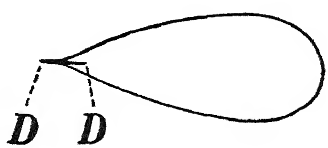


Fig. 105b.

72. Image distortion. — The image of an object formed by a lens is in general distorted. Thus, Fig. 106 represents a square network of lines, and Figs. 107 and 108 show two distorted

images of this network. In Fig. 107 the magnification of the image is greater near the edges of the field of view than it is near

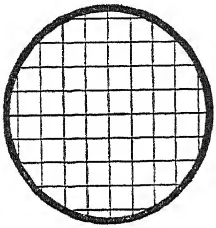


Fig. 106.

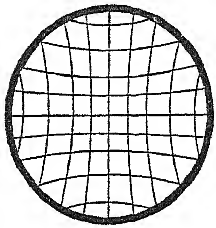


Fig. 107.

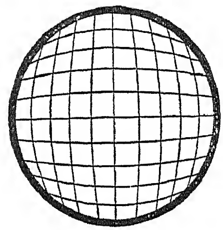


Fig. 108.

the center, and in Fig. 108 the magnification of the image is less near the edges of the field of view than it is near the center.

The distortion of an image by a lens may be shown in a very striking way as follows: A magic lantern is provided with a simple

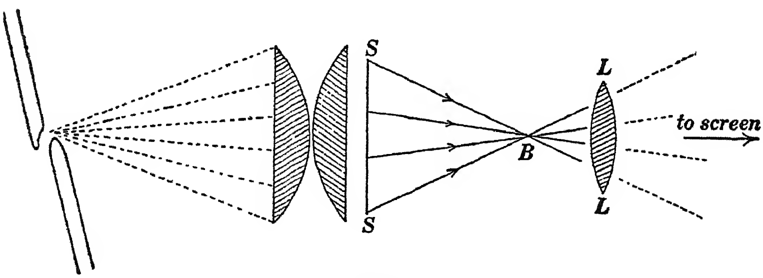


Fig. 109.

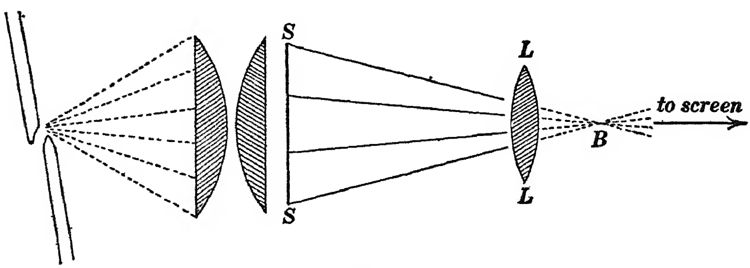


Fig. 110.

objective lens *LL*, Figs. 109 and 110, and an image of a cross-ruled lantern slide *SS* is projected upon the screen. The source of light in the lantern is adjusted so that the light, after passing

through SS, is concentrated at B, Fig. 109, *as if the point B were a small hole in a diaphragm in front of the object lens LL.* In this case the cross-rulings, as projected on the screen, appear like Fig. 108. The source of light in the lantern is then adjusted so that the light, after passing through SS is concentrated at the point B, Fig. 110, *as if the point B were a small hole in a diaphragm behind the object lens LL.* In this case the cross-rulings, as projected on the screen, appear like Fig. 107. These experiments show that the distortion of an image is of one kind or the other (like Fig. 107 or Fig. 108) according as the diaphragm, or stop, B is behind or in front of the lens, and one is able to see in a general way that one kind of distortion may be compensated by the other kind by placing the diaphragm between two lenses of a system, as shown in Fig. 111, in which the dia-

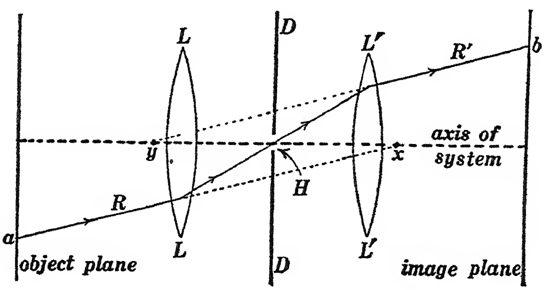


Fig. 111.

phragm, or stop, DD is behind one lens LL, and in front of the other lens $L'L'$. The action of the symmetrical double-lens of Fig. 111 may be understood from the following considerations. The hole H in the diaphragm is assumed to be small so that *all* incident rays like R which come from points in the object plane intersect at the point x which is conjugate to the point H with respect to the front lens LL, and *all* emergent rays like R' intersect at the point y which is conjugate to the point H with respect to the back lens $L'L'$. Furthermore every incident ray R is parallel to the corresponding emergent ray R'. Therefore the points like b in the image plane as seen from the point y

are distributed in exactly the same way as the corresponding points like a in the object plane as seen from the point x, or, in other words, the image is exactly similar to the object.*

A lens system which gives an undistorted image of an object is called a *rectilinear* or *orthoscopic* system. Such lenses are always used for photo-engraving, where it is desired to produce an accurate copy of a drawing, and for photographing buildings. See Art. 77.

73. Curvature of field. — In order to project upon a screen the most distinct image of an extended object which it is possible to form by means of a simple lens, the screen must be curved as shown by SS in Fig. 112. It is instructive to note the relation between Figs. 101 and 112. There is, between C and DD in

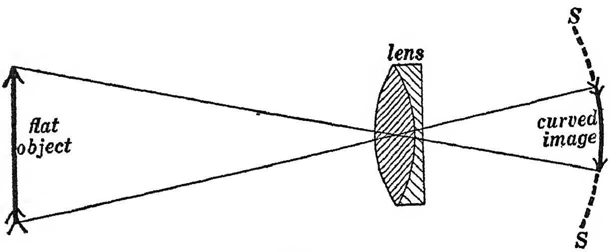


Fig. 112.

Fig. 101, a place where the astigmatic pencil would produce the smallest possible luminous spot, the circle of least confusion, so-called (see Art. 31), and the screen SS in Fig. 112 must pass between C and DD of Fig. 101.

The imperfection of a lens which is here described is called *curvature of field* and a lens system in which this error is corrected is said to have a *flat field*.†

* This discussion ignores the spherical aberration of the lenses LL and $L'L'$ in Fig. 111. The theory of the orthoscopic lens is quite fully discussed on pages 29–39 of Lummer's *Photographic Optics*, translated by S. P. Thompson.

† Curvature of field is intimately connected with astigmatism although von Seidel's theory gives two distinct conditions, one for the elimination of astigmatism and another for the elimination of curvature of field. See Winkelmann's *Handbuch der Physik*, Vol. VI, pages 139–143. Curvature of field is discussed on pages 10, 23, 57 and 61–67 of Lummer's *Photographic Optics*.

74. Chromatic aberration. — The five lens errors, axial spherical aberration, astigmatism, coma, image distortion and curvature of field, refer to the action of a lens when *light of one wave-length* (one color) is used. The use of white light introduces another complicated imperfection which is called *chromatic aberration* and which is due to the fact that a given sample of glass has different refractive indices for different wave-lengths (colors) of light. Thus, Fig. 113 shows the focal points v and r of a lens LL

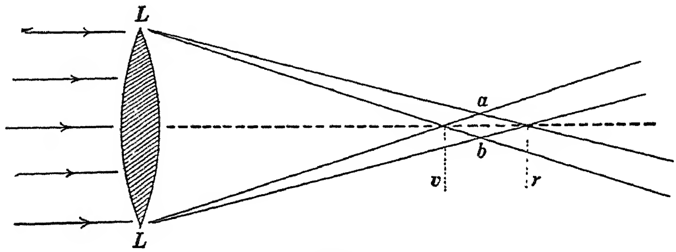


Fig. 113.

for violet light and for red light respectively; the focal points for the other colors lie between v and r in Fig. 113. The smallest diameter of the focal spot ab, Fig. 113, is about one thirty-third of the diameter of the lens.

The distance vr, Fig. 113, for a lens of given focal length varies greatly with different kinds of glass, and it is therefore possible to construct a converging lens of one kind of glass and a diverging lens of another kind of glass so that when the two lenses are combined and used as a lens system the chromatic aberration of one of the lenses is annulled (nearly) by the opposite chromatic aberration of the other, while the converging action of the one is not annulled by the diverging action of the other. A lens system (doublet) which is compensated for chromatic aberration in this way is called an *achromatic* doublet. The achromatic doublet was devised in 1758 by Dolland, who used such doublets for the object-glasses of his telescopes. A sketch of the theory of the achromatic doublet is given in the chapter on dispersion (Art. 86).

Some idea of the complexity * of chromatic aberration may be obtained by the following discussion of Figs. 114, 115 and 116. A lens system may be "achromatized" (*a*) so as to *form images of all colors* † *in one plane*, or (*b*) so as to *provide for equal sized images of all colors.*† In the first case the different colored images will be of different sizes, and in the second case the different colored images will be in different planes. The first is called the *achromatization of the focal plane* ‡ and the second is called the *achromatization of magnification.*§

Consider a converging lens *CC* and a diverging lens *DD* arranged as shown in Fig. 114. Let us for a moment ignore

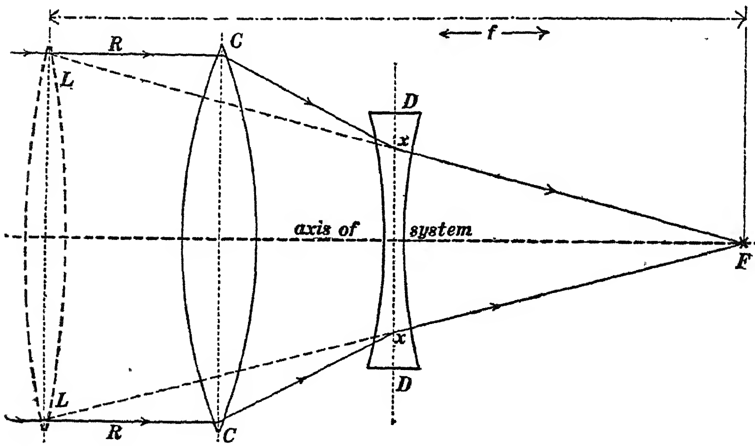


Fig. 114.

chromatic aberration and consider that the two lenses *CC* and *DD* focus a beam of parallel rays at the point *F*. Extend the two rays *Fx* backwards until they intersect the incident rays *RR* at *LL*. *The combined action of the two lenses CC and DD is equivalent to a single lens at LL of which the focal length is equal to f as shown.*

* This statement does not refer to the impossibility of compensating chromatic aberration for all colors. This matter is discussed in Art. 86.

† Strictly, two colors only. See Art. 86.

‡ Sometimes called *achromatization of the focal point.*

§ Sometimes called *achromatization of the focal length.*

Let CC, Fig. 115, represent a converging lens of crown glass and DD a diverging lens of flint glass, the two lenses being designed to bring both red light r and violet light v to a focus at the point F as shown. The two lenses CC and DD, in so far as their action on red light is concerned, are together equivalent to a single lens at A of which the focal length is f_r as shown; and the two lenses are equivalent to a single lens at B (focal

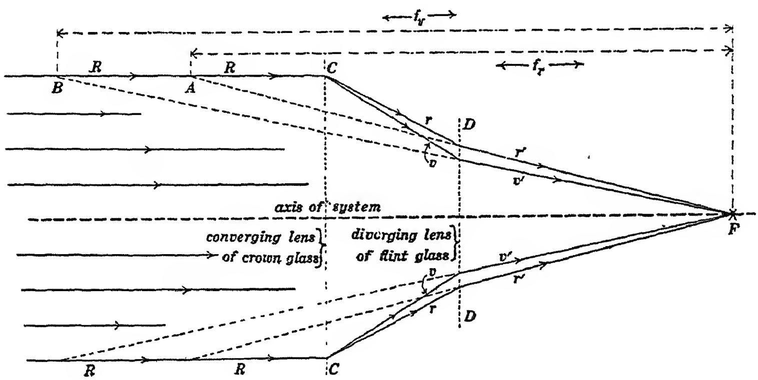


Fig. 115.

length f_v) in so far as their action on violet light is concerned. The lens system $CCDD$ of Fig. 115 produces red and violet images of a distant object in the same plane, namely, the plane containing the point F, but the violet image is larger than the red image in the ratio of f_v to f_r. The lens system $CCDD$, Fig. 115, is achromatized for focal point but not achromatized for focal length.

Figure 116 represents a combination of a converging crown glass lens and a diverging flint glass lens which is achromatized for focal length $(f_r = f_v)$. Such a system would give the same sized violet and red images of a distant object, but the images would not be in the same plane. In fact the red image would be in the plane containing F_r and the violet image would be in the plane containing F_v.

When the two lenses CC' and DD are very near together (near, that is, in comparison with their focal lengths), then

achromatization of the focal point carries with it the achromatization of focal length, as is evident from a careful study of Figs. 115 and 116. Complete* achromatization is therefore

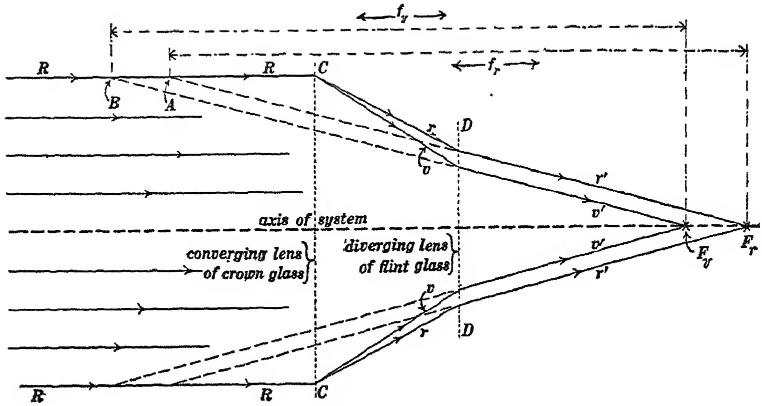


Fig. 116.

much more easily accomplished when the individual lenses of a system are near together than when they are far apart.

The two lenses of a telescope objective (see Figs. 120, 155 and 156) are very near together in comparison with their focal lengths and therefore the combination is achromatized for focal point and also for focal length.

In the modern high power microscope objective which is shown in Fig. 144 the front lens (next to the oil) is as near as possible to the lens next behind it, but its focal length is very short and therefore achromatization of the microscope objective for focal point does not carry with it the achromatization for focal length. See discussion of Fig. 144.

In photographic objectives it is usually desirable to separate the individual lenses as shown in Figs. 129–137, and in such cases *achromatic doublets* are used instead of simple individual lenses. Thus in the Petzval lens, Fig. 129, the front lens is a cemented

*Complete in the sense of including achromatization of focal point and achromatization of focal length, not in the senses of being achromatized for all colors of the spectrum. See Art. 86.

achromatic doublet and the back lens is an achromatic doublet the elements of which are only slightly separated.

A lens system which consists of two lenses of similar glass at a distance apart equal to half the sum of their individual focal lengths has *the same focal length for all wave-lengths* (all colors).* In this case, however, the lenses are at a great distance apart as compared with their focal lengths, and, although such combinations have the same focal length for all wave-lengths, they do not have the same focal point for all wave-lengths. Such a combination is therefore achromatized for focal length but not achromatized for the focal point. The doublets of Huygens and Ramsden which are so much used for eye-pieces for telescopes and microscopes are examples of partially achromatized systems of this type.

Chromatic differences of spherical aberration.—A lens system may be accurately aplanatic (with respect to its aplanatic points, of course) for a given wave-length (color) of light and non-aplanatic for other wave-lengths (colors). This error of a lens is called the chromatic difference of spherical aberration.

75. Wide-angle lenses versus wide-aperture lenses.—A lens cannot be made to give a wide field-angle and to have at the same time a large numerical aperture. The combination of these two things is impracticable. When a very wide field-angle is desired one must be content with small aperture, and when a very wide aperture is desired one must be content with small field-angle. Thus, a high-grade microscope objective with a numerical aperture of about 1.4 has a field-angle of not more than half a degree, and a wide-angle photographic lens having a field angle of 110 degrees has a numerical aperture of about $\frac{1}{36}$.

There is a demand in photography, however, for lenses having a moderately wide aperture and giving a moderately wide field, and many photographic lenses are now on the market which give fairly good definition over a field from 30 to 60 degrees wide with numerical aperture ranging from $\frac{1}{4}$ to $\frac{1}{8}$.

* See Appendix A.

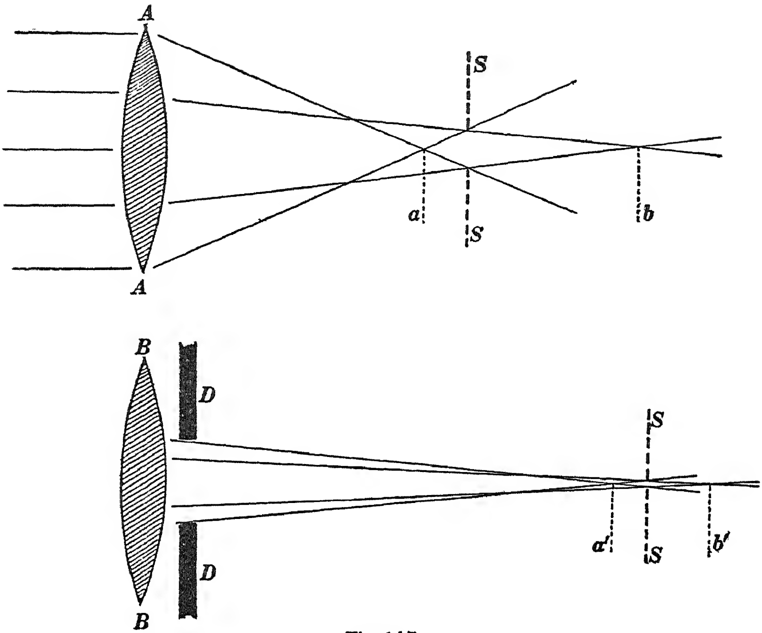


Fig. 117.

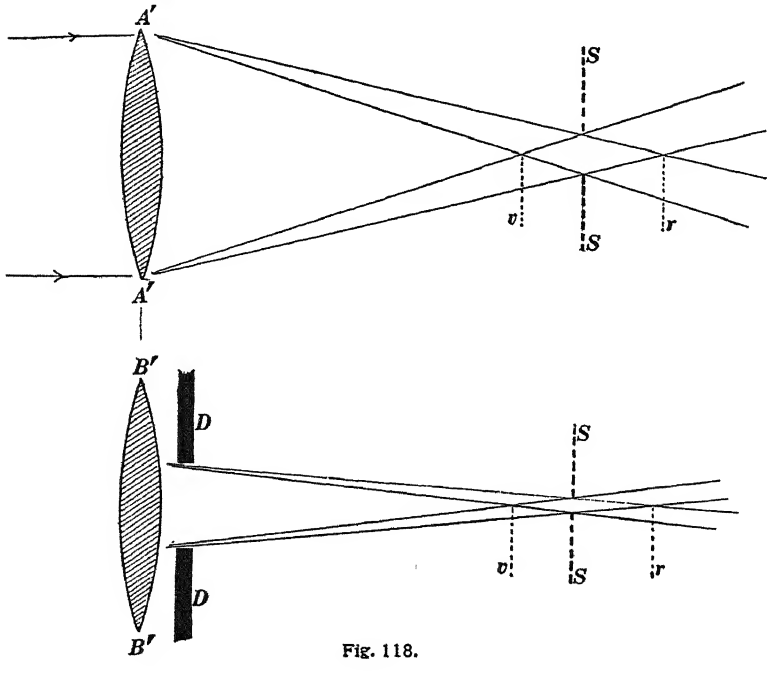


Fig. 118.

A wide-aperture lens must be, above all things, compensated for spherical aberration and for chromatic aberration. — The effect of spherical aberration in producing very great blurring at the focus of a large-aperture lens as compared with the blurring at the focus of a small-aperture lens is shown in Fig. 117. The same effect for chromatic aberration is shown in Fig. 118. In these figures SS represents the position of the screen for which the focal spot is the smallest possible. The distance ab in Fig. 117 is roughly proportional to the square of the aperture of the lens so that $a'b'$ in Fig. 117 is very much less than ab, and the size of the focal spot is extremely small for lens B as compared with its size for lens A. The distance rv in Fig. 118 is independent of the aperture of the lens, but the figure shows nevertheless that a beam of light is focused in a much smaller spot by lens B' than by lens A'.

A wide-angle lens must be, above all things, compensated for astigmatism, distortion, and curvature of field.

76. The wide-angle lens. — A lens in the form of a complete sphere having a diaphragm DD with a small hole at its center, as shown in Fig. 119, has a field angle of nearly 180°, but the field is very strongly curved, as shown. The wide-angle photographic objective involves the principle of the spherical lens, modified more or less to give flatness of field. This is exemplified by Fig. 130 which is a moderately wide-angle photographic objective and by Fig. 138 which is an extremely wide-angle photographic objective.

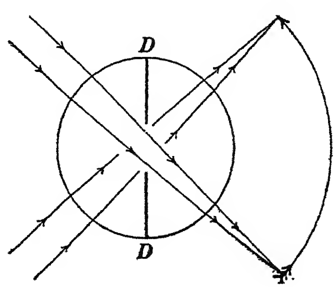


Fig. 119.

77. Examples of compensated lens systems. (*a*) *Telescope objectives.* — The telescope objective is generally used where a *small field angle* is desired, and the ordinary telescope objective is compenseated only for spherical aberration and for chromatic

aberration. The achromatization is accomplished by using a converging lens of crown glass and a diverging lens of flint glass, as explained in Art. 86. The condition which must be satisfied to produce achromatization leaves some freedom of choice as to the relative curvatures of the different surfaces of the two lenses, and these curvatures are chosen so that the spherical aberration of the converging crown lens may be compensated by the opposite spherical aberration of the diverging flint lens.* Figure 120 shows a sectional view, actual size, of an aplanatic achromatic telescope object-lens of 18 inches focal length as designed by Fraunhofer. It consists of a double convex crown glass lens *CC* and a concavo-convex flint glass lens *FF*. Figure 121 is a sectional view, actual size, of a modern high grade opera-glass of which the object-lens *O* and the eye-lens *E* are triplets. The object-lens of an opera-glass is much larger in numerical aperture (larger in diameter for a given focal length) than the object-lens of the astronomical telescope

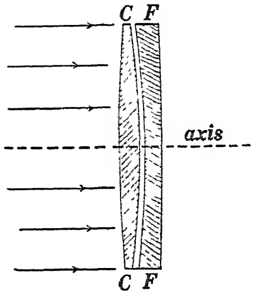


Fig. 120.

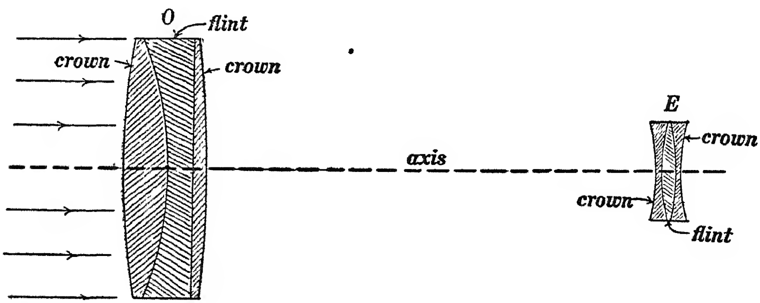


Fig. 121.

as usually constructed. It is for this reason desirable to use three lenses for the object-lens of an opera-glass, as shown in Fig. 121, in order that the various imperfections may be more completely

* See footnote on page 99.

compensated than would be possible by the use of two lenses, as shown in Fig. 120.

(*b*) *Magnifying glasses and eye-pieces.** — Magnifying glasses and eye-pieces are always used under conditions which involve *small numerical aperture* because of the delimiting action of the pupil of the observer's eye. Thus, Fig. 122 represents a very narrow pencil of rays passing from a point *a* of an object, through a magnifying glass, and through the pupil *p* of the observer's eye. The conditions of use of magnifying glasses and eye-pieces involve, however, comparatively wide field angles, in some cases as great as 50° or more.

Simple lenses are extensively used for magnifying glasses. When so used, the center of the field of view is usually quite

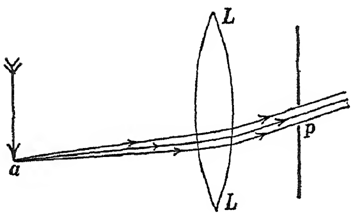


Fig. 122.

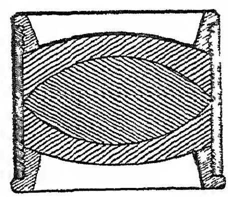


Fig. 123.

clearly defined but the edges of the field of view are more or less blurred. To illustrate, let the reader look at a printed page through an ordinary magnifying glass. In the center of the field of view the letters appear sharp and distinct but near the edges of the field of view the letters are more or less blurred and show fringes of color (chromatic aberration).

Figure 123 shows an achromatic form of a magnifying glass due originally to Brewster. It consists of a converging lens of crown glass between two diverging lenses of flint glass.

Figure 124 shows an eye-piece doublet which was designed by Ramsden in 1783. It consists of two similar plano-convex lenses of similar glass placed slightly nearer together than half

* A simple discussion of eye-pieces is to be found in Edser's *Light for Students*, pages 204–212.

the sum of their focal lengths. It is partially achromatic * and it has a fairly wide, flat field. It is extensively used with microscopes and telescopes at the present day, especially where it is

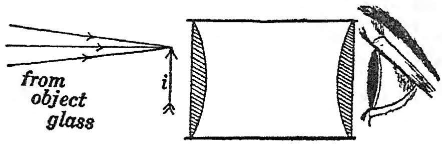


Fig. 124.

desired to use cross hairs in the focal plane of the object glass. The arrow *i* in Fig. 124 shows the position of the image formed by the object-glass of the telescope or microscope.

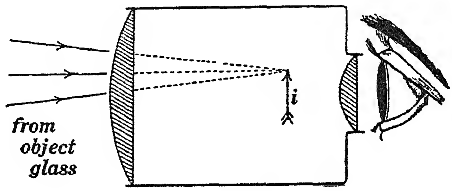


Fig. 125.

Figure 125 shows an eye-piece doublet which was designed by Huygens about 1680. It consists of two plano-convex lenses of similar glass of which the focal lengths are as three to one, placed

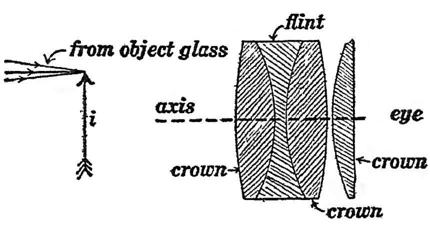


Fig. 126.

at a distance apart equal to half the sum of their focal lengths. It is partially achromatic * and it has a very wide flat field. It is extensively used with microscopes and telescopes at the present

* Achromatized for focal length but not for focal point. See Art. 74 and see Appendix A.

day, especially where it is not desired to use cross hairs in the focal plane of the object-glass. The arrow *i* in Fig. 125 shows the position of the image formed by the object-glass of the microscope or telescope.

Figure 126 shows the arrangement of lenses in a highly perfected, orthoscopic eye-piece designed by Abbe and manufactured by Carl Zeiss.

(*c*) *Photographic lenses.** — The first step in the development of the modern high-grade photographic lens was taken by Wollaston in 1812 who substituted the meniscus lens (No. 4 in the

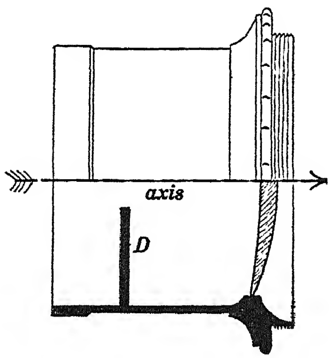


Fig. 127.

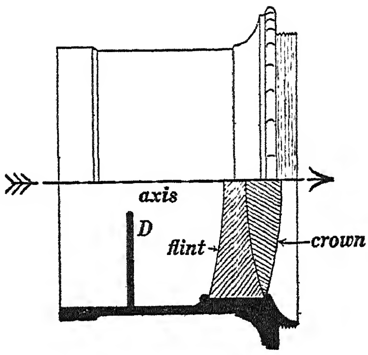


Fig. 128.

table on page 99) for the simple bi-convex lens which was used in the camera obscura † before his time, and he specified a particular position for the diaphragm or stop. The simple meniscus lens is extensively used at thé present day in cheap forms of photographic cameras, and it is arranged as shown in Fig. 127.

The achromatic doublet came into use for the camera obscura

* A good outline of photographic optics is given by Lummer in the *Zeitschrift für Instrumentenkunde*, 1897, pages 208, 225 and 264. These articles have been translated into English by S. P. Thompson and published as *Photographic Optics* by Macmillan & Co., 1900. A very complete discussion of the theory and history of the photographic objective is von Rohr's *Theorie und Geschichte des Photographischen Objectivs*, Berlin, 1899. See also *Photography for Students* by Louis Derr, The Macmillan Company, 1906.

† The camera obscura is a dark chamber, or box, with a lens at one side for projecting an image of an external object or landscape.

about 1835. This lens has been and is still extensively used for photographic purposes. Figure 128 shows the usual form of achromatic doublet when used as a photographic lens.

The demand for a quick-acting lens (wide numerical aperture) which came with the invention of photography was met by the

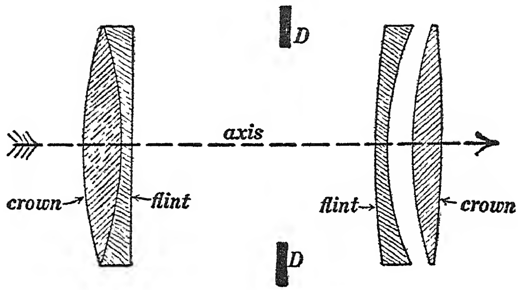


Fig. 129.

remarkable portrait lens of J. Petzval which was designed in 1840 and manufactured by Voigtländer. Figure 129 shows a sectional view, full size, of a Petzval portrait objective having a focal length of 10 centimeters. This lens has a numerical aperture of 1/3.5 and it is accurately aplanatic. It gives extremely good definition in the center of its field and fairly good definition over a field of about 25 or 30 degrees. This objective and the portrait objective of J. H. Dallmeyer, which was brought out in 1866, are still extensively used.

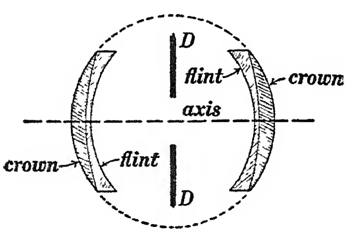


Fig. 130.

After the remarkable achievement of Petzval, the next notable improvement in the photographic lens was made in England, and the first symmetrical orthoscopic lens was the "Globe lens" of Harrison and Schnitzer, which was brought out about 1860. Figure 130 is a sectional view, full size, of a "Globe lens" of 10 centimeters focal length. In 1866 the symmetrical aplanatic orthoscopic lens of Steinheil was produced. A sectional view,

full size, of one of these lenses of 20 centimeters focal length, is shown in Fig. 131.

All converging lenses in Figs. 128 to 131 are of crown glass and all diverging lenses are of flint glass. The compensation of

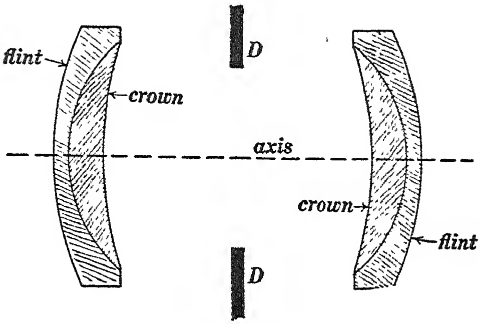


Fig. 131.

lens errors depends upon the combination of lenses made of different kinds of glass, and the development of photographic lenses (and also of microscope objectives) was greatly stimulated after

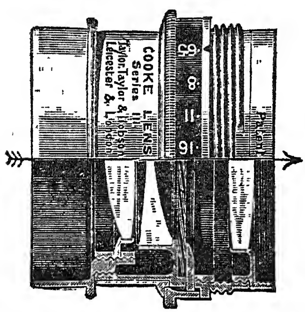


Fig. 132.—"Cooke Lens." Taylor, Taylor & Hobson. Patented 1893 by H. D. Taylor.

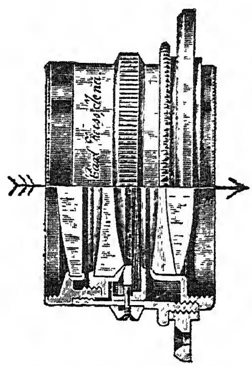


Fig. 133.—"Tessar Lens." Carl Zeiss. Patented 1902 by P. Rudolph

the establishment of the celebrated Jena Glass Works in 1885.* Figures 132, 133 and 134 show three wide-aperture (about 1/4.5), non-symmetrical lenses, all of which are approximately

* These glass works were established for the manufacture of a variety of new optical glasses which had been developed under the direction of Abbe.

orthoscopic. They are very completely corrected for axial and oblique spherical aberration and for chromatic aberration, and they give a fairly flat field with but little astigmatism.

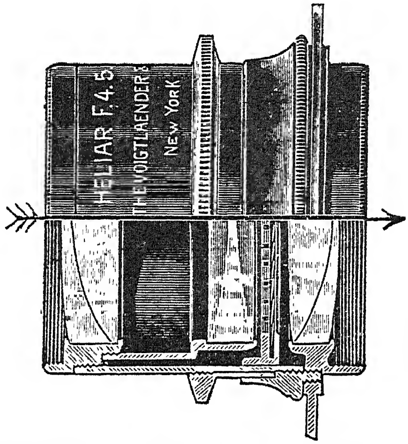


Fig. 134.—"Heliar Lens" Voigtländer & Son. Patented 1900 by H. Harting.

Figures 135 and 136 show two symmetrical double-objectives which are quite accurately orthoscopic but not so fully corrected

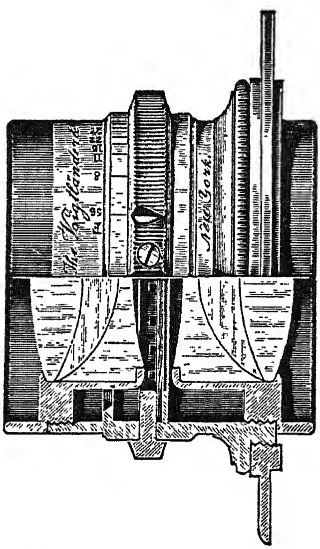


Fig. 135.—"Collinear Lens." Voigtländer & Son. Patented 1893 by R. Steinheil.

for other errors as the non-symmetrical lenses shown in Figs. 132, 133 and 134. The symmetrical double-objective is the most generally used photographic lens, and the cheaper forms are still made like Steinheil's "Aplanat" which is shown in Fig. 131.

Figure 137 shows a moderately wide angle (110° field angle) lens, and Fig. 138 shows an extremely wide-angle (135° field angle) lens. A very wide-angle lens must have a very narrow aperture, in which case chromatic and spherical aberrations are relatively unimportant. Thus, the wide-angle lens which is

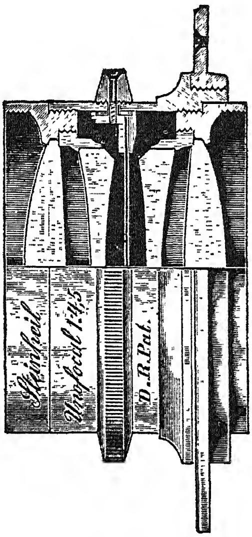


Fig. 136.—"Unofocal Lens" C. A. Steinheil & Sons. Patented 1903 by C. A. Steinheil.

Fig. 137.—"Orthostigmat Lens." C. A. Steinheil & Sons. Patented 1893 by R. Steinheil.

shown in Fig. 138 is orthoscopic and approximately anastigmatic, and it gives a flat field, but it is not corrected for spherical aberration nor for chromatic aberration.

Figure 139 shows a single-group photographic objective suitable for apertures up to 1/12.

For making photographs of very distant objects, a very long focal length lens is necessary if a fair size photograph is to be produced. In order to avoid the great inconvenience which would be involved in the use of a very long focal length lens of

the ordinary type, a combination called the *telephotographic lens* is used. Figure 140 shows a telephotographic lens. The equivalent focal length of this lens may be varied by changing the distance between the front and back combinations, and one may easily obtain an equivalent focal length of 8 or 10 feet with a camera length of 2 or 3 feet.

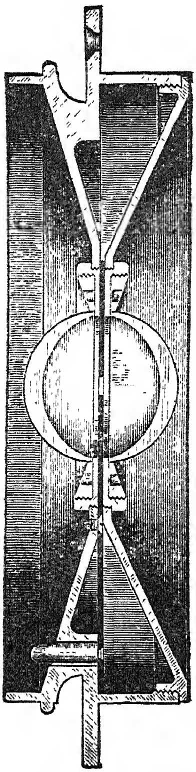


Fig. 138.—"Hypergon Lens" C. P. Goerz Optical Works. Patented 1900 by C. P. Goerz.

The action of the telephotographic combination may be understood with the help of Figs. 141 and 142 as follows: Figure 141 represents the object-glass O and the eye-lens E of an ordinary telescope. The eye-lens E has been drawn out (away from the object-glass) until the image i is beyond the focal point F of the eye-lens, as shown in the figure (the focal point F moves with E). The lens E then forms a greatly enlarged real image at I. The astronomical telescope is frequently used in this way to project a large image of the sun or moon upon a screen. Figure 142 shows a telescope of the opera-glass type in which the diverging eye-lens E has been drawn out (away from the object-glass) until its focal point F is beyond the image i, as shown. The lens E then forms a greatly enlarged real image at I. A comparison of Figs. 142 and 140 will serve to elucidate Fig. 140. The great focal length of the telephotographic lens may also be understood with the help of Fig. 114.

(*d*) *Microscope objectives.*—Low-power microscope objectives usually consist of one achromatic doublet. Figure 143 shows a low-power microscope objective consisting of two achromatic doublets combined to form one system. *In high power microscope objectives, however, the front lens of the object-glass (the lens*

nearest to the object) is always made in the form of a hemisphere, and in the most powerful microscope objectives the space between the front lens and the object which is being examined is filled with oil (the oil having the same index of refraction as the front lens of the object-glass) as shown in Fig. 144. A microscope objective when so used is called an *oil immersion objective.* The one case in which refraction at a spherical surface is entirely free * from spherical aberration is thus realized in the front lens of the oil immersion objective as explained in Art. 44.

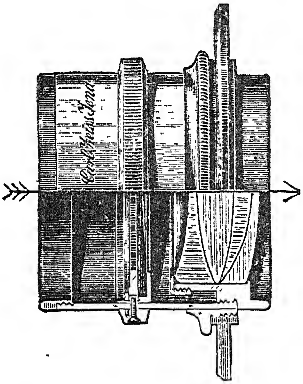


Fig. 139.—"Anastigmat Lens" Carl Zeiss. Patented 1894 by P. Rudolph.

Figure 144 shows (three times actual size) a highly perfected microscope objective of 2 millimeters (one twelfth inch) focal length. This objective was designed about 1886 by Abbe and it is known as the *apochromatic objective.* It has the largest possible numerical aperture

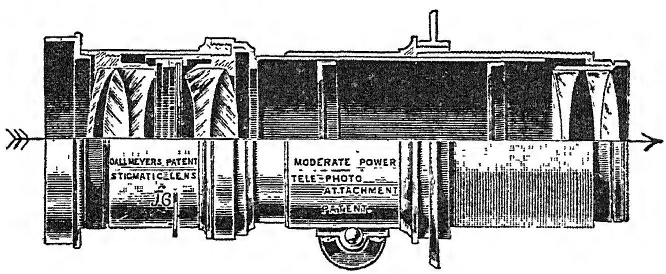


Fig. 140.—"Telephoto Lens." J. H. Dallmeyer, Limited. Patented about 1891 by T. R. Dallmeyer.

(for the sake of resolving power), and, although it is corrected only for spherical aberration and for chromatic aberration, it involves eleven separate compensating effects. It is achromatized for focal point for three colors, and its spherical aberration is eliminated for each of two colors for oblique rays

* For one color.

as well as for rays parallel to the axis. The lack of achromatization of focal length causes the objective to form colored images

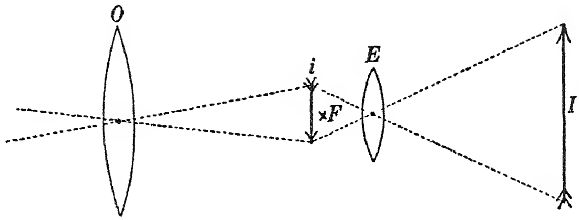


Fig. 141.

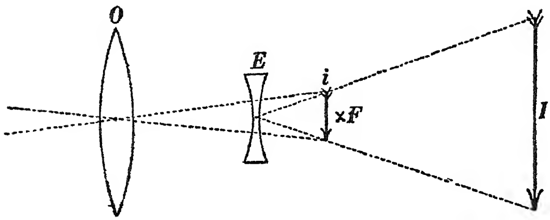


Fig. 142.

of different sizes, and this imperfection is compensated by using a specially designed non-achromatic eye-piece which magnifies the

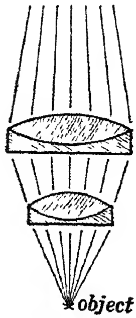


Fig. 143.

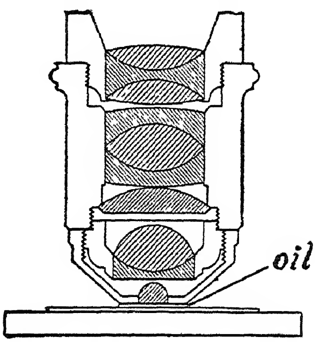


Fig. 144.

different colored images differently thus giving a single colorless resultant image. The excellence of this objective is such that the limit of possible resolving power may be considered as actually attained by it.

CHAPTER VII.

DISPERSION. SPECTRUM ANALYSIS.*

78. Newton's experiment.† Homogeneous light. Non-homogeneous light. — A beam of parallel rays of white light, such as sun light or lamp light, is changed into a fan-like beam by passage through a prism. Thus, the beam of parallel rays B, Fig. 145, is changed into the fan-like beam B' by passage through the prism P. This fan-like beam in falling upon a screen SS produces an illuminated band RV called a *spectrum* which is red at the end R and passes by insensible gradations through orange, yellow, green, and blue to violet at the end V. The beam B of white light is said to be *dispersed* by the prism. A photographic plate reveals the existence of invisible rays beyond V. These rays are called the *ultra-violet* rays. A thermopile or bolometer (see Appendix B)

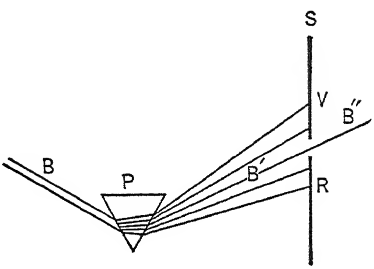


Fig. 145.

*A good discussion of dispersion is given in Edser's *Light for Students*, pages 375–387. See also Wood's *Physical Optics*, pages 85–99 and 308–348. The modern theory of dispersion is given in Drude's *Theory of Optics* (translated by Mann and Millikan), pages 382–399.

The *rainbow* is produced by the dispersion of sun light in drops of rain. Other interesting optical phenomena of the atmosphere are *mirage*, *coronas and halos*, *scintillation* and the *color of the sky*. These various matters are discussed in Hastings' *Light*, pages 111–153 (Scribner's, 1901); Preston's *Theory of Light*, pages 529–541 (Macmillan & Company, 1901); Edser's *Light for Students*, pages 101–107 (Macmillan & Company, 1902); and Wood's *Physical Optics* (The Macmillan Co., 1905), pages 69–78. Wood's discussion of mirage is especially interesting. The sharp-edged appearance of the sun is a mirage effect. See A. Schmidt, *Physikalische Zeitschrift*, Vol. IV, pages 282–285, February, 1903.

† An interesting account of Newton's original experiments with the prism may be found in Preston's *Theory of Light*, pages 117–122.

shows the existence of rays below R. These rays are called *infra-red* rays. The portion of the spectrum between R and V is called the *visible spectrum.*

The beam of white light B, Fig. 145, is deflected by the prism and also spread out or dispersed. On the other hand, the beam of light B'', Fig. 145, which passes through a small hole in the screen, can be deflected by a prism but it cannot be spread out or dispersed. The beam of white light is evidently made up of dissimilar parts because these parts are unequally deflected by a prism. Therefore, white light is called *non-homogeneous* light. The beam B'', Fig. 145, on the other hand, consists of but one kind of light because it is deflected by the prism without being dispersed. Therefore, the beam B'' is called *homogeneous* light.* Since the prism P deflects the different parts of the non-homogeneous beam B differently, it is obvious that the glass of which the prism is made has a different index of refraction for each of the homogeneous parts or components of white light.

The phenomena of interference which are described in the next chapter show that *a beam of homogeneous or monochromatic light is a simple wave-train of definite wave-length* or, more strictly speaking, *a simple wave-train of definite frequency*, inasmuch as the wave-length is halved, for example, when a given wave-train passes into a medium in which the velocity is halved. *A beam of white light, on the other hand, is an utterly irregular succession of wave-pulses and short sections of wave-trains of every variety of wave-length.* In a vacuum, all these different waves travel at the same velocity, and this is approximately true in air also. In substances like glass, wave-trains of different wave-lengths have distinctly different velocities and therefore the different wave-trains which make up a beam of non-homogeneous light are differently refracted by a glass prism.†

* Homogeneous light is sometimes called *monochromatic light*, or light of one color.

† The action of a prism in resolving a beam of white light into a series of simple wave-trains is a phenomenon of resonance. See Drude's *Theory of Light* (translated by Mann & Millikan), pages 382–399.

The velocity of ordinary water waves, especially of the variety known as ripples, varies greatly with the wave-length, and a phenomenon which is closely analogous to the dispersion of light by a prism (due to the different velocities of the wave-trains of light of different wave-lengths) is the following: An oar is dipped gently into the smooth surface of a pond and the irregular wave which is produced by the oar is quickly resolved into a series of fine ripples, as shown at *A*, Fig. 146. The fine ripples of short wave-length travel faster than the coarse ripples of long wave-length and are thus separated from them, as shown.

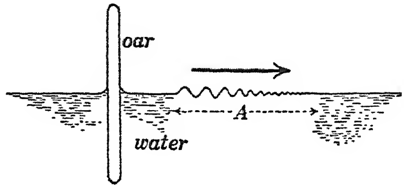


Fig. 146.

79. The spectroscope. — In the spectrum as obtained by Newton (see Fig. 145), the beam of white light has some breadth and the various beams of homogeneous light into which the white light is resolved by the prism are each as wide as the original beam of white light. Therefore the various beams of homogeneous light overlap each other greatly, that is to say, each point on the screen in Fig. 145 is illuminated by several over-lapping beams of homogeneous light. The *spectroscope* is an instrument for separating as completely as may be the homogeneous components of a beam of non-homogeneous light.

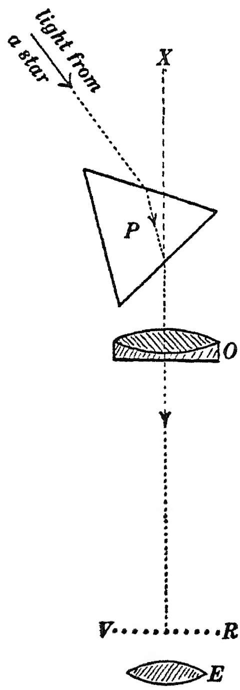


Fig. 147.

The spectroscope is exemplified in its simplest form by placing a large prism *P* in front of the object-glass *O* of a telescope *OE*, as shown in Fig. 147; the light from a star is then deflected by the prism

and appears to have come from *X*. In fact, the different wave-lengths of light from the star are differently deflected and the light appears to have come from a number of stars near *X*, one star for each wave-length of light. The result is that the object-glass *O* of the telescope forms a row of images of the star at *RV*, one image for each wave-length. This row of images constitutes the *spectrum* of the star and it may be examined by the magnifying glass (eye-piece) *E*.

If the attempt is made to use the arrangement shown in Fig. 147 for looking at a point-source of light near at hand instead of looking at a star, then the images at *VR* will not be sharply defined because the light from the near source *S* will enter the prism *P* as a series of spherical waves, as shown in Fig. 148; these spherical waves are refracted at the plane surface of the prism *subject to spherical aberration* as explained in Art. 43 and as represented in Figs. 52 and 53; and the result is that the waves which emerge from the prism *P* are non-spherical and they cannot be sharply focused by the lens *O*. In order to overcome this difficulty a lens is placed between the point-source *S* and the prism *P* in Fig. 148, the point-source *S* being at the focal point of the lens. With this arrangement, which is shown in Fig. 149, the beam of divergent rays from *S* is converted into a beam of parallel rays by the lens (that is, the spherical waves from *S* are converted into plane waves by the lens) and this beam of parallel rays (plane waves) is refracted by the prism without spherical aberration.

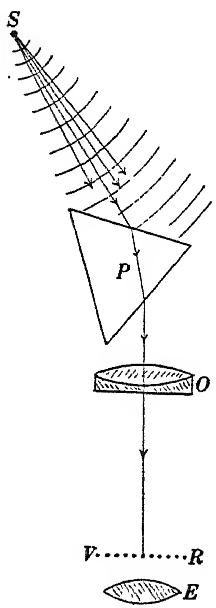


Fig. 148.

The light which is to be analyzed passes through a very narrow slit *S*, Fig. 149, between two straight metal edges. This slit is then, in effect, the source of the light, and it is at

the focal point of an achromatic lens C, which is called the *collimating lens* of the spectroscope. After passing through the collimating lens, the light passes through the prism P and

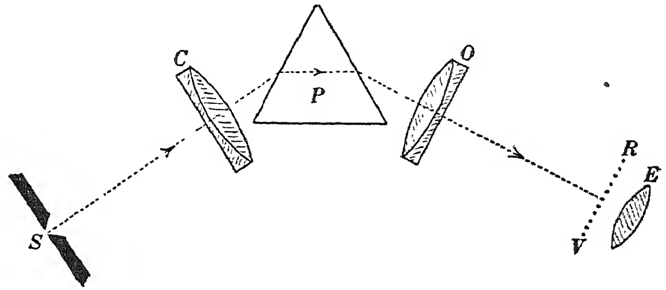


Fig. 149.

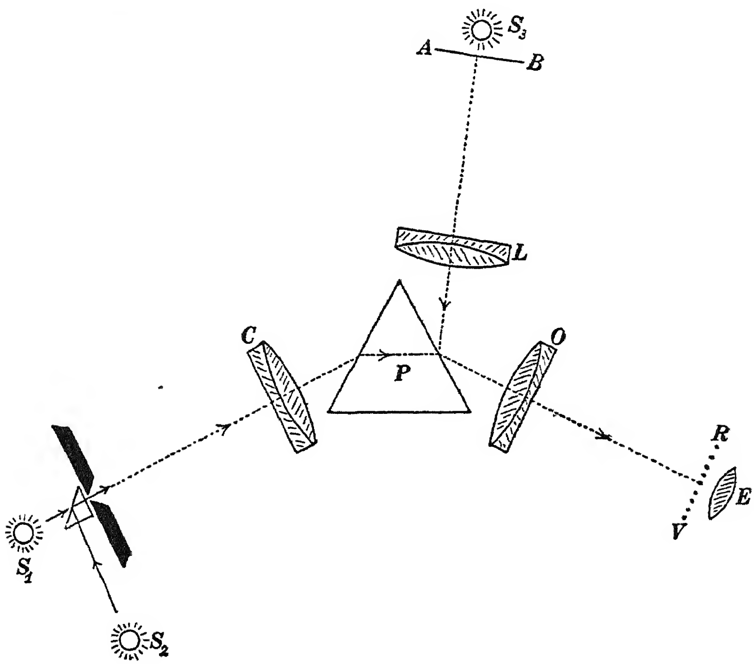


Fig. 150.

is then focused by the lens O at a series of points (images of the slit) at RV. These images of the slit are examined by the magnifying glass (eye-piece) E. The band of images at RV

is called the *spectrum* and the individual images of the slit are called the *lines of the spectrum.* The slit S and the lens C are mounted at the ends of a short tube which is called the *collimator*, and the lens O and the eye-piece E are mounted at the ends of a tube which is called the *telescope.*

The usual arrangement of the spectroscope is shown in Fig. 150 in which two independent sources S_1 and S_2 are arranged to send light through the slit by covering one end of the slit with a total reflecting prism so that light from S_1 passes through the upper end of the slit and light from S_2 passes through the

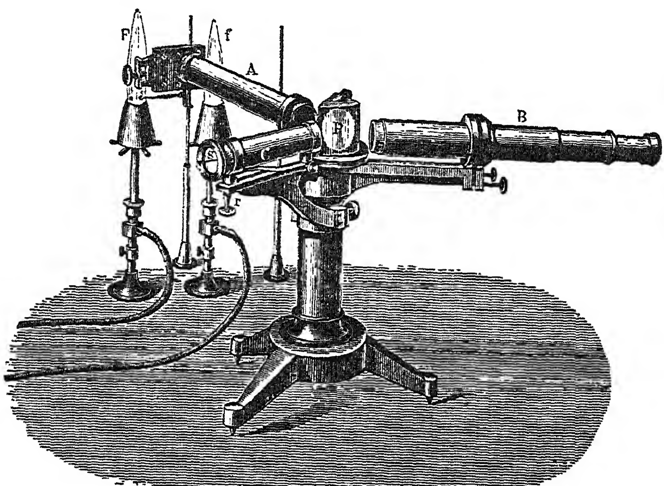


Fig. 151.

lower end of the slit. The two spectra (the spectrum of S_1 and the spectrum of S_2) are then seen side by side in the spectroscope and they may be compared with great ease. The total reflecting prism is often called the *comparison prism.*

In order to be able to read off the positions of the images of the slit at RV, a lamp S_3 is arranged to send light through a transparent scale AB which is at the focal point of the lens L. The light which passes through the scale AB is reflected from the face of the prism into the telescope lens O, and an image of

the scale AB is formed in the plane RV so that the positions of the images of the slit may be read off. A general view of a spectroscope is shown in Fig. 151.

80. Continuous spectra. — When light from a hot solid or liquid is analyzed by a spectroscope, a continuous band of images of the slit is produced at RV, Fig. 149. Such a spectrum is called a *continuous spectrum.* Light which gives a continuous spectrum contains wave-trains of every gradation of wave-length. The candle flame, the petroleum flame, and the gas flame give continuous spectra. The light from such flames is given off by hot particles of solid carbon.

81. Bright-line spectra. — When the light which is emitted by a hot vapor or gas is analyzed by a spectroscope, a group of distinctly separate images of the slit is produced at RV, Fig. 149. Such a spectrum is called a *bright-line spectrum* inasmuch as the separate images of the slit appear as bright lines. Light which gives a bright-line spectrum contains wave-trains of certain definite wave-lengths only.

Every gas or vapor has a characteristic spectrum, that is, a characteristic grouping of images of the slit in a spectroscope.

82. Dark-line spectra. — When an intense beam of white light, containing all wave-lengths, is passed through a relatively cool vapor or gas and then analyzed by the spectroscope, dark lines (missing images of the slit) are seen where bright lines would be located in the direct spectrum of the vapor or gas. That is to say, a relatively cool vapor absorbs those particular wave-trains which it would itself give off if hot. This relation between the bright-line spectrum of a vapor or gas and the dark-line spectrum of the vapor or gas was discovered by Bunsen and Kirchoff in 1865. Using the flame of a Bunsen burner and charging it with sodium vapor by the vaporization of common salt, they obtained the ordinary bright-line spectrum of sodium. Then passing an intense beam of white light from a lime-light through the Bunsen flame into the slit, it was found that the absorption of the

sodium vapor was such as to leave relatively dark lines in place of the bright lines given by the flame alone. This experiment has become classical under the name of *the reversal of the sodium lines*.

The most striking dark-line spectrum is the solar spectrum, which shows a great number of dark lines. The dark lines in the solar spectrum were first studied by Fraunhofer in 1819 and they are called *Fraunhofer's lines*. The more prominent of these lines are designated by the letters B, C, D, E, b, F, G, H_1 and H_2, in order, beginning at the red end of the spectrum as shown in Fig. 152, which shows the visible portion of the solar

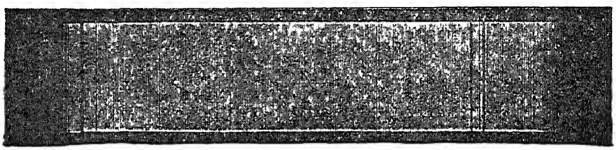


Fig. 152.

spectrum as it appears through a small spectroscope. Fraunhofer's A line is at the extreme red end of the spectrum and it is visible only when intense direct sunlight is passed through the slit.

Groups of dark lines in the solar spectrum are found to coincide with groups of bright lines given by iron vapor, sodium vapor, hydrogen and other elements, which is an indication that relatively cool vapors of these substances exist in the gases which surround the sun. Certain dark lines in the solar spectrum vary in intensity with the altitude of the sun above the horizon. These lines are due to the absorption of the earth's atmosphere.

Many stars give dark-line spectra like the solar spectrum, from which we infer that their chemical and physical constitution is like that of our sun, which is a comparatively dense and extremely hot radiating body surrounded by relatively cool vapors.*

* A very good simple discussion of spectrum analysis and its teachings is to be found on pages 330–360 of Edser's *Light for Students*, Macmillan, 1902. One of

83. **The spectrometer** is a spectroscope which is provided with a divided circle by means of which the position of the axis of the telescope *OE* in Fig. 149 may be read off. The center of the circle is beneath the prism *P*, Fig. 149, the telescope *OE* is carried on an arm which is pivoted at the center of the circle, and cross-wires are stretched across the focal plane *RV*.

Determination of index of refraction. — An important use of the spectrometer is the determination of indices of refraction. The glass to be studied is made into a prism of which the angle ϕ, Fig. 49, is measured, and the deflection (minimum) α, Fig. 49, is determined by means of the spectrometer; whence the index of refraction of the glass for the wave-length of the light used, may be calculated in Art. 41.*

84. The spectrophotometer. — This instrument is a spectroscope arranged for determining the composition of light quantitatively, that is, for determining the relative intensities of the various homogeneous components or wave-lengths of a given light. The measurements consist in comparing the intensity of each part of the spectrum of a given source of light with the intensity at the same part of the spectrum of a constant light source which has been selected as a standard. A simple form of spectrophotometer is described in the chapter on Photometry.

85. The direct-vision spectroscope. — Consider a prism of flint glass and a prism of crown glass which give the same deflection for the middle of the spectrum (by deflection is meant the angle

the best small works in English on spectrum analysis is *Spectroscopy* by E. C. C. Baly, Longmans, Green & Co. See also Landauer's *Spectrum Analysis* translated by J. Bishop Tingle, John Wiley & Sons. A very interesting discussion of the application of the interferometer to spectroscopy is given on pages 60–83 of Michelson's *Light Waves and their Uses*, University of Chicago Press, 1903. The interesting effect of magnetic field upon the spectrum of a hot gas (the Zeeman effect) is discussed by Michelson on pages 107–126. See also a brief note in Franklin and MacNutt's *Elements of Electricity and Magnetism*, page 319, The Macmillan Co.

The most complete work is Kayser's *Spectralanalyse*, and perhaps the best resumé is Kayser's chapter (pages 654–784 of volume VI) in Winkelmann's *Handbuch der Physik*.

* See *Practical Physics*, Franklin, Crawford and MacNutt, Vol. III, pages 36–38.

α in Fig. 49). Then the flint-glass prism gives a much longer spectrum than the crown-glass prism as shown in Fig. 153, and the two prisms, if arranged as shown in Fig. 154*a*, disperse a beam

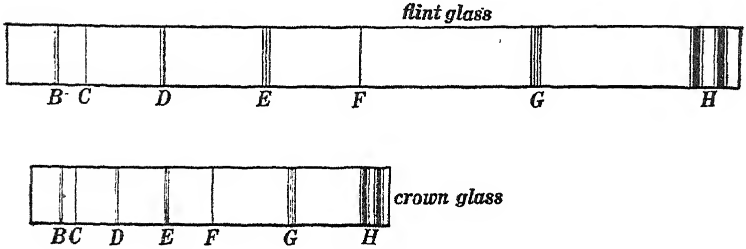


Fig. 153.

of white light without deflecting the middle portion of the spectrum.

The ordinary spectroscope would be an awkward instrument to use if one were to attempt to hold it in the hands and keep the *collimator* *CS*, Fig. 149, directed towards a source of light

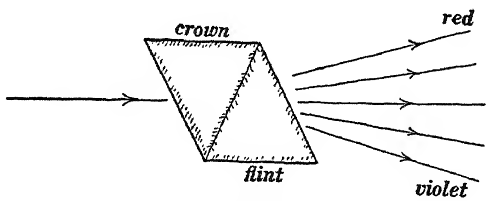


Fig. 154a.

while looking into the *telescope* *OE*, Fig. 149. Figure 154*b* shows the optical parts of a *direct-vision spectroscope*; *S* is the slit, *CL* is the collimating lens, *CFC* is a combination of crown-

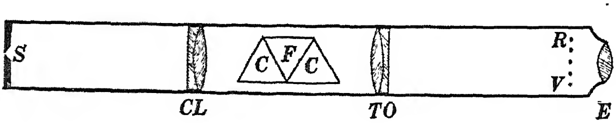


Fig. 154b.

glass and flint-glass prisms which does not deflect the middle part of the spectrum, but which gives a moderate amount of dispersion, *TO* is the telescope objective, *VR* is the spectrum

(row of images of the slit), and E is the eye lens. The eye lens is generally a lens system like Fig. 124 or Fig. 125.

86. The achromatic lens. — Consider a prism of flint glass and a prism of crown glass which give spectra of the same length. Then the crown-glass prism gives a much greater deflection (by deflection is meant the angle α in Fig. 49) than the flint-glass prism. Two such prisms arranged as shown in Fig. 154 would deflect a beam of light without perceptibly dispersing it. Such an arrangement might be called an *achromatic prism.* An achromatic lens consists of a converging lens CC of crown glass and a diverging lens FF of flint glass arranged as shown in Fig. 155 in which WW represents a series of plane waves of

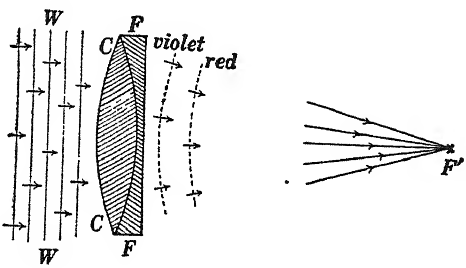


Fig. 155.

white light. After passing through the compound lens, waves of all wave-lengths have approximately the same curvature and are focused at the point F'. Short waves (violet light) travel slower in both kinds of glass than long waves (red light), but the difference between the velocity of red light and the velocity of violet light is much greater in flint glass than in crown glass and the result is that the middle portion of a wave is retarded with respect to the edge portions by an amount which is independent of the wave-length. However, the short wave-lengths (violet light) fall behind the long wave-lengths (red light) as shown by the two dotted circles in Fig. 155.

Let C_r and C_b be the indices of refraction of crown glass for red light and for blue light, respectively, and let F_r and F_b

be the indices of refraction of flint glass for red light and for blue light, respectively. Consider a converging lens of crown glass C, Fig. 156, and a diverging lens of flint glass F. The crown lens is assumed to come to a sharp edge and the flint lens is assumed to be of zero thickness at the center for the sake of simplicity. Let h be the thickness of the crown lens at its center and k the thickness of the flint lens at its edge. Consider a plane wave of red light passing through the two lenses. The central part of the crown lens is equivalent to a distance $C_r h$ to be traveled in air, and the edge portion of the flint lens is equivalent to a distance $F_r k$ to be traveled in air. Therefore, the central portion of a red wave is retarded with respect to the edge portions by an amount which is equal to $(C_r h - F_r k)$. Consider a plane wave of blue light passing through the two lenses. The central portion of the crown lens is equivalent to a distance $C_b h$ to be traveled in air, and the edge portion of the flint lens is equivalent to a distance $F_b k$ to be traveled in air. Therefore the central portion of the blue wave is retarded with respect to the edge portion by an amount which is equal to $(C_b h - F_b k)$. If both sets of waves, red and blue, are to be focused at the same point, the relative retardation of the central portion with respect to the edge portions must be the same for both, that is,

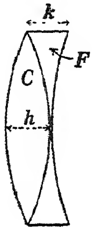


Fig. 156.

$$C_r h - F_r k = C_b h - F_b k$$

whence

$$\frac{k}{h} = \frac{C_r - C_b}{F_r - F_b} \tag{9}$$

This is the necessary relation between h and k to give achromatism. The absolute values of h and k depend upon the diameter of the lens and the desired focal length. The curvatures of the various surfaces in so far as they are not fixed by the absolute values of h and k and the diameter of the lenses may be chosen so as to eliminate spherical aberration.*

* An example of the complete calculation of an achromatic, aplanatic telescope objective, according to Gauss, is given by Otto Lummer in Müller-Pouillet's *Lehrbuch der Physik*, Vol. II, Part I, pages 573–579.

It is impossible to completely compensate the dispersion of a converging crown-glass lens by the dispersion of a diverging flint-glass lens for reasons which are evident from Fig. 157. This

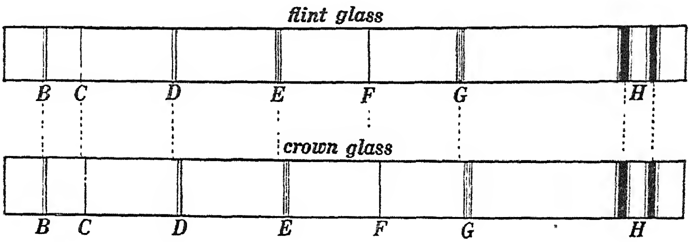


Fig. 157.

figure shows spectra of the same total length (Fraunhofer's *B*-line to Fraunhofer's *H*-lines) formed by flint-glass and crown-glass prisms, and it also shows that *the spectra are not of the same length* as measured between other pairs of lines of the spectrum.

CHAPTER VIII.

INTERFERENCE AND DIFFRACTION.*

87. Interference of waves from similar sources. — Two sources which send out waves in exactly the same rhythm are called *similar sources.* Thus, two prongs O and O', Fig. 158, which

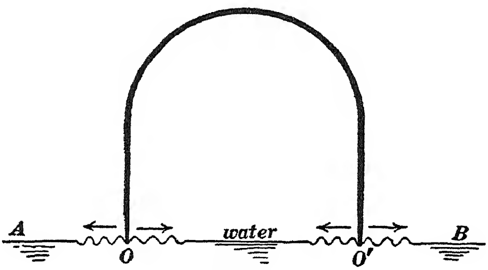


Fig. 158.

are moved up and down together, constitute similar sources of water waves on the surface AB. A small whistle W set into one side of a large tube TT, Fig. 159, produces waves which issue from the open ends OO' of the tube, and these open ends of the tube OO' constitute similar sources of sound waves. Consider a source of light O, Fig. 160, and its image O' in a mirror AB; O and O' constitute similar sources of light.

Consider two similar sources OO', Fig. 161, and let us assume that these sources give out simple wave-trains of which the wave-length is λ. Consider any point q whose distance from

* A very good general discussion of interference is to be found in Preston's *Theory of Light*, pages 139–210. An extremely interesting book and one that is easily readable is Michelson's *Light Waves and their Uses*, University of Chicago Press, 1903. This book is devoted exclusively to the phenomena of interference and to the uses of the interferometer.

A very good general discussion of diffraction is to be found in Preston's *Theory of Light*, pages 211–293. See also Drude's *Theory of Optics* (translated by Mann and Millikan), pages 159–241.

O is equal to its distance from O' or differs from it by a whole number of wave-lengths $n\lambda$. The wave-trains from O and O' are continually alike in phase* at such a point and they work together to produce disturbance there.

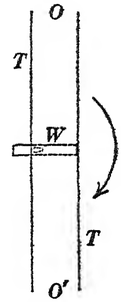


Fig. 159.

Consider any point p whose distance from O differs from its distance from O' by an odd number of half wave-lengths. The wave-trains from O and O' are continually opposite to each other in phase* at such a point and therefore the wave-trains tend to annul each other at such a point.

All the points q which satisfy the above condition lie on the dotted hyperbolas† in Fig. 161 of which O and O' are the foci. These dotted hyperbolas intersect the line OO' at equal intervals, namely, $\frac{1}{2}\lambda$ apart.

All the points p which satisfy the above condition lie on the full-line hyperbolas, midway between the dotted hyperbolas, in Fig. 161.

Everywhere on the dotted lines in Fig. 161 the disturbance due to the two sources O and O' is great, and everywhere along the full lines the disturbance is small, or, in some cases, zero; because the waves from O and O' are in phase with each other (crest corresponding with crest and hollow with hollow) everywhere along the dotted lines; and the waves from O and O' are opposite to each other in phase (crests of waves from O coinciding with hollows of waves from O') at every point along the full lines.

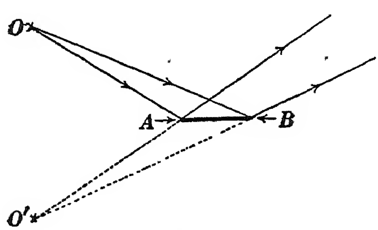


Fig. 160.

If O and O', Fig. 161, are similar sources of light and if the light from O and O' falls on a screen AB, then bright bands of illumination will be produced where AB intersects the dotted

* See Art. 23.

† The hyperbola is the locus of a point whose distances from two fixed points have a constant difference.

hyperbolas (hyperboloids of revolution about OO' as an axis), and dark bands will be left along the intersections of AB with the full-line hyperbolas. This phenomenon is called *interference* and the light and dark bands on the screen AB, Fig. 161, are called *interference fringes.*

It is important to remember that the strengthening of light or sound at one place by interference always involves simultaneous weakening at some other place, and *vice versa.* The total quantity of light or sound is not altered by interference, but its distribution is changed.

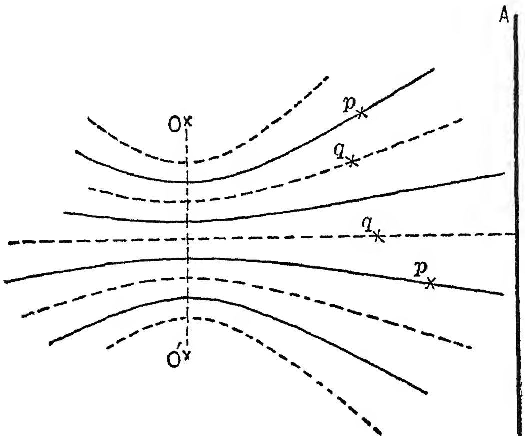


Fig. 161.

Example.—A large brass tube TT, Fig. 159, about a yard long has at its center a small shrill-sounding whistle which can be blown from the outside of TT. When the whistle is blown the open ends OO' of the large tube become similar sources of sound waves, and if there are no large objects near to introduce complications by the reflection of the waves, the state of affairs represented in Fig. 161 will be produced, namely, the sound will be loud along a series of hyperboloidal sheets qqq and weak along a series of intervening hyperboloidal sheets ppp. If the person who blows the whistle turns slowly around so as to rotate the tube TT as indicated by the curved arrow in Fig. 159, then

the whole system of hyperboloidal sheets will rotate with OO' and a second person standing at some distance will hear pulsations of loudness as the p and q regions pass by him. This is a very striking experiment and must be performed out of doors, preferably near the center of a large lawn.

The regions of maximum and minimum loudness of sound near the tube TT, Fig. 159, may be shown by means of the sensitive flame.* Such a flame is shown in Fig. 162. When undisturbed, the flame is long and smooth like b, and when it is disturbed, it changes to the short, rough form a. Such a sensitive flame rises and falls rapidly when one moves the whistle tube TT of Fig. 159 in its neighborhood.

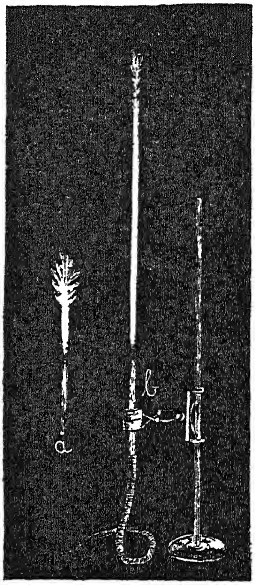


Fig. 162.

Color interference fringes. — If the similar sources OO', Fig. 161, give off white light instead of light of one wave-length, then a separate set of fringes will be produced on the screen by each homogeneous component of the white light, that is, by each wave-length of light, and the effect on the screen will be the superposition of all the fringes thus produced. The central fringe will be white, for this is a bright fringe for every wave-length. Passing out in either direction (up or down in Fig. 161) from this bright center we come first to the region where violet light (having the shortest wave-length) is extinguished, leaving the illumination of the complementary † hue; and so on, for blue, green, yellow, and red, in order.

* A very interesting account of sensitive flames and jets is given in Tyndall's *On Sound*, pages 214–256.

† Two colors which give white light when mixed are said to be complementary to each other.

88. Arrangements for producing interference fringes. (*a*) *Young's arrangement.*—Young, who was among the first to observe interference phenomena, employed an arrangement which is shown in Fig. 163. Light from a lamp *L* was allowed to pass

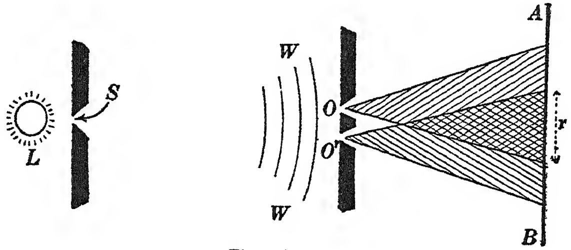


Fig. 163.

through a very narrow hole or slit *S*. This slit then becomes, in effect, the source of light for the region beyond, and, since it is a *small* source, *clearly defined wave-fronts exist in the region WW*, as explained in Art. 26. To say that the waves *WW* have clearly defined fronts is to say that these waves are almost exactly alike where they pass through two very small holes or slits *OO′* which are close together. Therefore the holes or slits *OO′* act as *similar sources* for the region beyond them, and the region *r* where the two beams of light

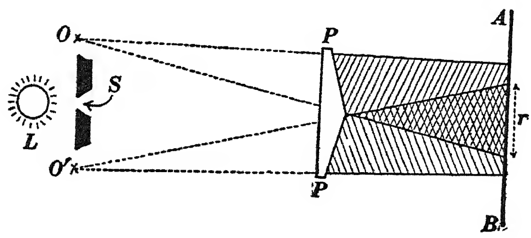


Fig. 164.

from *O* and *O′* overlap is the region where interference takes place, producing fringes on the screen *AB*.

The two holes *OO′*, Fig. 163, must be very close together, because the overlapping of the two beams as shown in Fig. 163, depends upon the spreading action, which is represented by the central spot of light in Fig. 95, as explained in Arts. 66 and 92.

(*b*) *Fresnel's biprism.*—Light passes from a lamp L through a small hole or slit S and thence through a double prism PP as shown in Fig. 164. After passing through the two halves of the double prism PP the light appears to have come from two

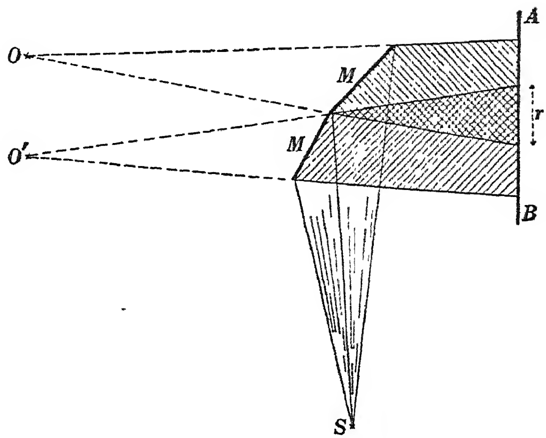


Fig. 165.

points O and O', and the region r, where the beams from O and O' overlap, is the region where interference takes place.

(*c*) *Fresnel's mirrors.*—Light from a fine hole or slit S is reflected from two mirrors MM as shown in Fig. 165. After

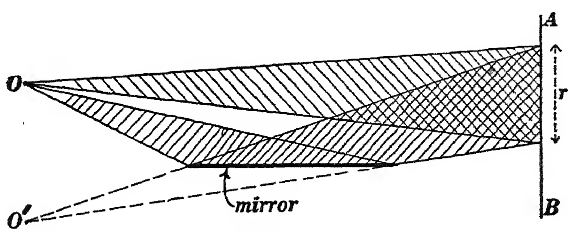


Fig. 166.

reflection from the mirrors the light appears to have come from two points O and O' and the region r, where the two beams overlap, is the region where interference takes place.

(*d*) *Lloyd's mirror.*—The portion r of a screen AB, Fig. 166, receives light directly from a small source O and the light

from O is also reflected upon the portion r by the mirror. The portion r of the screen is therefore illuminated by light from two similar sources O and O', and interference fringes are produced.

89. Beats.—An interesting effect is produced when the two sources O and O', Fig 161, *give out musical tones which differ very slightly in wave-length* (that is to say, in frequency or pitch). Suppose, for example, that source O gives out 251 waves per second and that source O' gives out 250 waves per second. Then the hyperboloidal sheets $ppp \cdots qqq \cdots$ will recede slowly from O and slowly approach O'; a given sheet moving from a p position to a q position in Fig. 161 in half a second, that is, while source O gains one half of a vibration on source O'. The slow movement of the hyperboloidal sheets $ppp \cdots qqq \cdots$ produces pulsations of loudness as they sweep by the ear of a stationary observer. These pulsations of loudness are called *beats*. Beats are produced most satisfactorily by sounding simultaneously two organ pipes of nearly the same pitch, and in a large auditorium where there is not too much reflection from the walls, the sweeping of the hyperboloidal sheets $ppp \cdots qqq \cdots$ over an audience might be made visible by having each person in the audience wave his hand slowly up and down with increase and decrease of loudness of the sound. The movement of hands would sweep across the room like a wave.

90. The colors of thin plates.—Thin transparent plates or films produce very striking interference phenomena. The colors of soap films and of films of oil on water are familiar examples. The action of a thin transparent film in producing interference may be understood, in general, by considering the two images of a source of light, one produced by reflection at the front of the film, and the other produced by reflection at the back of the film, thus giving two similar sources; the following discussion, however, applies to the case in which the incident light consists of a bundle of parallel rays (plane waves) and it is not necessary to use ex-

plicitly the idea of similar sources. Consider a simple train of waves T, Fig. 167, of wave-length λ. These waves strike a transparent plate PP, a portion T' of the wave-train is reflected from the surface A (with reversal of phase, see Arts. 20 and 25), and the remainder of the train passes through the plate, reaches the second surface B, and is again partly reflected (without reversal of phase). The portion of the wave-train which is reflected from the back of the plate after passing back through the plate emerges (in part) into the air and travels as the train T'' which is parallel to and partly overlaps the wave-train T'. It is in the overlapping portions of T' and T'' that interference is produced.

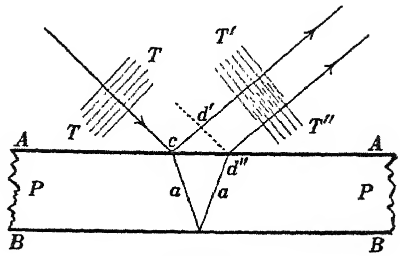


Fig. 167.

Suppose, for example, that the overlapping portions of T' and T'' enter the eye of an observer, being focused, of course, at a point on his retina so that the observer sees the distant point-source of the wave-train T. If the overlapping portions of T' and T'' correspond crest to crest and hollow to hollow, the resultant of the two trains will be an intense wave-train and the observer will see a brilliant point of light. If the overlapping portions of the wave-trains T' and T'' correspond crest to hollow and hollow to crest, then the resultant will be a greatly weakened wave-train and the observer will see a dim point of light, or no light at all. Thus when the light which is reflected from a soap bubble enters the eye one sees an image of the window from which the light comes, and this image is usually brilliantly colored as explained below.

When a given wave of the incident train T has reached the point c, the portion which is reflected at c has a distance cd' to travel in air to reach the dotted line $d'd''$, whereas the portion which is reflected from the back of the plate has the distance

$2a$ to travel in glass in order to reach the dotted line $d'd''$. The distance $2a$ in glass is equivalent to a distance $2a\mu$ in air. Therefore the wave-train T'' falls behind the wave-train T' by the amount $2a\mu - \overline{cd'}$. If reflection had taken place at both faces of the plate without phase reversal, then a relative retardation equal to a whole number of wave-lengths would cause the two wave-trains T' and T'' to correspond crest to crest and hollow to hollow, and a relative retardation equal to an odd number of half wave-lengths would cause the crests of one wave-train T' to coincide with the hollows of the other wave-train T''; but reflection from the front face A is accompanied by phase reversal, which means that the crests of wave-train T' are where the hollows would be if reflection had taken place without phase reversal. Therefore, *the two wave-trains* T' *and* T'' *tend to annul each other when the total retardation* $2a\mu - \overline{cd'}$ *is a whole number of wave-lengths, and they tend to intensify each other when the total retardation* $2a\mu - \overline{cd'}$ *is an odd number of half wave-lengths.*

If the incident light T, Fig. 167, is non-homogeneous (white light), then all of those homogeneous components (wave-lengths) of the white light are greatly strengthened in the reflected beam $T'T''$ whose half wave-lengths are contained an odd number of times in the distance $2a\mu - \overline{cd'}$, and those homogeneous components are greatly weakened whose half wave-lengths are contained an even number of times in the distance $2a\mu - \overline{cd'}$.

If the plate PP in Fig. 167 is moderately thick, then a great number of the homogeneous components of white light ($\lambda = 75 \times 10^{-6}$ cm. for red, to $\lambda = 39 \times 10^{-6}$ cm. for violet) will satisfy the above conditions. In this case the strengthened components and the weakened components will be distributed evenly throughout the spectrum and the plate will show no perceptible color. If, however, the plate is very thin, that is, if the distance $2a\mu - \overline{cd'}$ is small, then only one or two of the homogeneous components of white light will satisfy the above conditions and the plate will appear brilliantly colored. The existence

of interference in the case of a thick plate is beautifully shown by reflecting the light of a lamp into a spectroscope by a thin plate of mica (much too thick to give visible colors). The spectrum will be seen to be crossed by numerous dark bands corresponding to the wave-lengths which are weakened.

The value of a in Fig. 167 depends partly upon the obliquity of the incident wave-train and partly upon the thickness of the plate.

Newton's rings.—The thin film of air between two glass plates which are laid together often presents a fine show of interference colors. Newton made a very careful study of this effect, using the air film between a flat glass plate and the convex surface of a lens resting upon it. In this case the film increases in thickness from the point of contact outwards. With homogeneous light a succession of light and dark rings surround the point. With white light these rings are colored.

91. The interferometer.*—An instrument which has come into quite extensive use for extremely accurate measurement of small distances is Michelson's interferometer. The essential features of the interferometer are shown in Fig. 168*a*. Homogeneous (monochromatic) light TT, from a sodium flame, for example, strikes a half-silvered† glass plate AA and is partly reflected and partly transmitted. The part that is reflected strikes the silvered mirror D, is thrown back, and a portion of it passes through A and enters the telescope P as the wave-train T'.

The part of the original light that passes through A strikes the silvered mirror C, is thrown back, and a portion of it is reflected by A and enters the telescope P as the wave-train T''.

The light T' has traversed the glass plate A three times, whereas the light T'' has traversed A but once. Therefore a glass plate B similar to A is placed between A and C so

*See *Light Waves and their Uses* by A. A. Michelson, University of Chicago Press, 1903. (A series of eight semi-popular lectures which were delivered at the Lowell Institute in 1899.) This is a very interesting discussion of the more important researches which have been made with the help of the interferometer.

†A glass plate with an extremely thin coating of silver so as to transmit about half of the light which falls upon it and reflect the other half.

that after traversing B twice the light T'' will have passed through the same total thickness of glass as the light T'.

The light T' is reflected by A internally (without change of phase), and the light T'' is reflected by A externally (with

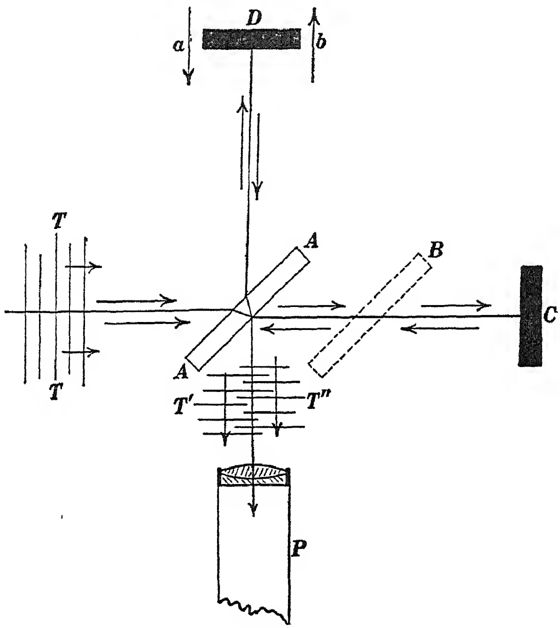


Fig. 168a.

reversal of phase). Therefore, if the distance $2AD$* is equal to the distance $2AC$, or if the distance $2AD$ differs from the distance $2AC$ by a whole number of wave-lengths, the wave-trains T' and T'' will correspond crest to hollow and hollow to crest and the field of the observing telescope P will appear dark. If the distance $2AD$ differs from the distance $2AC$ by an odd number of half wave-lengths, the two wave-trains T' and T'' will correspond crest to crest and hollow to hollow and the field of the observing telescope P will appear bright.

* It may seem that "the distances AD and AC" are not definite; in fact it is the distance from any given point on the mirror A that is here referred to. Another way to look at the matter is to consider the *image* of D in the mirror A; the distance between C and this image of D is the relative retardation between the wave-trains T' and T''.

Figure 168*b* is a perspective view of a Michelson interferometer as constructed by Wm. Gaertner & Co., of Chicago.

The mirror D is mounted on a sliding carriage which is actuated by a micrometer screw so that it may be moved at will in the direction of the arrow a or in the direction of the arrow b, Fig. 168*a*. If the mirror D is adjusted until the field of the telescope is dark, then, as the mirror is moved, the field becomes alternately light and dark, and the distance that the mirror is moved is equal to $n\lambda/2$, where n is the number of times that the field has become dark during the movement and λ is the wave-length

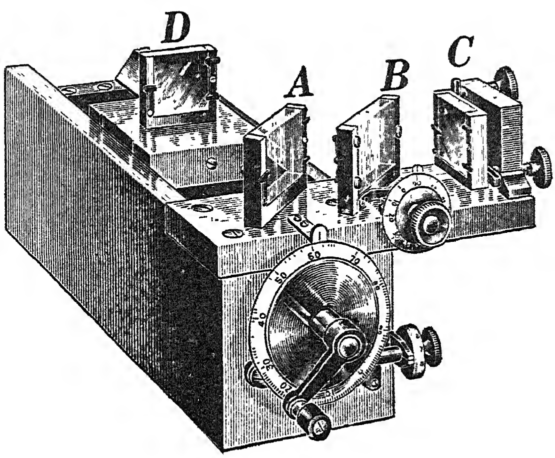


Fig. 188b.

of the light *TT*. If the pitch of the micrometer screw is known, then the value of λ may be determined. The most accurate wave-length determinations have been made in this way. If, on the other hand, the value of λ is known, the movement of the mirror is known because it is equal to $n\lambda/2$, as above explained.

In practice, the mirrors C and D are not exactly at right angles to each other so that the field of view of the telescope is crossed by bright and dark bands. Under these conditions bands move across the field of view when the mirror C is moved in the direction of a or b, and the movement of the mirror is equal

to $n\lambda/2$ where n is the number of dark bands which pass by a given point of the field during the movement.

92. Diffraction. — The spreading of a wave disturbance into the region behind an obstacle is called *diffraction.* This spreading action is very prominent in the case of water waves and sound waves, but in the case of light waves it requires special arrangements to make it perceptible. The slight tendency of light waves, as compared with the great tendency of sound waves and water waves to spread into the region behind an obstacle, is due to the very short wave-length of light as compared with the long wave-lengths of water waves and of sound waves.*

93. The diffraction grating. — When a train of plane waves TT, Fig. 169, strikes an obstacle AB in which there is an extremely narrow slit, the portion of the disturbance which passes through the slit spreads out, as indicated by the semicircles and divergent arrows in Fig. 169. The *diffraction grating* is an opaque plate in which there is a large number of equidistant parallel slits.† Thus, AB, Fig. 170, represents a diffraction grating, the slits of which are numbered 0, 1, 2, 3, 4, etc., and TT represents a simple train of plane waves, of wave-length λ, approaching the grating as shown. The figure shows the state of affairs after twelve successive waves of the train TT have struck the grating, that is to say, the figure shows twelve successive spherical‡ waves

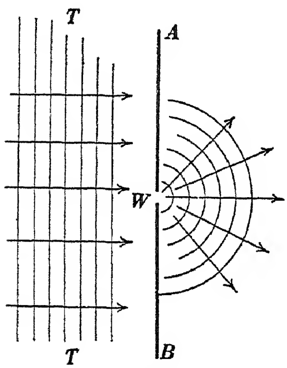


Fig. 169.

* The theory of diffraction was first developed by Fresnel in 1815. See Fresnel's *Œuvres Complètes*, Vol. I, pages 1–382. A very good discussion of this subject is to be found in Edser's *Light for Students*, pages 427–470. Discussions of diffraction are given in Preston's *Theory of Light*, pages 211–393, and in Drude's *Theory of Optics*, translated by Mann and Millikan, pages 159–241. No attempt is given in this text to discuss the theory of diffraction except in so far as the diffraction grating is concerned.

† This description applies to the transmission grating. The reflection grating is described in a subsequent article.

‡ Cylindrical waves, in fact.

which have emanated from each slit of the grating. *According to Huygens' construction (see Art. 28), every possible surface which is tangent to series of wavelets in Fig. 170 is a wave-front.* Thus, there is a series of wave-fronts parallel to the grating in Fig. 170 at a distance apart equal to λ. Consider the wavelets which are indicated in Fig. 170 by the heavy circles, namely, the second

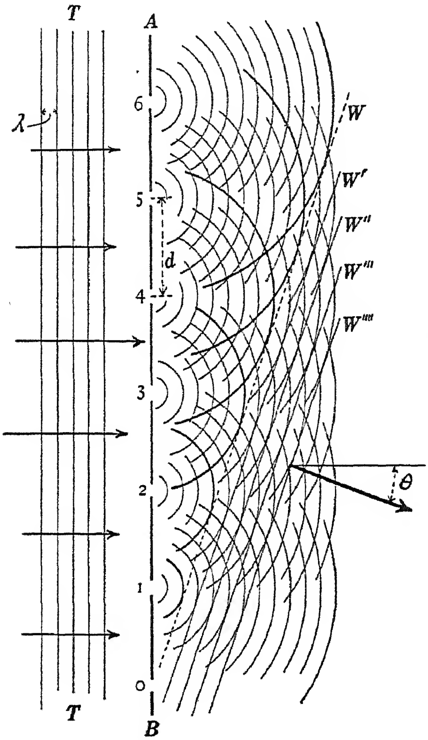


Fig. 170.

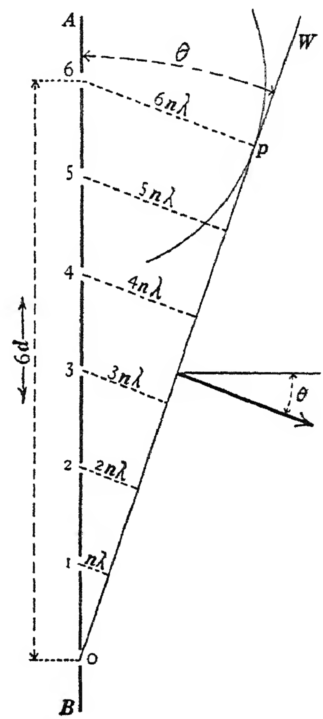


Fig. 171.

wavelet from slit No. 1, the fourth wavelet from slit No. 2, the sixth wavelet from slit No. 3, the eighth wavelet from slit No. 4, or, in general, the $2m$th wavelet from slit No. m. These wavelets have the common tangent plane which is indicated by the dotted line W, and there is a series of tangent planes W', W'', W''', etc., which are parallel to W and at a distance apart equal to λ. The existence of this series of tan-

gent planes means a train of plane waves traveling out from the grating in the direction of the heavy arrow in Fig. 170. The value of the angle θ in Fig. 170 is shown more clearly in Fig. 171. Consider the right triangle $o6p$ of which the hypothenuse is equal to $6d$, where d is the distance between centers of adjacent slits. From this triangle, we have

$$\sin \theta = \frac{n\lambda}{d} \tag{10}$$

in which n is any whole number, λ is the wave-length of the light which is striking the diffraction grating (see Fig. 170), d is the distance between centers of adjacent slits, and θ is the angle between the plane of the diffraction grating and the wave-fronts W, W', W'', etc., in Fig. 170, or θ is the angle between the normal to the diffraction grating and the direction of progression of the wave-train W, W', W'', W''', etc., as shown in Fig. 171.

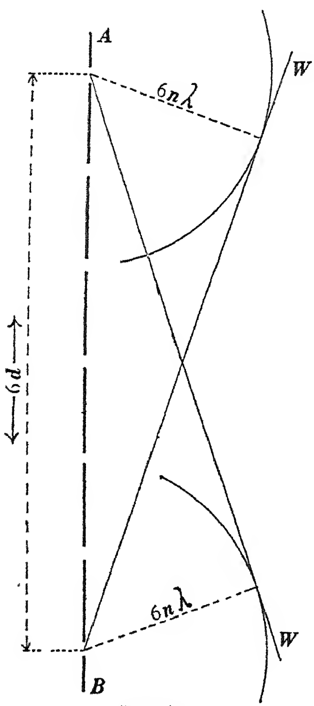


Fig. 172.

The wave-train W, W', W'', W''', etc., which corresponds to $n = 1$, is called the wave-train of the *first order;* the wave-train W, W', W'', W''', etc., which corresponds to $n = 2$ is called the wave-train of the *second order*, and so on. There are two wave-trains of each order as may be seen by a careful study of Figs. 170, 171, and 172.

94. The grating spectroscope.—The essential features of the grating spectroscope are shown in Fig. 173. A simple train of plane waves TT of wave-length λ approaches a grating AB behind which is a lens LL. The figure shows one of the wave-

trains of the second order, W, W', W'', etc. ($\sin\theta = n\lambda/d$, where $n = 2$). This train of plane waves is focused in the focal plane PP of the lens at the point F'' where the ray r cuts the plane PP, as explained in Art. 46. The wave-trains (like W, W', W'', etc.) of the various orders ($n = 1$, $n = 2$, $n = 3$, etc.) are focused at the points F', F'', etc., as shown in Fig. 173. The wave-train TT may be thought of as coming from an infinitely distant point and the points F', F'', etc., are images

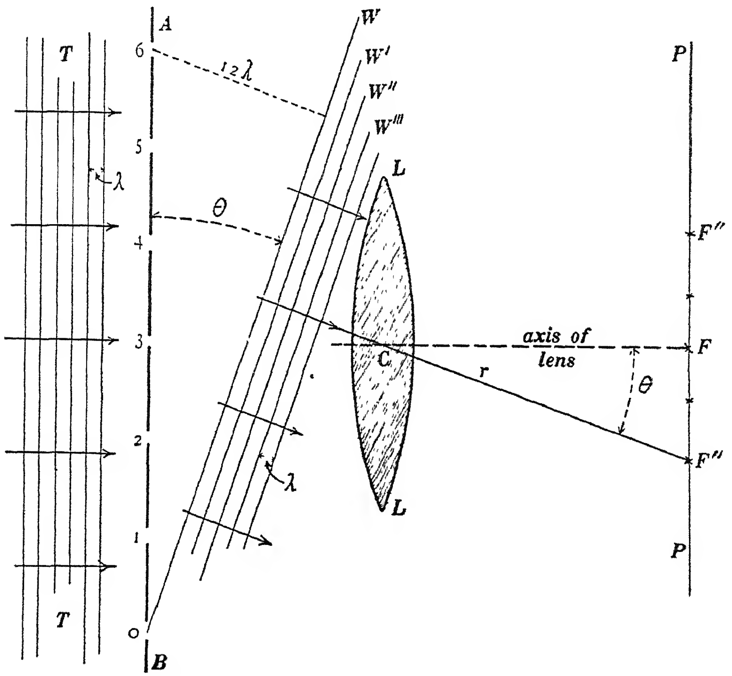


Fig. 173.

of this distant point source, or, the train of plane waves TT may come from a very narrow slit S and pass through a collimating lens CC, as shown in Fig. 174. In this case the focal point at F' is an image of the slit S. The arrangement in Fig. 174 is called a *grating spectroscope.*

If the incident light TT in Figs. 173 and 174 contains but a single wave-length, then there will be a single image of the slit at

F'. If, however, the light TT contains many wave-lengths, then there will be a group of images of the slit near F', a separate image for each wave-length, and this group of images VR, Fig. 174, constitutes a spectrum of the incident light TT. The foregoing statement applies to the spectrum of the first order ($n = 1$) or, indeed, to *one* of the spectra of the first order, because there is another like it at vr in Fig. 174. In addition to these two spectra of the first order, the diffraction spectroscope gives two spectra of the second order, two spectra of the third order, two spectra of the fourth order, etc.

Example. — A striking illustration of the action of the diffraction grating as above described is furnished by looking through a

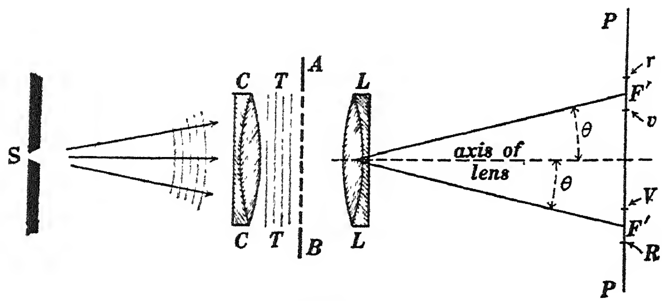


Fig. 174.

finely and regularly woven umbrella web towards a distant arc lamp. The umbrella web is the grating, the lens of the eye takes the place of the lens LL in Fig. 173, and the retina of the eye is the focal plane PP. In this case each of the points F', F'', F''', etc., is a spectrum, and these spectra are duplicated above and below the plane of the paper in Fig. 173, on account of the fact that the umbrella web has two sets of slits at right angles to each other.

95. The action of the diffraction grating upon obliquely incident light. — Consider the obliquely incident train of plane waves TT, Fig. 175. In this case the wavelets do not pass out from all the slits simultaneously. The line tt represents the position of a wave which has just reached the slit at O, and in order that

the line (plane) W may be tangent to the wavelets from the various slits the sum of the distances a and b must be equal to a whole number of wave-lengths. But $a = d \sin i$ and $b = d \sin \delta$, where d is the distance from center to center of slits, and i and δ are the angles shown in Fig. 175. Therefore, if W is tangent to wavelets from all the slits, we have

$$d \sin i + d \sin \delta = n\lambda$$

or

$$\sin i + \sin \delta = \frac{n\lambda}{d} \tag{11}$$

The lens LL in Fig. 175 gives a central focus (white) at the point

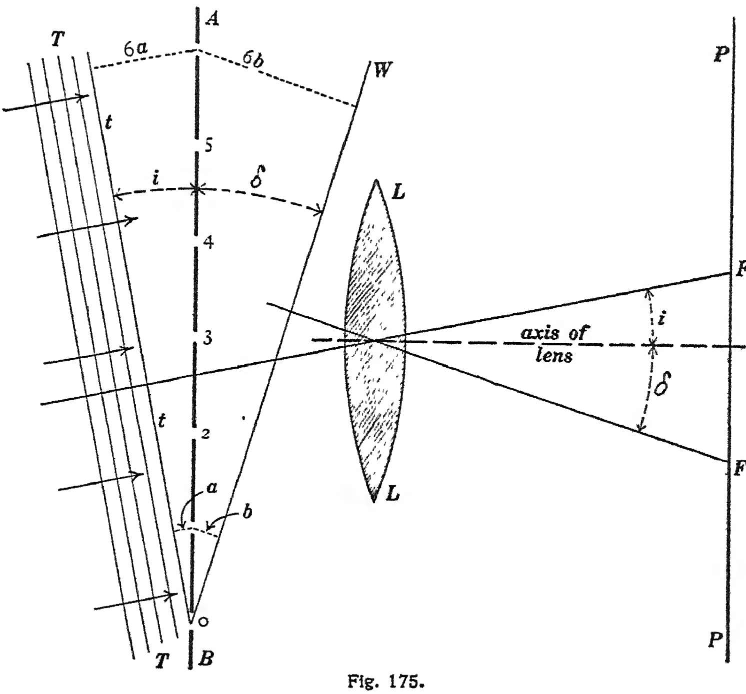


Fig. 175.

F, and the spectra of the various orders are near the points F', F'', etc. Figure 175 is constructed for $n=2$, that is, $a+b=2\lambda$.

96. Transmission gratings and reflection gratings. — Diffraction gratings for optical purposes are usually made by ruling fine equi-

distant lines on a plate of glass or speculum metal. The former is called a *transmission grating* and the latter is called a *reflection grating*. The action of the transmission grating is discussed in Arts. 93, 94 and 95. The reflection grating is a number of parallel equidistant reflecting strips separated by non-reflecting spaces. The action of the reflection grating is shown in Fig. 176, in which 0, 1, 2, 3, 4, 5 and 6 represent reflecting strips, TT is an incident wave-train, and W is one of the resultant wave-fronts as in Figs. 173 and 175. The discussion given in Art. 95 applies to Fig. 176 by changing the word *slit* to *strip*.

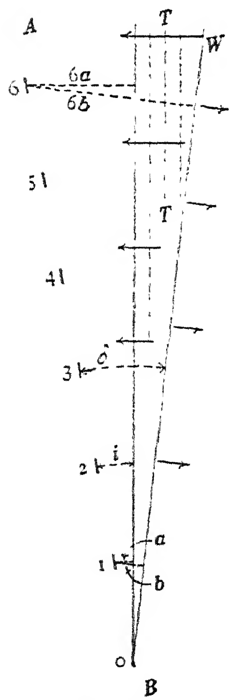


Fig. 176.

The reflection grating is generally used in grating spectroscopes, and the modification of Fig. 174 which is involved in the use of the reflection grating is sufficiently explained in the following articles on the *grating spectrometer* which is merely a grating spectroscope with a divided circle by means of which the angles θ, i and δ may be measured.

97. The grating spectrometer.—The grating spectrometer is a grating spectroscope provided with a divided circle by means of which the angles i and δ, Fig. 175 or 176, may be measured. The arrangement of the grating spectrometer (with reflection grating) is shown in Fig. 177. The grating AB is mounted upon an arm R which is pivoted at the center of the divided circle DD. The slit S and collimating lens CC are mounted at the ends of a tube which is also carried on an arm which is pivoted at the center of the circle, and to this arm a vernier is attached by means of which the angle i may be read off. The lens L forms a group of images of the slit at VR and this group of images is examined

through the magnifying glass (eye-piece) E. The lens L and the lens E are mounted at the ends of a tube which is fixed to an arm and this arm is pivoted at the center of the circle and it carries a vernier by means of which the angle δ may be measured.

Determination of wave-length. — If the grating space d is known and if the angles i and δ are measured by means of

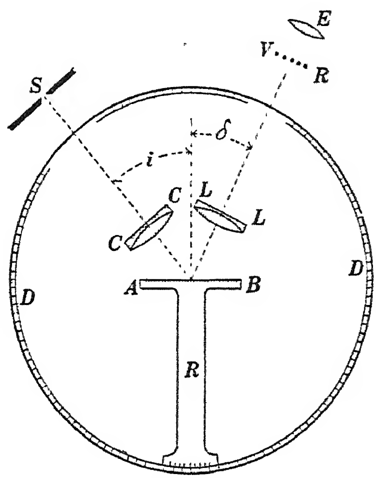


Fig. 177.

the grating spectrometer for a spectrum of a given order (n known), then the wave-length of the light may be calculated from equation (11). The following table gives the wave-lengths corresponding to the principal Fraunhofer lines in the solar spectrum as measured by Bell, using a Rowland * grating.

* The most perfect diffraction gratings hitherto made are those of H. A. Rowland. The concave grating, which does away with the lenses CC and LL in Fig. 177, is due to Rowland. A simple discussion of the concave grating is given on pages 459–463 of Edser's *Light for Students*. The theory of the concave grating is discussed at length by H. A. Rowland, *American Journal of Science*, Series 3, Vol. XXVI, 1883; by R. T. Glazebrook, *Philosophical Magazine*, Series 5, Vol. XVI, 1883; and by J. S. Ames, *Philosophical Magazine*, Series 5, Vol. XXVII, 1892. A very complete discussion of the diffraction grating is given by C. Runge in Kayser's *Handbuch der Spektroskopie*, Vol. I.

Wave-length units. — The micron (symbol μ) is one millionth of a meter or one ten-thousandth of a centimeter. The Ångstrom unit (symbol Å) is one hundred-millionth of a centimeter. Wave-lengths are frequently expressed in microns and less frequently perhaps in Ångstrom units.

TABLE.

Wave-lengths corresponding to Fraunhofer's Lines in the Solar Spectrum.

Line.		Line.	
A	7594.06 × 10^{-8} cm.	F	4861.49 × 10^{-8} cm.
B	6867.46 "	G	4308.07 "
C	6563.06 "		4307.90 "
D_1	5896.15 "	H_1	4101.85 "
D_2	5890.18 "	H_2	3968.62 "
E_1	5270.50 "		
E_2	5269.72 "		

CHAPTER IX.

PHOTOMETRY AND ILLUMINATION.*

98. Radiant heat. Light. — The radiation† from a hot body may be resolved into simple component parts each of which is a train of ether waves of definite wave-length. All of these component parts of the total radiation have one common property, namely, they generate heat in a body which absorbs them. Therefore every portion of the radiation from a hot body is properly called *radiant heat*. The intensity of a beam of radiant heat is measured by the heat it delivers per second to an absorbing body. Thus, the radiant heat emitted by a standard candle represents a flow of about 450 ergs of energy per second across one square centimeter of area at a distance of one meter from the candle.

Radiant heat of which the wave-length lies between 39 and 75 millionths of a centimeter affects the optic nerves and gives rise to sensations of light. Therefore radiant heat of which the wave-length lies between these limits is called *light*. These limits, which are called the limits of the visible spectrum, are not sharply defined, but with light of given intensity they vary considerably with different persons and with the degree of fatigue of the optic nerves. With very intense light the spectrum is faintly visible beyond these limits.

The physical intensity of a beam of light is measured by its perfectly definite thermal effect, that is, by the heat energy it de-

*This chapter is adapted from Franklin and Esty's *Elements of Electrical Engineering*, Vol. I. Standard text-books on Photometry are H. Krüss's *Die elektrotechnische Photometrie*, Vienna, 1886. A. Palaz's *Traité de Photomètrie industrielle*, Paris, 1892; English translation by G. W. and M. R. Patterson, Van Nostrand, 1894. Wilbur M. Stine's *Photometrical Measurements*, The Macmillan Company, New York, 1900.

† The general subject of radiation is discussed in Appendix B.

livers per second to an absorbing body. Thus, those parts of the radiation of a standard candle which lie within the visible spectrum represent a flow of about 9.3 ergs per second across an area of one square centimeter at a distance of one meter from the candle. Comparing this with the flow of energy which is represented by the total radiation from a standard candle, namely, 450 ergs per second across an area of one square centimeter at a distance of one meter from the candle, it follows that only about two per cent. of the energy radiated by a standard candle lies within the limits of the visible spectrum, that is, only about two per cent. of the radiation from a standard candle is light.

The luminous intensity of a beam of light is presumably measured by the intensity of the light sensation it can produce, but the intensity of the light sensation which is produced by a given beam of light is extremely indefinite. A given beam of light entering the eye may produce a strong or weak sensation depending upon manifold individual peculiarities of the person and upon the degree of fatigue of the retina; and the vividness of the sensation depends upon the extent to which it is enhanced by attention. Our sensations are not quantitative in the physical meaning of that term; in fact, they enable us merely to distinguish objects, to judge whether things are alike or unlike, and the certainty and precision with which we can do this is exemplified in every outward aspect of our daily life. *The ratio of the luminous intensities of two beams of light is measured by using a device to alter*, **in a known ratio,** *the physical intensity of one beam until it gives, as nearly as one can judge, a degree of illumination on a screen which is equal to (like) the illumination produced by the other beam.* Thus, if the physical intensity of a beam A has to be reduced in the ratio of 3 : 1 in order that it may produce the same degree of illumination on a screen that is produced by another beam B, then the ratio of the luminous intensities of the two beams is taken as 3 : 1.

99. Simple photometry. Spectrophotometry.—The measurement of the light emitted by a lamp is called photometry. This

measurement is always made by comparing a beam of light from the given lamp with a beam of light from a standard lamp as explained in Art. 98, and the physical device there referred to is called a photometer.

The comparison of the total light in a beam from a given lamp with the total light in a beam from a standard lamp is called *simple photometry;* whereas the comparison, wave-length by wave-length, throughout the spectrum, is called *spectrophotometry.* A fundamental difficulty in simple photometry is that different lamps usually show differences of color, and these differences of color do not disappear when the attempt is made to adjust a photometer so that the two lamps give equal (like) illumination on a screen.

100. Standard lamps.—The *British standard candle* is a sperm candle made according to exact specifications.* When this candle burns 120 grains of sperm per hour it is a standard candle, and the actual candle power during a given test is taken to be $a/120$ where a is the number of grains of sperm actually burned per hour during the test. The British standard candle is now obsolete.

The *Hefner lamp,*† so-called from its inventor, is a lamp which burns pure amyl acetate; the wick and its containing tube are of prescribed dimensions, and the wick is turned up to give a flame of prescribed height. Figure 178 is a sectional view of the Reichsanstalt ‡ form of the Hefner lamp, showing the dimensions in millimeters. The height of the flame is indicated by a sighting arrangement K. Most of the lamps are also provided with a

* See *American Gas Light Journal,* Vol. 60, page 41, 1894.

† A full discussion of the Hefner lamp may be found in *Photometrical Measurements* by Wilbur M. Stine. In particular, see the discussion of Influence of Atmospheric Moisture, Influence of Carbon Dioxide, Influence of Atmospheric Pressure, and Influence of Atmospheric Temperature on the brightness of the Hefner lamp on pages 153–157.

‡ The *Reichsanstalt* is the German physical laboratory corresponding to the *National Physical Laboratory* in England, and the *Bureau of Standards* in the United States.

Krüss optical flame-gauge which consists of a simple lens and a ground glass screen upon which the image of the top of the flame is projected.

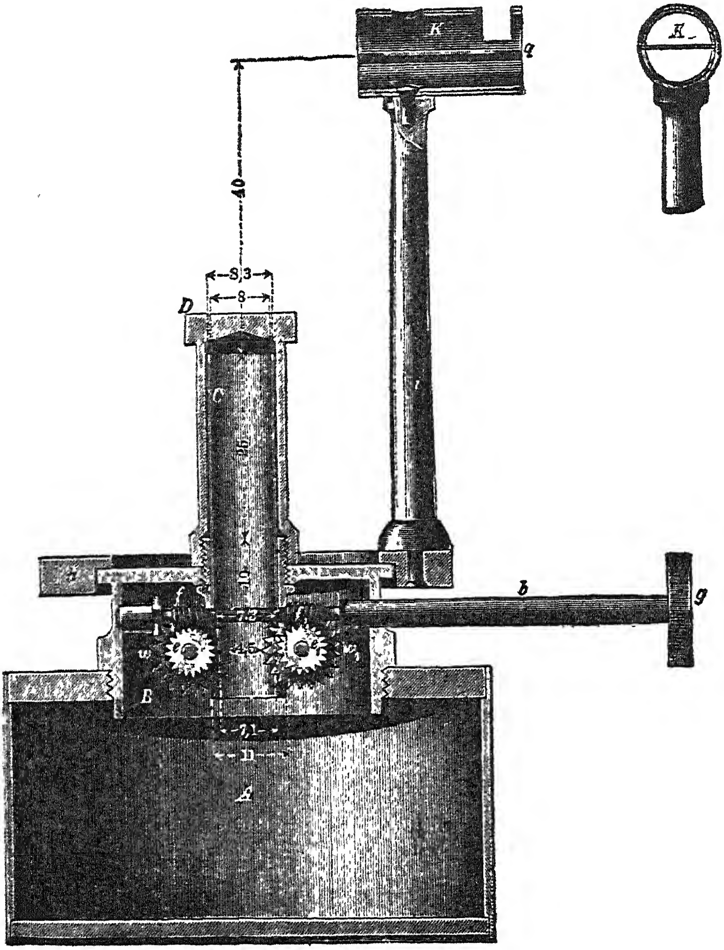


Fig. 178.

By means of a bolometer (see Appendix B) and a sensitive galvanometer it is possible to follow the continual fluctuations of brightness of a candle flame or of the Hefner lamp flame from moment to moment and to draw a curve showing graphically the

character of these fluctuations. Figure 179 * gives a portion of such a curve showing the irregular fluctuations of intensity of a British standard candle, and Fig. 180 gives a portion of such a curve showing the irregular fluctuations of intensity of a Hefner lamp.

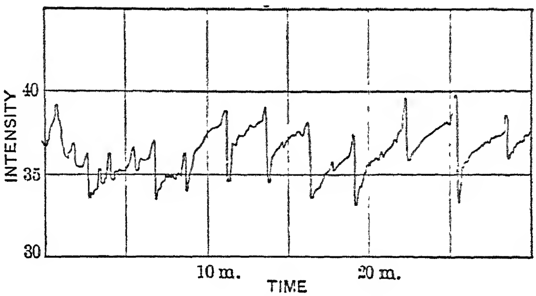


Fig. 179.

The carbon-filament electric lamp is extensively used as a working standard in photometric measurements. When so used, this lamp is previously standardized for a particular voltage and in a particular direction by comparing it with a Hefner lamp; and when used it is operated at this particular voltage.

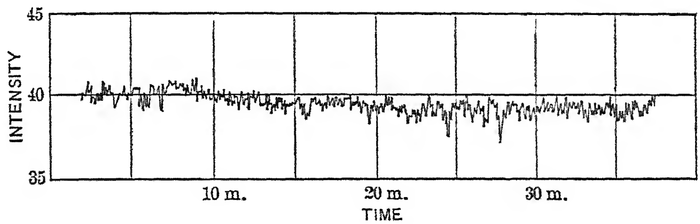


Fig. 180.

101. Light units.—The intensity of a horizontal beam of light from a Hefner lamp is called a *hefner-unit*, or a *hefner*. If a lamp were to give one hefner-unit of light intensity in every direction, the amount of light, or the so-called flux of light emitted by the lamp, would be what is called one *spherical-hefner*.

The great reliability of the Hefner lamp as compared with the

* Figures 179 and 180 are from a paper by C. H. Sharp and W. R. Turnbull, *Physical Review*, Vol. 2, page 1, 1894.

standard candle has led to the definition of the candle in terms of the hefner-unit. The *candle*, or *candle-unit*, is a beam of light of which the intensity is 1.136 hefner-units, and of course the spherical-candle is equal to 1.136 spherical-hefners.

102. Conical intensity and sectional intensity of light. — The expression, *intensity of a beam of light*, which is used in the above definitions of the hefner-unit and candle-unit, refers to the amount of light in a unit-sized cone of rays. This *conical intensity*, which it may be called for brevity, is expressed in hefners or candles; it is independent of distance, since the light in a given cone of rays remains in that cone;* and it depends only upon the area and brightness of the luminous surface of the lamp. The hefner-unit and the candle-unit are units of conical intensity.

The intensity of a beam of light may also refer to the amount of light per unit sectional area of the beam. This *sectional intensity* of a beam of light, which it may be called for brevity, decreases as the square of the distance from the lamp increases, because the amount of light in a given cone of rays remains constant and the sectional area of the cone increases as the square of the distance. The sectional intensity of a beam of light is expressed in terms of a unit called the *lux*, which is the sectional intensity of a horizontal beam from a Hefner lamp at a distance of one meter from the lamp.

The sectional intensity I of a given beam in luxes at a distance of d meters from a lamp is given by the equation

$$I = \frac{h}{d^2} \tag{12}$$

in which h is the conical intensity of the beam in hefner-units.

* It is here assumed that the light source is very small in size; it is very difficult to establish the fundamental ideas of photometry unless this assumption is made. Some matters relating to light sources which are not negligibly small are discussed in connection with Figs. 183, 184, and 185. A good example of the application of the fundamental ideas of photometry to large luminous sources is the paper, "Geometrical Theory of Radiating Surfaces with Discussion of Light Tubes," by E. P. Hyde, *Bulletin of the Bureau of Standards*, Vol. 3, pages 81–104.

103. Intensity of illumination. — The amount of light per unit sectional area of a beam, that is, the sectional intensity of the beam, measures the *intensity of illumination* of a surface upon which the beam falls perpendicularly. Therefore, the intensity of illumination of a surface may be expressed in luxes. Thus, the intensity of illumination required for easy reading is the intensity of illumination produced by a standard candle at a distance of one foot which is equal to 12.21 luxes, according to equation (12) inasmuch as one standard candle is equal to 1.136 hefners [equals h in equation (12)] and one foot = 0.305 meter [equals d in equation (12)].

104. Discussion of light units. — To understand the relationship of the various photometric units one must understand what is called solid or spherical angle. Consider a cone, and a sphere with its center at the apex of the cone. The solid or spherical angle of the cone is measured by the ratio of the area of the portion of the spherical surface within the cone to the square of the radius of the sphere. Thus, the unit of solid angle is subtended by one square centimeter of the surface of a sphere of one centimeter radius, and the complete surface of a sphere represents 4π units of solid angle. This method of measuring the solid or spherical angle of a cone is analogous to the method of measuring ordinary plane or circular angles in terms of the ratio of the arc of a circle to the radius, and the circumference of a circle represents 2π units (radians) of plane or circular angle.

The lumen. — Consider a lamp placed at the center of a sphere of unit radius so that one unit of area of this sphere may represent one unit of solid angle, or one unit-cone. Imagine the lamp to give one hefner of conical intensity in every direction. Then the amount of light (light flux) passing out in one unit-cone (through unit area of the sphere) is called one *lumen* of light flux. The given lamp emits one spherical-hefner of light flux, because the conical intensity is assumed to be the same in every direction; but the whole spherical surface represents 4π units of

solid angle or 4π unit-cones. Therefore the lamp emits 4π lumens of light flux. That is to say, a spherical-hefner of light flux is equal to 4π lumens.

Relation between the lux and the lumen.—The lux, which is defined above as the intensity of illumination at a distance (horizontal) of one meter from a Hefner lamp, represents, of course, a certain amount of light flux falling upon each square centimeter of the illuminated surface. In fact, one lux is one ten-thousandth of a lumen per square centimeter, or $1/40{,}000\pi$ of a spherical-hefner per square centimeter. This is evident from the following considerations: Given a lamp of one hefner intensity in all directions. Such a lamp gives out one spherical-hefner or 4π lumens. At a distance of one meter, this lamp gives an intensity of illumination of one lux; but the area of a sphere of which the radius is one meter is $40{,}000\pi$ square centimeters. Therefore dividing the total light which passes through the spherical surface, namely, 4π lumens, by the area of the spherical surface, namely, $40{,}000\pi$ square centimeters, gives the number of lumens per square centimeter in one lux.

105. Change of conical intensity of light by lenses and concave mirrors.—A beam of light, of which the conical intensity is h, passes through a lens LL as shown in Fig. 181. The lens brings the light to a focus at S' and the light spreads out again as a cone of rays beyond S'. It is evident, in the first place, that the conical intensity of the beam of light has the same value h' in the regions R and R' because the total amount of light is the same in both regions and the size of the cone is unchanged. The relation between h and h' is established as follows, the diameter of the lens being thought of as small in comparison with the distances a and b: Let A be the area of the lens. Imagine a sphere of radius a with its center at S and a sphere of radius b with its center at S'; the areas of the portions of these spherical surfaces which are included within the cones cc and dd are sensibly equal to A. Therefore the solid angle of the cone

cc is equal to A/a^2 and the solid angle of the cone *dd* is equal to A/b^2. Neglecting the amount of light lost by reflection from the surfaces of the lens and by absorption in the lens, we may consider the amount of light to be the same in the two cones *cc* and *dd*. Therefore the conical intensities of the two beams

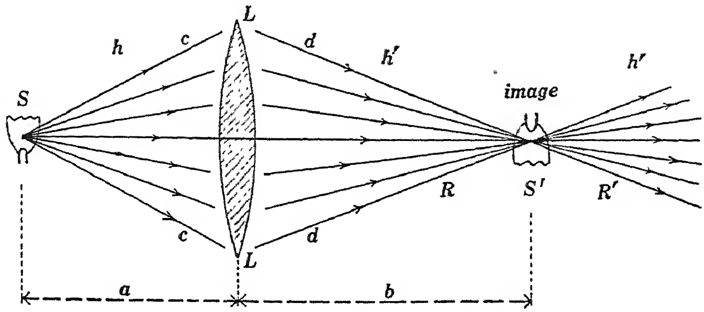


Fig. 181.

cc and *dd* are inversely as their solid angles, that is, the conical intensity of *cc* is to the conical intensity of *dd* as a^2 is to b^2.

The above discussion assumes that the light source *S* is a point, and the above conclusion is meaningless when the

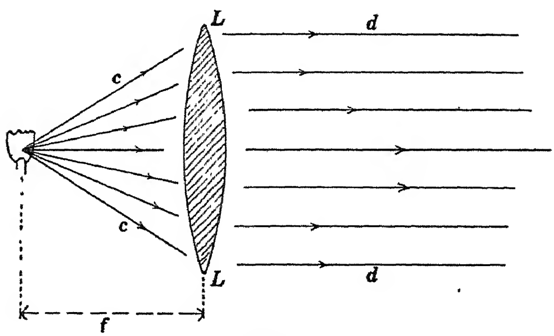


Fig. 182.

lamp *S* is at the focal point of the lens *LL* (that is, when $a = f$ and $b =$ infinity) as shown in Fig. 182. The beam *dd* is *not* a parallel beam of rays in this case as shown in Fig. 182, because the cone of rays *cc* does not come from a single lumi-

nous point as indicated, but from the entire luminous surface of the lamp. The light from any given point p, Fig. 183, is converted by the lens into a beam of rays which are parallel to the

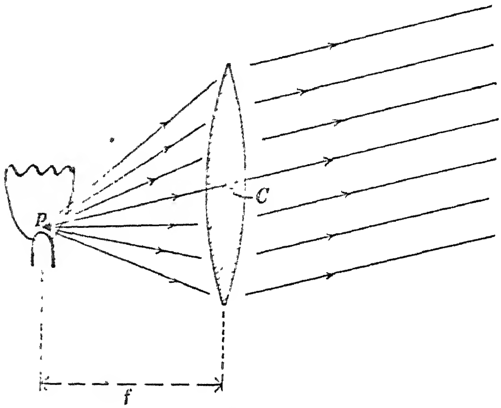


Fig. 183.

line pC, as shown. Therefore the light from the entire luminous surface of the lamp after passing through the lens LL in Fig. 184 is contained * within the cone rr, and the solid angle of this cone is equal to the area of the luminous surface of the

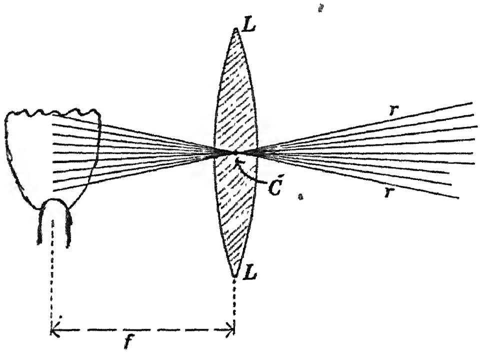


Fig. 184.

lamp, as seen from the center of the lens, divided by f^2. The light which enters the lens in Fig. 185 is contained within the

* This statement applies to the character of the beam rr at a great distance from the lens. Near the lens a large portion of the light is outside of the cone rr.

cone *cc*, and after passing through the lens the light is contained within the cone *dd* (at a great distance from the lens). Therefore the effect of the lens in Fig. 185 is to increase the conical intensity in the ratio of the area of the lens to the area of the luminous surface of the lamp.

Example. — The powerful arc lamp of a certain search-light emits light of 10,000 candles conical intensity towards every portion of a lens (or mirror) 24 inches in diameter. The luminous surface of the lamp is 0.24 inch in diameter, as seen from the center of the lens. Therefore the conical intensity of the beam of rays from the search-light is $(24/0.24)^2 \times 10{,}000$ candles, which equals 100,000,000 candles of conical intensity. In this example the lens is assumed to be perfect (free from the various kinds of aberration), and the loss of light by reflection from the surfaces of the lens and by absorption in the lens is ignored.*

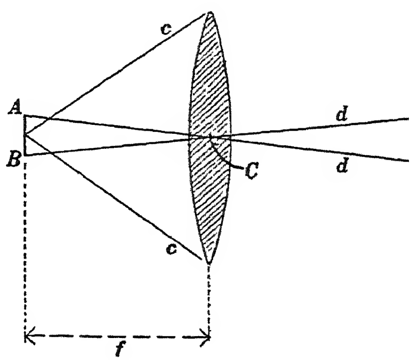


Fig. 185.

106. The Bunsen photometer is a device for measuring the conical intensity of a beam of light from a given lamp in terms of the conical intensity of a beam of light from a standard lamp. It is the photometer that is almost universally used in simple photometric measurements. The given lamp and the standard lamp are placed at the ends of a horizontal bar, and a screen of thin, unsized paper is moved along the bar until the two sides of the screen are equally illuminated by the two lamps. The intensities of illumination (sectional intensities of the beams of light) due to the respective

* One who considers this example carefully will be impressed with the fact, which is not by any means generally recognized, that *the candle is not a unit of quantity of light.* The candle and the hefner are units of conical intensity.

lamps are h/d^2 and h'/d'^2, according to equation (12), and since these are equal, we have

$$\frac{h}{d^2} = \frac{h'}{d'^2}$$

or

$$\frac{h}{h'} = \left(\frac{d}{d'}\right)^2 \tag{13}$$

in which h and h' are the conical intensities, in hefners or in candles, of the light which is sent towards the screen by the respective lamps, and d and d' are the respective distances of the lamps from the screen.

An irregular grease spot on the thin paper screen enables one to judge better when the illumination is the same on the two sides.

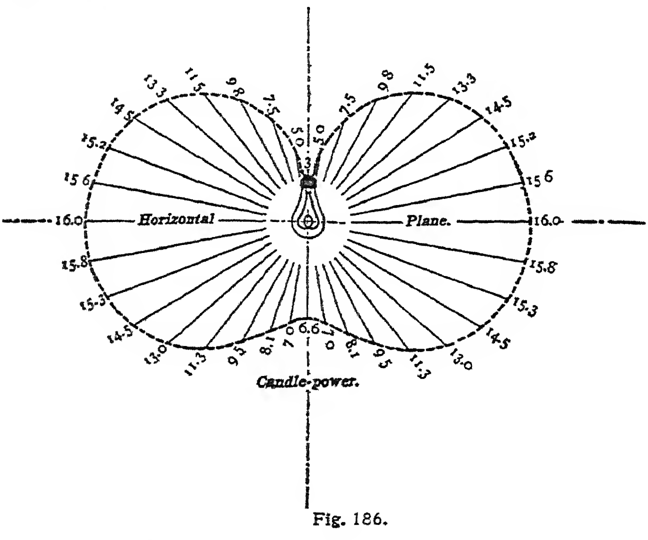


Fig. 186.

This spot should be made with clean paraffine and the excess of paraffine should be drawn out of the screen by placing it between folds of absorbent paper and applying a hot flat-iron.

The bar of the Bunsen photometer is generally divided to read the values of $(d/d')^2$ directly, and the product of this reading and the intensity of the beam from the standard lamp (h') gives the intensity of the beam from the other lamp (h).

In judging the equality of illumination on the two sides of the Bunsen photometer screen, one eye only should be used. In using both eyes, one unconsciously looks at one side of the screen with one eye and at the other side of the screen with the other eye and the difference between the two eyes leads to a constant error of setting.

The Lummer-Brodhun photometer is exactly similar to the Bunsen photometer except that an elaborate optical device is used for showing portions of the two sides of an opaque screen side by side in the same field of view.*

107. Distribution of light around a lamp. — In the definition of the spherical-hefner the idea of uniformity of distribution of light around a lamp was introduced for the sake of simplicity. In fact, however, no lamp gives complete uniformity of distribution, but the conical intensity in hefners or candles is always greater in certain directions and less in other directions. Thus, Fig. 186,† shows the distribution of light around an ordinary "16 candle-power" carbon-filament lamp without a shade, and Fig. 187 shows the distribution of light around the same lamp when it is placed in an aluminum cone reflector. In these figures the conical intensity of the light in each direction in candles is represented to scale by the length of the corresponding radius vector of the dotted curve.

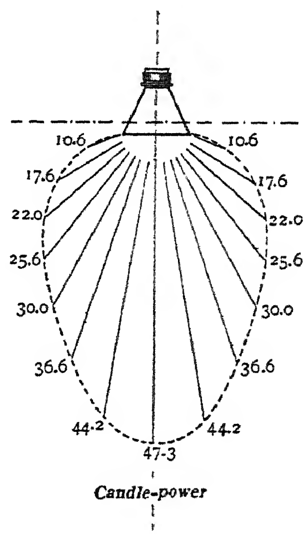


Fig. 187.

The distribution of light about a lamp, which, like a carbon-

*The Lummer-Brodhun photometer is described on pages 70–78 of Wilbur M. Stine's *Photometrical Measurements.*

† Figures 186 and 187 are taken from a paper by J. R. Cravath and V. R. Lansingh on Reflectors, Shades and Globes, *Electrical World and Engineer,* Vol. 46, pages 907, 947, 991, 1033, and 1074, November 25 to December 23, 1905.

filament lamp, can be held in any position, may be determined by mounting the lamp in a universal holder at one end of the photometer bar, turning it step by step into various positions, and taking the photometer reading for each position.

In some cases a lamp is symmetrical with respect to an axis, so that a complete knowledge of the distribution of light around the lamp may be obtained by determining the intensities of the light in different directions in a single plane which contains the axis of symmetry. In many cases, a lamp is approximately symmetrical with respect to an axis so that the slight variations of the intensity of the light around the axis of approximate symmetry are of no importance. In such a case the lack of symmetry may be averaged out, as it were, by rotating the lamp at a speed of three or four revolutions per second about its axis of

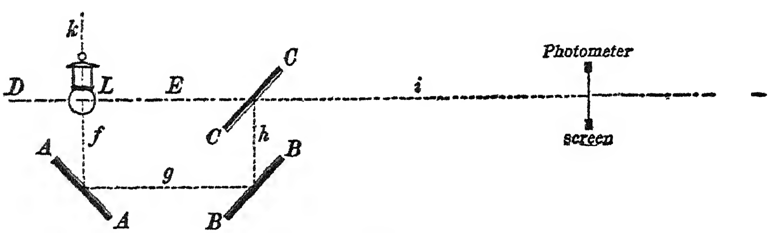


Fig. 188.

approximate symmetry while the photometric readings are being taken. The data for Figs. 186 and 187 were obtained in this way.

In the case of a lamp which must be held in a fixed position, one or more mirrors are used to reflect the different beams from the lamp along the photometer bar. Thus, Fig. 188 shows three mirrors *AA*, *BB* and *CC* arranged to reflect the light from a fixed lamp *L* along a photometer bar. The mirrors are supported in a rigid frame which may be rotated about the line *DE* as an axis. The figure shows the mirrors in the position to reflect the downward beam from the lamp along the photometer bar. The mirrors *AA*, *BB* and *CC*, Fig. 188, must be large enough so that with the eye placed at the photometer screen, one can see the entire luminous surface of the lamp including the

globe or shade; and the distance [*d* in equation (13)] of the lamp from the photometer screen must be taken as the sum of the distances *f*, *g*, *h* and *i* in Fig. 188.

The mirrors in Fig. 188 reflect a certain fractional part only of the light from the lamp, and therefore the photometer reading must be multiplied by a correction factor. This correction factor may be found by observing the photometer readings of the horizontal beam from the lamp with and without the mirrors, making due allowance for the effective distance from lamp to screen in each case.

If it is feasible, the lamp should be rotated steadily about the vertical axis *kf* in Fig. 188 while the photometer readings are being taken.

108. Measurement of total light flux from a lamp.*—If a lamp were to emit light of the same conical intensity in all directions, then the conical intensity of the light in hefners (or candles) would be numerically equal to the total light flux from the lamp in spherical-hefners (or spherical-candles), and a single measurement of such a lamp by means of a Bunsen photometer would give not only the conical intensity of the light in hefners (or candles), but also the total light flux in spherical-hefners (or

*Two forms of photometer have been devised for measuring the total light flux from a lamp directly, that is, with one setting of the photometer. See The Integrating Photometer, by C. P. Matthews, *Trans. American Institute of Electrical Engineers*, Vol. XVIII, pages 677–697, 1901, and Vol. XX, pages 59–70, 1902. A simple exposition of the theory and use of one form of the Matthews integrating photometer is given on pages 14–17 of Franklin, Crawford and MacNutt's *Practical Physics*, Vol. III.

In the globe photometer of Ulbricht, the lamp to be measured is placed at the center of a hollow sphere which is painted white on the inside, and the light which is emitted from a small translucent window is measured by means of an ordinary photometer, the window being screened from the direct light from the lamp by means of a small piece of white cardboard placed inside the globe. A very thorough investigation of the reliability of the globe photometer has been made by L. Bloch, see *Electrotechnische Zeitschrift*, pages 1047–1052 and pages 1074–1078, November 16 and 23, 1905. In this paper the details of construction of the photometer are fully described and it is shown that the photometer gives results which are sufficiently reliable for most practical purposes.

spherical-candles).* In general, however, light is emitted by a lamp unequally in different directions and it is necessary to distinguish between conical intensity in hefners (or candles) and total light flux in spherical-hefners (or spherical-candles). The total light flux in spherical-hefners emitted by a lamp is determined by measuring the conical intensity in hefners in every direction and taking the average which gives the light flux in spherical-hefners. If this average is to be calculated in the ordinary way by adding and dividing, the directions in which the separate readings are to be taken must be distributed uniformly over the surface of a sphere with its center at the lamp. This sphere is called the *reference sphere* for brevity. If the readings are not so distributed, then each reading must be multiplied by the spherical area which may be properly assigned to it, and the sum of such products must be divided by the total area of the reference sphere to give the correct average.

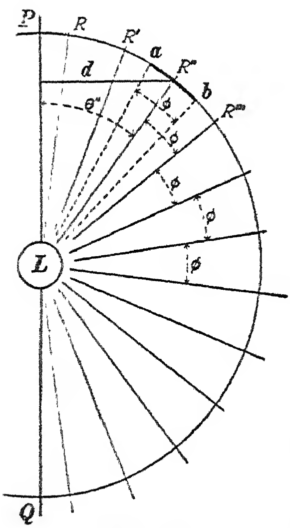


Fig. 189.

When a lamp can be rotated at a speed of three or four revolutions per second about its axis of approximate symmetry, the total light flux may be determined accurately from readings of the conical intensity in different directions *in one plane only*, namely, a plane which includes the axis of rotation. Thus, the lamp L, Fig. 189, is rotated about the vertical axis PQ, and the conical intensities R, R', R'', R''', etc., at equal angular distances

*In the absence of exact data concerning the distribution of light around a given lamp, the irregularities of the distribution are ignored, and the intensity of a horizontal beam, in candle-power, is used as the easiest and simplest approximate measure of the total light given off by the lamp, as if the light were of the same conical intensity in all directions. It is this fact which has led to the confusion of the unit of conical intensity, the candle, and the unit of light flux, the spherical-candle. The light flux emitted by a lamp is often called the *spherical candle-power* of the lamp.

ϕ are measured. On account of the rotation of the lamp each setting of the photometer gives the average conical intensity along a parallel of latitude, as it were. Each reading R, R', R'', etc., represents, therefore, the conical intensity over a zone of the reference sphere; so that the readings must be multiplied by the areas of the respective zones and the sum of these products must be divided by the total area of the reference sphere to give the average conical intensity in all directions.

Usual methods of rating lamps.—Incandescent electric lamps are usually rated in terms of their mean horizontal candle-power which is determined by rotating the lamp about a vertical axis while the photometer reading is being taken. This rating of an incandescent lamp in terms of mean horizontal candle-power instead of in terms of spherical-candles has come into vogue because of the difficulty of determining mean spherical candle-power. The mean spherical candle-power of an ordinary carbon-filament incandescent lamp is about 0.85 of its mean horizontal candle-power.

Street lamps are frequently rated on the basis of what is called their mean hemispherical candle-power, that is, the mean candle-power in all directions below the horizontal.

109. The flicker photometer is a device for eliminating, to some extent, the error in setting of a photometer due to differences of color of the lamps which are being compared. The following is the principle upon which the elimination of color error is based: When one looks at a thing, such as a photometer screen, one has a sensation of *brightness* and a sensation of *color*. Both of these sensations persist for an appreciable interval of time after light ceases to enter the eye, but sensations of color persist much longer than sensations of brightness. Therefore, if the two sides of the photometer screen are brought into the same field of view in rapid succession (with high frequency of interchange), the color sensations produced by the two sides of the screen and also the brightness sensations produced by the two sides of the screen will be completely blended, whereas a much lower frequency of interchange will suffice to blend the color sensations and leave a flick-

ering sensation of brightness unless the two sides of the screen have the same brightness. Therefore, if a fairly low frequency of interchange be used, the two sides of the photometer screen can be brought to equal brightness by adjusting the photometer until the flicker disappears.

The device employed by Whitman * in his flicker photometer is as follows: A white cardboard A, Figs. 190 and 191, is

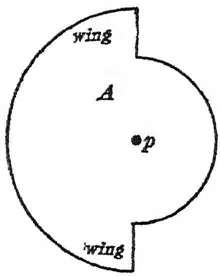


Fig. 190.

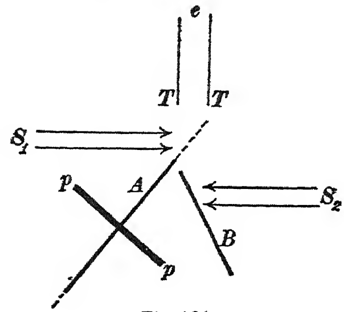


Fig. 191.

mounted on an axis pp, and a stationary piece of the same cardboard is placed at B as shown in Fig. 191. When the disk A is rotated the wing of the disk and the cardboard B are seen in succession by the eye which is placed at e and shielded by the tube TT, Fig. 191. The wing of the disk is illuminated by light from the source S_1 and the cardboard B is illuminated by the source S_2. The disk A is driven at sufficient speed to blend the color sensations, and then the distances of the two lamps S_1 and S_2 from A and B, respectively, are adjusted until the flickering sensation of brightness disappears. The flicker photometer is not much used in commercial photometry.

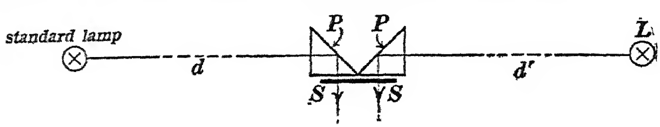


Fig. 192.

110. The spectrophotometer is a combination of a spectroscope and a photometer for comparing the intensities of two beams of light wave-length by wave-length. A simple form of spectro-

* F. P. Whitman, *Physical Review*, Volume 3, page 241.

photometer is shown in Figs. 192 and 193. Figure 192 shows a standard lamp and a given lamp *L* at the ends of a photometer bar. Two reflecting prisms *PP*, Fig. 192, reflect light from the respective lamps into the slit *SS* of a spectroscope so that the spectra of the two lights are seen side by side in the spectroscope. Figure 193 shows the spectroscope (direct-vision type) mounted on a car so that it can be easily moved along the photometer bar, thus changing the relative intensities of the two spectra at will. Observations are taken with this instrument as follows: The observer's attention is fixed on a certain region of the two spectra, for example, the extreme red, and the car is moved until the two spectra are of the same intensity in this region. The ratio of brightness of the two lights for this region of the spectrum is then equal to the ratio of the squares of the distances d and d' in Fig. 192. Each of these distances should be measured as the total length along the path of the rays from lamp to slit. This operation is repeated step by step throughout the spectrum. The attention of the observer is directed to a certain region of the two spectra by placing in the focal plane of the eye-piece of the spectroscope a diaphragm with a narrow slit in it so that a narrow strip only of the two spectra is visible.

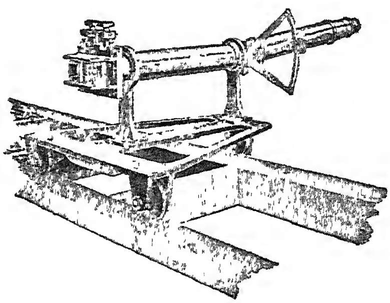


Fig. 193.

The objection to the above-described spectrophotometer is that the two luminous fields which are compared (the two parts of the narrow slit in the focal plane of the eye-piece) are very small. This difficulty is obviated in the spectrophotometer of Lummer and Brodhun, the essential features of which are shown in Fig. 194. Two right-angled glass prisms *XX* are cemented together forming a cube, one prism being cut away slightly so as to leave a central patch *c* in actual contact whereas the portions *dd* of the prisms do not come into contact. The light from the stand-

ard lamp L' illuminates a paper or magnesia plate P' from which it enters the slit S', and, after passing through the collimator C', it is totally reflected by the portions dd of the double prism XX into the prism P of the spectroscope; whereas the light from L' passes through c without any

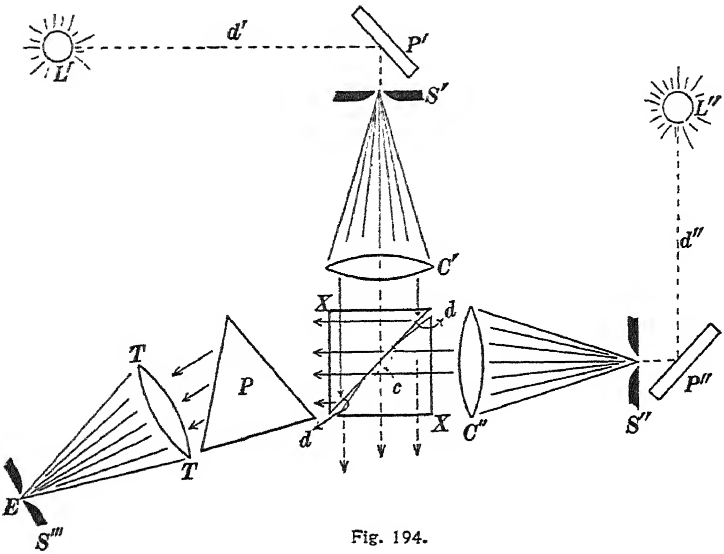


Fig. 194.

reflection at all. The light from the lamp L'', reflected into the slit S'' by the diffusing plate P'', passes through c into the prism of the spectroscope, but is totally reflected by the portions dd of the double prism. After passing through the prism P of the spectroscope, the light is brought to a focus by the telescope objective TT, and a slit S''' is placed at the focus of this lens so that light of a single wave-length enters the eye at E, and the eye, focused upon the face dcd of the double prism XX, *sees the central portion of the field uniformly illuminated by light of one wave-length from lamp L'' and the edge portions of the field uniformly illuminated by light of one wave-length from the lamp L'.* The distances d' and d'' are then adjusted until the field is uniformly bright. Then the intensity of the given wave-length from L' is to the intensity of the same wave-length

from L'' directly as the squares of the distances d' and d''.

Examples of spectrophotometric measurements.—The results of the spectrophotometric comparison of gas light, lime light and day light are shown by the curves g, l and d, respectively, in Fig. 195.* The curves refer to beams of gas light, lime light, and day light *all of which have the same intensity at Fraunhofer's D line* (the middle of the yellow region of the spectrum); and the curves show, for example, that the beam of day light is about six times as bright as the beam of gas light at Fraunhofer's F line (in the blue region of the spectrum) and only about one-quarter as bright as the gas light at Fraunhofer's B line (in the red region of the spectrum).

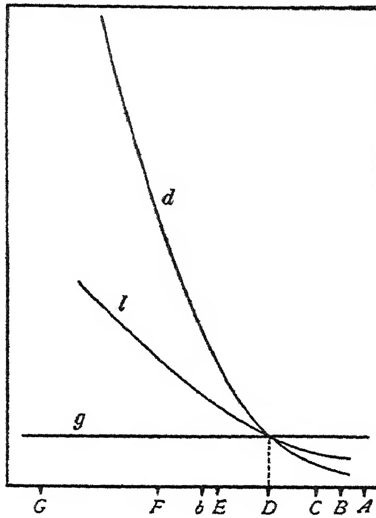


Fig. 195.

111. The problem of illumination.†—A room may be said to be well lighted when the eye is easily able to distinguish, in minute detail of perception, the various objects in the room. Completeness of visual perception depends upon three conditions, namely,

* Taken from a paper on A Spectrophotometric Comparison of Sources of Artificial Illumination, by E. L. Nichols and W. S. Franklin, *American Journal of Science*, Vol. 38, pages 100–114, December, 1889.

† An extremely interesting discussion of the conditions which determine visual perception is given by Helmholtz in his popular lecture on The Relation of Optics to Painting which is translated (by E. Atkinson) in the second series of Helmholtz's Popular Lectures, Longmans, Green & Co., 1903. Everyone who is concerned with the practical problems of illumination should read this lecture. Helmholtz's Popular Lectures are published in German under the title Vorträge und Reden, 2 volumes, Braunschweig, Vieweg und Sohn, 1884.

Three lectures in Helmholtz's first series (translated by Dr. Pye-Smith; Longmans, Green & Co., 1873), On the Theory of Vision, also have a bearing upon the important practical subject of illumination.

(*a*) a sufficient brightness of illumination, (*b*) a proper location of the light sources so as to bring out that combination of soft shadows which is so essential to the perception of form, and (*c*) a proper composition* of the light so as to bring out those physical differences in objects which the eye perceives as variations of color.

A room may be intensely illuminated by a single arc lamp, but such illumination is ineffective even when the eye is shaded from the direct light of the lamp because the excessive harshness of the shadows renders the perception of form almost impossible. On the other hand, a sufficiently intense and properly distributed light which contains certain wave-lengths in great excess may be ineffective because of the unusual or weak color effects produced thereby. For example, the light of an ordinary kerosene lamp is very deficient in the shorter wave-lengths; these shorter wave-lengths have much to do with the bringing out of blue and violet tints, and consequently a deep blue or violet piece of cloth appears almost black by kerosene lamp light. False color values are produced in a very striking way by the light from the mercury-vapor lamp on account of the almost complete absence of longer wave-lengths (red) in the light from this lamp. The brilliant white light of the carbon-arc lamp, on the other hand, contains all wave-lengths in about the same proportion as sun light, and all colors show up well by the light of such a lamp.

Glare. — Excessive contrast of light and shade in the field of vision tends to hinder visual perception. The eye adapts itself automatically to the brightest lights in the field of view and all perception of detail in the shadows is lost. This blotting out of detail in the shadows by excessively brilliant lights in the field of vision is called *glare* and it is especially marked when the field of vision includes a bright unshaded lamp. The explanation of glare is as follows: In the first place, the pupil of the eye contracts greatly when there is a bright light in the field of vision and this contraction lessens the effective brightness not only of

* The composition of light refers to the relative intensities of the various wave-lengths which are present in the light.

the "high lights" but also of the deep shadows; in the second place, the sensitiveness of the retina seems to be greatly reduced in bright light and although this reduction of sensitiveness may leave the eye able to perceive detail in the more brilliantly illuminated portions of the field of vision, it tends to obliterate all detailed perception in the shadows; and in the third place an intense beam of light entering the eye from a bright source illuminates the whole interior of the eye just as a beam of sun light entering a window illuminates a room, and this diffused light in the eye illuminates and excites the entire retina including those portions where the images of the deeper shadows fall and thereby obliterates all detail of perception.

An interesting case involving excessive contrast is that in which a workman at a loom, for example, has his immediate work illuminated to a fair degree of brightness while the remainder of the room is left in darkness. If the workman could keep his eyes upon his work incessantly, it is conceivable that this kind of illumination might be satisfactory; but the eye moves about in spite of everything one can do, and under the assumed conditions, the workman would be unable to see when he glanced about the room and he would be blinded when he glanced back at his work. To avoid this impracticable situation a general illumination of the room is necessary.

It is very important in arranging for the illumination of a room to place the lamps outside of the field of vision, if possible, so that no light can enter the eye directly from the lamps and render the eye insensible to the delicate shading of surrounding objects. The excessive discomfort that is produced by the glare of improperly located lamps, such, for example, as the exposed foot-lights of a poorly arranged stage, is due not only to the physical pain that is associated with long-continued looking at a bright light but more especially to the incessant strain of trying to peer into the dark region beyond.

Where a lamp cannot be removed from the field of vision the bad effects of glare may be greatly reduced by enlarging the

effective luminous surface of the lamp by means of a translucent globe or shade. A translucent globe always absorbs a considerable portion of the light of a lamp, but the effectiveness of the globe in eliminating the glare is primarily due to the fact that a given amount of light coming from a small brilliant source produces a much greater glare than the same amount of light coming from a large faint source.

Dim lamps versus brilliant lamps. — A much more satisfactory distribution of light in a space to be illuminated may be obtained by using several lamps of moderate brightness than by using one or two lamps of great brightness. Thus, very bright lamps are not suitable for illuminating small rooms because the one or two lamps required to produce the desired quantity of light give a very unsatisfactory distribution, and the number of lamps required to give a satisfactory distribution would produce an excessive amount of light. To give a satisfactory distribution of light over the field of vision in a lower portion of a room, very bright lamps should be raised to a considerable height overhead. In general, therefore, very bright lamps are unsatisfactory except for lighting very large, high rooms and for street lighting; such lamps may, however, be used with good effect in moderately small rooms when the indirect system of illumination is employed. In this system, all of the direct light from the lamp or lamps is thrown upon the ceiling of the room and the diffused light which is reflected from the ceiling produces a beautiful soft illumination in the lower portion of the room.

Influence of absorption upon illumination. — An illuminated surface such as the wall, ceiling, or floor of a room, or the surface of an object in the room, absorbs a definite fractional part of the light which falls upon it. This fraction is called the coefficient of absorption of the illuminated surface. The lamps in a room emit a given flux of light and at the instant the lamps are turned on, the intensity of the illumination in the room is quickly increased by repeated reflections of the light from the illuminated surfaces until *the rate of absorption of light by the illuminated sur-*

faces is equal to the rate of emission of light by the lamps. Given, for example, two rooms, *A* and *B*, illuminated by the same number of lamps. Suppose that the two rooms have the same area of walls and objects to be illuminated but suppose that the illuminated surfaces in room *A* absorb an average of 40 per cent. of the light which falls upon them, whereas the illuminated surfaces in room *B* absorb an average of 80 per cent. of the light which falls upon them. In both rooms, the same amount of light is emitted by the lamps according to the assumption, and therefore the same amount of light is absorbed by the illuminated surfaces; but this absorbed light is only 40 per cent. of the mean intensity of illumination in room *A* whereas it is 80 per cent. of the mean intensity of illumination in room *B*; therefore the mean intensity of illumination in room *A* is twice as great as it is in room *B*. In general, the total light flux (spherical-hefners or spherical-candles) required to produce a given intensity of illumination in a room is proportional to ab, where a is the combined area of all the surfaces to be illuminated and b is the mean coefficient of absorption of these surfaces.

Flux of light required for effective illumination.—The total flux of light in spherical-candles required for the effective illumination of a given sized room depends upon the manner in which the light is distributed, upon the composition (color) of the light, and upon the mean coefficient of absorption of the illuminated surfaces, as pointed out above. Interior lighting is usually accomplished by lamps which give a soft yellow light, the lamps are usually distributed overhead so as to be as much as possible out of the field of vision, and walls and ceiling are usually yellowish-white. Under these conditions, about 0.2 of a spherical-candle is required for each square foot of floor area to give a degree of illumination that would be considered satisfactory in a reception room or in a lecture hall. If the ceiling is very high a greater candle-power is required inasmuch as the area of the walls is increased. If the ceiling and walls are very dark, if they are made of stained oak or cherry paneling, for example, effective

illumination may require 0.4 or 0.5 of a spherical-candle per square foot of floor area.

112. Lamp efficiency.* — In practice the efficiency of an electric lamp is always specified by giving the watts of power consumed in the lamp per spherical-candle † of light flux emitted. Thus, the carbon-filament glow lamp of the kind that is at present most widely used, consumes about 3.6 watts per spherical-candle and the tungsten-filament glow lamp consumes about 1.25 watts per spherical-candle. The actual efficiency of a lamp is of course the ratio of the light energy (luminous part of the radiation) emitted by the lamp to the total energy supplied to the lamp, and the actual efficiency is of course greater the smaller the so-called efficiency in watts consumed per spherical-candle.

Angström ‡ has found that that part of the horizontal beam of a Hefner lamp which lies within the limits of the visible spectrum represents a flow of 8.1 ergs per second across one square centimeter at a distance of one meter from the lamp, so that one spherical-hefner corresponds to a flow of $4\pi \times 100^2 \times 8.1$ ergs per second, or 0.102 watt. This corresponds to 0.115 watt for one spherical-candle. The power value of the spherical-hefner (or candle) depends however to some extent upon composition (color) of the light. The value of 0.115 watt per spherical-candle applies to the orange-colored light from the Hefner lamp, and this value can be considered only as an approximation to the power value of one spherical-candle of yellowish light or of pure white light, or of bluish light.

Taking 0.115 watt as the approximate power value of one spherical-candle, it follows that the ordinary carbon-filament glow lamp which takes 3.6 watts per spherical-candle has an actual efficiency of about 3 per cent., an ordinary enclosed-arc lamp which takes 2.2 watts per spherical-candle has an actual efficiency of about 5.2 per cent., and the tungsten-filament glow lamp which takes 1.25 watts per spherical-candle has an actual efficiency of about 9.4 per cent.

* This matter is discussed briefly in Appendix B.

† Engineers usually specify efficiency, approximately, in terms of watts per mean-horizontal-candle.

‡ *Physikalische Zeitschrift*, Vol. III, page 257, 1902.

CHAPTER X.

COLOR. *

113. Sensations of brightness and sensations of color. — As explained in Art. 109, a beam of light which falls upon the retina produces two distinct sensations, namely, a *sensation of brightness* and a *sensation of color*. The intensity of the sensation of brightness depends upon the physical intensity of the light, and the character and vividness of the sensation of color depends upon the wave-length of the light, or when the light is not homogeneous, upon the relative intensities of the various wave-lengths which are present in the light.

114. Luminosity of various parts of the spectrum. — Any part of the spectrum, of gas light, for example, may be bright or dim according to the intensity of the original source of light. The different portions of the spectrum from a given source of light, however, differ greatly in brightness and by no means in proportion to the energy that is present in the various parts of the spectrum. The relative brightness of the various parts of the spectrum of a given source of light of given intensity is called the *luminosity* of the various parts of the spectrum. The ordinates of the dotted curve in Fig. 196 represent the energy intensities, or heating effects, of the various parts of the spectrum of gas light, and the ordinates of the full-line curve represent the degrees of brightness (luminosities) of the various parts of the spectrum as perceived by

* The most complete treatise on color from both the physical and physiological points of view is to be found in Helmholtz's *Handbuch der Physiologischen Optik*, pages 275–384.

An extremely interesting discussion of brightness and color effects in their bearing upon painting is to be found in Helmholtz's popular lecture on The Relation of Optics to Painting, see footnote to Art. 111.

An interesting book is Ogden Rood's *Text Book of Color or Modern Chromatics*, New York, 1881.

the eye, the abscissas represent wave-lengths in hundred-millionths of a centimeter. The luminosities of the various parts of the spectrum may be most easily determined by the flicker photometer as follows:* One screen *B* of the flicker photometer (see Fig. 191) is illuminated by a definite part of the spectrum of the gas light and the other screen *A* of the photometer is illuminated by a standard light source of which the distance from the screen

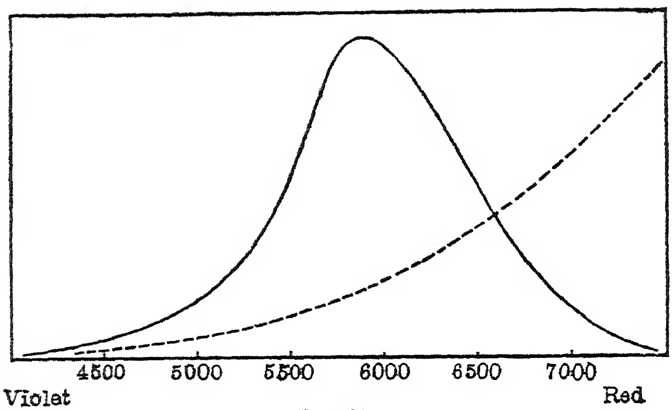


Fig. 196.

is adjusted until the two screens are of the same brightness, as indicated by the absence of the flicker as explained in Art. 109. The screen *B* is then illuminated by another part of the spectrum of the gas light and the same adjustment is repeated, and so on. Then the degrees of brightness (ordinates of the full-line curve in Fig. 196) of the various parts of the gas-light spectrum are to each other inversely as the squares of the respective distances of the standard light source from the photometer screen *A*.

The ordinates of the dotted curve in Fig. 196 were determined by the bolometer as explained in Appendix B.

115. Color sensations due to homogeneous light. — The various wave-lengths of homogeneous light produce the familiar series of colors which are visible in the spectroscope. Newton recognized and named seven colors in the spectrum: red, orange, yel-

* See *Practical Physics*, Franklin, Crawford and MacNutt, Vol. III, pages 26–27.

low, green, blue, indigo and violet. As a matter of fact, however, about 150 steps are made in going through the spectrum from one tint to the next which can barely be distinguished from it. That is to say, there are about 150 distinguishable tints in the spectrum. The most vivid colors in the spectrum are the extreme red, the green and the blue or violet.

116. Color sensations due to mixed light. — It is evident that there is a possibility of an infinite variety of mixtures of the various wave-lengths of light, according to the relative intensities of the various wave-lengths that are present in the mixture, and the number of distinguishable color sensations which may be produced by various light mixtures is, according to Titchener,* about 30,000, and, according to Rood,† a much larger number.

White light. — Sun light, or any light approaching sun light in composition, is called *white light.* The sensation produced by such light, aside from complications growing out of contrast effects, is called *white.*

Saturated and diluted colors. — Mixed light which contains a great excess of one wave-length or group of wave-lengths generally produces a vivid sensation of color. Such a color is sometimes called a *saturated color.* Mixed light which approaches white light in composition, having only a slight excess of one wave-length or group of wave-lengths, gives a pale sensation of color. Such a color is sometimes called a *diluted color.*

117. The cause of color in natural objects. — Colored objects occurring in nature owe their color to the fact that they send to the eye light of which the composition (relative intensities of the different wave-lengths) differs more or less widely from the composition of white light.

Examples. — (*a*) Hot gases and vapors give off light which differs widely in composition from white light as described in Art. 81. Thus, most of the color effects in fireworks are pro-

* Titchener, *An Outline of Psychology*, page 66.

† Rood, *A Text Book of Color*, Chapter IX.

duced by the use of salts of various metals such as strontium and copper. These salts are vaporized and the hot vapors give off brilliantly colored lights.

(*b*) Many substances reflect or transmit in excess certain wave-lengths of the light which falls upon them. Thus, powdered ultra-marine blue reflects about 24 per cent. of the light in the neighborhood of the *G* line of Fraunhofer and only about two or three per cent. of the light in the neighborhood of the *E* and *D* lines of Fraunhofer, as shown by the ordinates of the curve in Fig. 197. A moderately dilute solution of potassium chromate transmits about 80 per cent. of the light in the yellow and red (in the neighborhood of the *B*, *C* and *D* lines of Fraunhofer) and an extremely small per cent. of the light in the neighborhood of the *G* and *H* lines of Fraunhofer, as shown by the ordinates of the curve in Fig. 198.

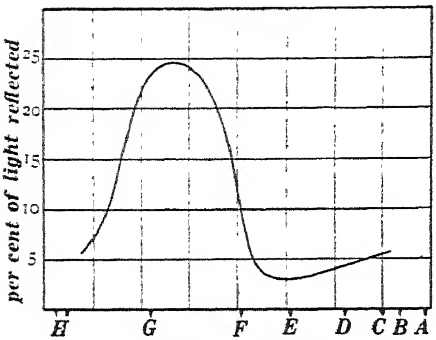


Fig. 197.

The curves of Figs. 197 and 198 were determined by the spectrophotometer. Thus, for example, two identical lamps may be used in Fig. 192, and a glass cell containing a so-

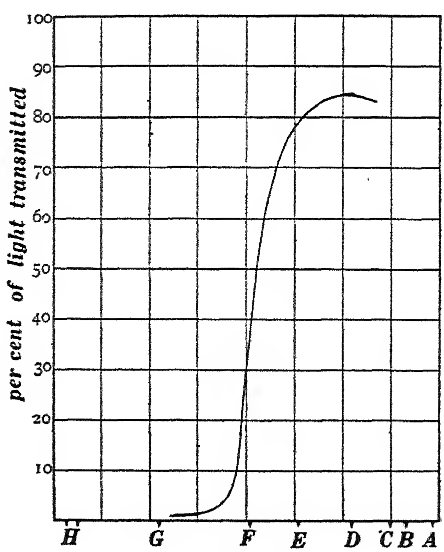


Fig. 198.

lution of potassium chromate placed in front of one of the lamps. With this arrangement the intensity of each wave-length of the light transmitted by the potassium chromate is compared with the intensity of the same wave-length from the other lamp.

The action of highly colored glasses in transmitting certain wave-lengths in great excess may be shown very strikingly as follows: Direct a spectroscope towards a gas flame and then insert a colored glass in front of the slit and withdraw it repeatedly. The selective action of the faintly colored objects can be shown only by the use of the spectrophotometer.

118. Color mixing. Dichroic vision and trichroic vision. — It has been known since the early days of painting that two colors completely blend when mixed, giving a single resultant color or tint. The mixing of colored lights and the mixing of pigments are, however, essentially different. Thus, ordinary yellow glass transmits a large amount of red and yellow and green light, and ordinary blue glass transmits a large amount of violet and blue and green light; and if two such glasses are laid one upon the other, the two together will transmit only green light because this light can pass through both. In general, colored pigments owe their colors to selective transmission, as in the case of colored glasses, and the mixture of a yellow pigment and a blue pigment gives a green pigment. When, however, *the light which is transmitted by red glass* is mixed with *the light which is transmitted by blue glass*, the resultant mixture contains every wave-length in approximately the same relative proportion as in white light, and, as a matter of fact, the resultant mixture is white light. * That is to say, the mixture of yellow and blue *lights* gives white, whereas the mixture of yellow and blue *pigments* gives green. The following discussion refers to the mixing of lights, not to the mixing of pigments, and two lights are said to be mixed when

*A light may contain two wave-lengths only and give the sensation of white. Thus yellow light (of one wave-length) and blue light (of one wave-length), when mixed in proper proportions, give a light which appears white.

they are allowed to enter the eye and fall upon the same portion of the retina.

The simplest device for mixing two lights is that which is shown in Fig. 199, in which *a* is a plate of unsilvered glass, and *b* and *c* are bits of colored paper or colored paint. The light from *b* passes through the glass plate *a* into the eye, and the light from *c* is reflected by the glass plate into the eye, as indicated by the dotted lines.

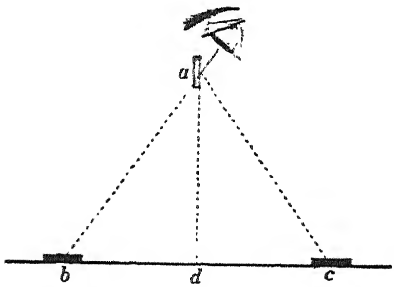


Fig. 199.

The most convenient arrangement, perhaps, for mixing colored lights is the *color top*. This consists of a rotating spindle upon which are mounted disks of colored paper which are slitted in such a way that any desired sector of the face of each disk may be exposed to view. When this composite disk is rotated rapidly the colors blend and give a single sensation of color, the tint of which may be modified at will by varying the amounts of the respective disks that are exposed.

Complementary colors. — Two colors which produce white light when mixed are called complementary colors. Thus, red and greenish-blue are complementary colors, yellow and indigo-blue are complementary colors, green and purple are complementary colors, and so on.

Matching of colors by mixtures. — *Any color may be matched by a mixture, in proper proportions, of a saturated red light, a saturated green light, and a saturated violet light.* This is an experimental fact which was first pointed out by Thomas Young.

For some persons (red blind) any color may be matched by a mixture, in proper proportions, of a saturated green light and a saturated violet light. For other persons (green blind) any color may be matched by a mixture, in proper proportions, of a saturated

red light and a saturated violet light. Persons for whom any color may be matched by mixing two saturated colors are said to have *dichroic vision.* Persons for whom the matching of any color requires the mixing of three saturated colors are said to have *trichroic vision.* Dichroic vision is commonly called *color blindness.* About four per cent. of the male population and about four tenths of one per cent. of the female population of the civilized world are color blind.

119. The Young-Helmholtz theory of color. — The fact that any color can be matched by a proper mixture of three saturated colors led Thomas Young, in 1801, to infer the existence of three primary color sensations. Helmholtz attributed each of these primary sensations to a distinct set of nerves in the retina of the eye. The nerves which upon excitation give the primary sensation of red are called the *red nerves*, those which upon excitation give the primary sensation of green are called the *green nerves*, and those which upon excitation give the primary sensation of violet are called the *violet nerves.* Simultaneous excitation of all three sets of nerves gives a blended sensation the character of which depends upon the relative intensities of excitation of the respective sets of nerves.

A person having dichroic vision has, according to the Young-Helmholtz theory, only two sets of color nerves, and therefore only two primary color sensations. Persons who do not have the primary sensation of red are said to be *red blind;* and persons who do not have the primary sensation of green are said to be *green blind.* No clearly defined case of *violet blindness* has ever been found.

The Young-Helmholtz theory of color is not generally accepted. Physiologists are inclined to reject it for lack of microscopical evidence of the existence of the three sets of nerves, and psychologists are inclined to reject it mainly because of the difficulty of explaining the great number of distinguishable color sensations by the varying intensities of sensation of three sets of nerves.

The theory gives, however, a very clear representation of the experimental facts of color mixing and a very satisfactory explanation of contrast effects and of color blindness.

Sensitiveness of the color nerves for different wave-lengths of light. — When a given sensory nerve is excited by different means, the sensation is always the same, as explained in Art. 2. Thus excitation of the red, green or violet nerves always gives sensations of red, green or violet, respectively, however the excitation may be produced. The ordinates of the three curves in Fig. 200 represent the relative degrees of sensitiveness of three sets

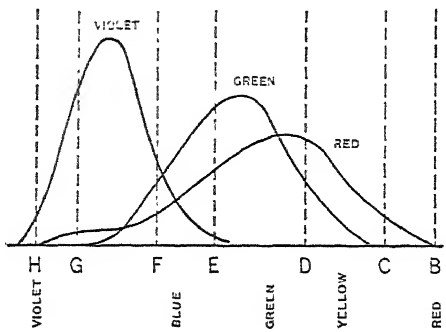


Fig. 200.

of nerves to the various wave-lengths of light. These curves are from measurements made by Koenig. Thus, every wave-length in the spectrum is capable of affecting the red nerves and in some degree producing the sensation of red. The green nerves are more or less affected by all wave-lengths between the Fraunhofer lines C and G, and the violet nerves are more or less affected by all wave-lengths between the Fraunhofer lines E and H. Any particular wave-length in the spectrum affects all three nerves to some extent and produces a blended sensation.

120. Contrast effects.*— Two complementary colors placed side by side tend to intensify each other mutually as the eye glances

* See Helmholtz's popular lecture on The Relation of Optics to Painting. See footnote to Art. 111.

from the one to the other. Any two colors placed side by side, if they are at all different, tend to become complementary in appearance as the eye glances from one to the other. This *contrast effect*, as it is called, may be shown very strikingly by taking two small pieces of pale green paper exactly alike and placing one on a large sheet of red paper and the other on a large sheet of blue paper. The two pieces of green paper are so greatly changed by the opposite contrasts that one can scarcely believe that they are physically alike. A bit of white or grayish paper placed upon a broad sheet of brilliantly colored paper assumes a very distinct hue complementary to the surrounding color.

The explanation of contrast effects in terms of the Young-Helmholtz theory of color is as follows: White light affects all three sets of color nerves in certain relative proportions. A colored light also affects all three sets of nerves, but the vividness of the color sensation which is produced depends upon the preponderating excitation of one or two sets of nerves. When one looks at a brilliant color, one or two sets of color nerves become more or less fatigued, and, under these conditions, a neutral tint produces a decreased excitation of these fatigued nerves or a relatively greater excitation of the unfatigued set of nerves, thus tending to produce the complementary color sensation.

121. Color blindness. — Color-blind persons show marked peculiarities in their sensations of brightness and of color. Thus, the ordinates of the dotted curve in Fig. 201 represent the luminosities of the various parts of the spectrum of gas light to a person who has normal trichroic vision, and the ordinates of the dotted curve* represent the luminosities of the various parts of the same spectrum to a person who is red blind. No data are available concerning the luminosities of the various parts of the spectrum to a person who is green blind.

Color sensations due to homogeneous light. — Figure 202 shows roughly the appearance of the spectrum to persons with normal

* From measurements made by Ferry, *American Journal of Science*, Vol. 44, 1892.

trichroic vision, to green-blind persons, and to red-blind persons respectively. The red and violet nerves of the green-blind person are both affected more or less by every wave-length, and that

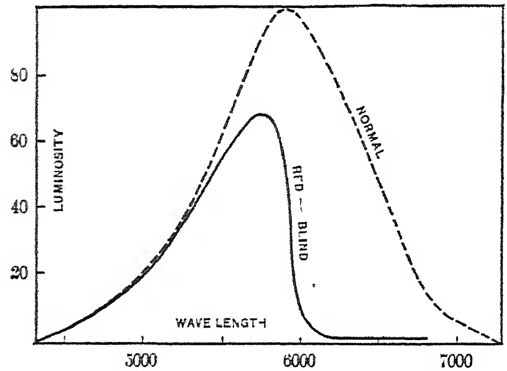


Fig. 201.

particular wave-length in the spectrum which affects these two sets of nerves in the same way that ordinary white light affects them, gives the sensation which the green-blind person calls white. This wave-length lies between the *E* and *F* lines of Fraunhofer, according to Fig. 200.

The green and violet nerves of the red-blind person are both affected more or less by every wave-length, and that particular

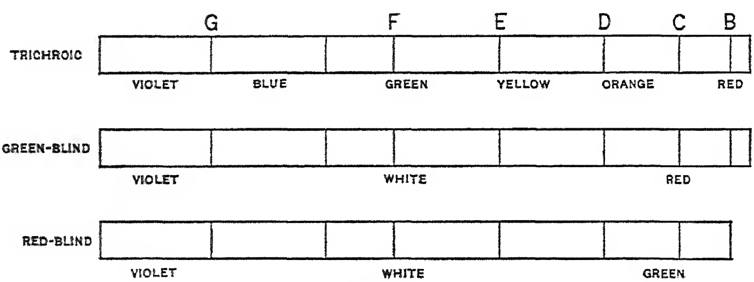


Fig. 202.

wave-length which affects these two sets of nerves in the same way that ordinary white light affects them, gives the sensation which the red-blind person calls white. This particular wave-

length lies between the E and F lines, according to Fig. 200. The red end of the spectrum does not extend much beyond the C line for a red-blind person.

Color sensations due to mixed light. — Color-blind persons show marked peculiarities of color sensations due to mixed lights. These peculiarities are very complicated, in fact, they are hardly the same for any two persons. The following article which describes the Holmgren test for color blindness, gives some idea of the peculiar color sensations of color-blind persons.

122. The Holmgren test for color blindness. — All systems of signalling upon railways and at sea, depend more or less upon the recognition of colored lights, and consequently the character of the color sense of employees is a matter of vital importance. It is fortunately possible to detect dichroic vision with certainty even when the existence of the peculiarity is unsuspected by the subject, or when the subject attempts to conceal the matter. The simplest method of performing such tests was invented by Holmgren, of Upsala, after the occurrence (in Sweden) of a dreadful railway accident due to the color blindness of an employe. The Holmgren apparatus consists simply of a collection of colored worsteds* which includes a large variety of colors of every degree of saturation and a number of neutral grays of various degrees of brightness. There are also three larger skeins which are called the *confusion samples.* One of these is a pale green, corresponding very nearly in tint (but of course not in saturation) to the portion of the spectrum which appears white to both red-blind and green-blind persons, as described in Art. 121. Another confusion sample is pale magenta, which is a blend of red and violet with much white, and the third confusion sample is a brilliant red.

Both red-blind and green-blind persons invariably select a variety of neutral or gray worsteds as corresponding most closely with the pale green confusion sample.

* For sale by E. B. Meyrowitz, East 23d St., New York City.

Red-blind persons invariably select violets and blues as corresponding most closely with the magenta confusion sample.

Both red-blind and green-blind persons are inclined to select brilliant greens as well as brilliant reds in matching the bright red confusion sample.

The selections made by a color-blind person in matching the confusion samples in the Holmgren test are very striking to an observer with normal color vision. A university student who was red blind, but who showed great power of discrimination in his choice of colors as viewed from his own standard, and who was fully aware of the unusual character of his color sense, made the following selections of worsteds which seemed to him most nearly related to the confusion samples:

Of the twelve skeins of worsteds selected to go with the pale green confusion sample, only one contained any green pigment. The remainder were pale yellows and browns almost colorless, and two skeins were of a very pale rose color.

All the worsteds which were selected to go with the magenta sample were purples in which blue predominated, some being very nearly pure blue.

In matching the bright red confusion sample a few nearly pure reds were chosen together with a large number of dark browns and brilliant greens.

CHAPTER XII.

POLARIZATION AND DOUBLE REFRACTION.

123. Polarization of transverse waves. — In longitudinal waves the medium always moves to and fro in the direction of progression of the waves, whereas in transverse waves the medium moves to and fro in a direction at right angles to the direction of progression of the waves, as explained in Art. 18. Transverse waves are said to be *plane polarized* when the oscillations are all in one direction or in one plane. Transverse waves are said to be *circularly polarized* when a particle of the transmitting medium describes a circular path as the waves pass by. Transverse waves are said to be *elliptically polarized* when a particle of the transmitting medium describes an elliptical path as the waves pass by.

These particular types of transverse waves may be most clearly understood by considering the transverse waves produced on a stretched rubber tube one end of which is held in the hand. If the hand is moved rapidly up and down (in a vertical plane), a train of plane polarized waves is produced, the oscillations all along the rubber tube being in a vertical plane. If the hand is moved rapidly in a small circular path around the stretched rubber tube as an axis, a train of circularly polarized waves is produced, and any given particle of the tube describes a circular path as the wave-train passes by. If the hand is rapidly moved in a small elliptical path, a train of elliptically polarized waves is produced, and any given particle of the tube describes an elliptical path as the wave-train passes by.

When one end of the stretched rubber tube, in the above illustration, is moved up and down, then to and fro sidewise, then in a circular or elliptical path in irregular succession, an irregular series of waves will pass along the tube.

Separation of plane polarized waves from irregular waves. —

A plane polarized series of waves on a rubber tube may be separated out from an irregular series of waves, that is, from a series devoid of any persistent and simple type of oscillation, by stretching the rubber tube through a narrow slit in a board. Those waves of which the oscillations are parallel to the slit, pass through the slit freely; those waves of which the oscillations are at right angles to the slit, do not pass through the slit at all, but are turned back or reflected; and those waves of which the oscillations are inclined to the direction of the slit are partly transmitted and partly reflected. In the latter case the oscillations are resolved by the slit into two components, parallel to the slit and perpendicular to the slit, respectively; the former component-oscillations pass through the slit and the latter component-oscillations are turned back by the slit, or reflected. The slit thus resolves an irregular series of waves into two series of plane polarized waves with their planes of oscillation at right angles to each other, one series being transmitted and the other series being turned back or reflected.

If the rubber tube is stretched through two slits, the waves which are transmitted by the first slit pass freely through the second, if the slits are parallel; the waves are partially transmitted by the second slit if the slits are inclined to each other; and the waves are completely stopped by the second slit if the slits are at right angles to each other.

124. The optical behavior of tourmaline crystals. Polarized light. — A plate of tourmaline, cut parallel to the axis* of the crystal, transmits only a portion of the light which falls upon it. The light which passes through such a plate passes freely through a second similar plate when the axes of the plates are parallel, but is shut off when the axes of the plates are at right angles. The beam of light transmitted by the first plate is *plane polarized.* When the second plate is slowly turned about the beam of light as an axis, the intensity of the beam which is transmitted by the pair

* See latter part of Art. 128.

of plates changes slowly from maximum intensity when the axes of the plates are parallel, to zero intensity when the axes of the plates are crossed. This is shown by the shading in Fig. 203*a*. A convenient mounting for two tourmaline plates is shown in Fig. 203*b*. This arrangement is called the tourmaline tongs.

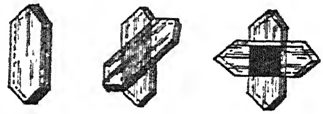


Fig. 203a.

The action which takes place in a tourmaline crystal and *which results in the absorption of light oscillations in a certain direction* is analogous to the action of the device shown in Fig. 204. A block of wood *AA* has two knife edges which rest upon shelves *SS*, and a pendulum swings about these knife edges without friction, as shown in the side view. A rod *RR* is fixed to the block *AA*, and the pendulum swings on this rod as a pivot subject to very considerable friction. If the pendulum is set oscillating in a plane which does not include the axis of the rod *RR*, then the component-oscillations of the pendulum which are in the plane of the knife edges die away rapidly because of the friction about the rod *RR*, leaving, after a short time, the component-oscillations which take place about the knife edges as an axis.

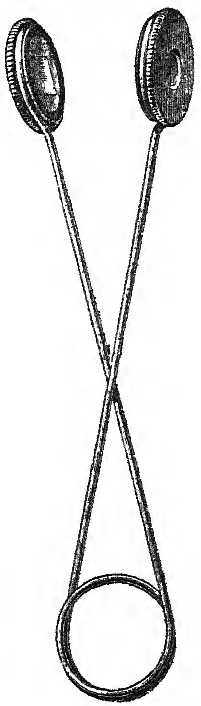


Fig. 203b.

125. Description of ordinary light, of plane polarized light, of circularly polarized light, and of elliptically polarized light, in terms of the elastic-solid theory. — Imagine a large number of thin flat boards arranged side by side as shown in Fig. 205, and imagine the spaces between the boards to be filled with elastic jelly. If the board *AB* be moved rapidly up

and down, a train of plane-polarized plane * waves pass out from AB to the right, as indicated by the dotted arrows, and each

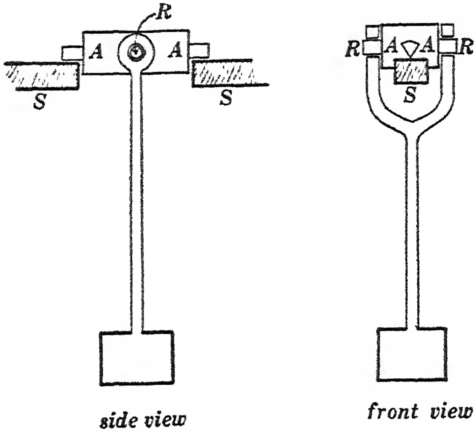


Fig. 204.

board oscillates up and down (in a vertical plane) as the wave-train passes by.

If the board AB is moved so that each point of the board describes a small circular or elliptical path in the plane of the board, a circularly or elliptically polarized train of plane waves passes out from AB to the right, as indicated by the arrows, and each board oscillates in a manner similar to the original oscillations of AB as the wave-train passes by. This motion of the system of the boards in Fig. 205 represents the motion of the ether in a beam of parallel rays (plane waves) of plane polarized, circularly polarized, and elliptically polarized light, respectively. In a beam of ordinary light, these and other types of oscillation follow each other with the utmost irregularity, any given type of oscillation persisting only

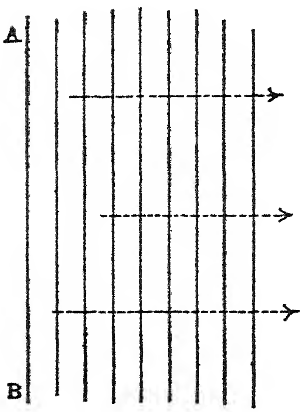


Fig. 205.

* Waves with plane fronts.

for an extremely short interval of time, not longer, perhaps, than a thousand-millionth of a second.

A beam of light is said to be *partially polarized* when oscillations in a certain direction preponderate. Such a beam of light varies in brightness when viewed through a slowly rotating tourmaline plate, but is not entirely extinguished by the plate in any position.

126. Polarization by reflection. — The theory of reflection which is discussed in Chapter III considers only the direction of the reflected ray, whereas a complete theory of reflection considers not only the direction of the reflected ray but its intensity and the character of its oscillations as to wave-length and polarization.* When ordinary light is reflected from the polished surface of a transparent substance, such as water or glass, the reflected light is partially or totally polarized. The degree of polarization varies with the angle of incidence. At normal incidence the reflected beam is not polarized at all, as the incidence becomes more and more oblique, the degree of polarization increases, and at a certain obliquity of incidence the polarization is complete if the polished surface is perfectly clean.† As the incidence becomes still more oblique the degree of polarization again falls off. When the reflected beam is completely polarized it is at right angles to the refracted beam as shown in Fig. 206. The angle of incidence i, Fig. 206, for which the polarization of the reflected beam is complete is called the *polarizing angle*. Its tangent is equal to the refractive index of the reflecting substance.‡ Thus for ordinary glass the polarizing angle is about 57° as shown in Fig. 207.

* To give even a bare outline of the fundamental ideas which are involved in the complete physical theory of reflection is beyond the scope of this text. This matter is discussed in a very satisfactory manner in Drude's *Theory of Optics*, translated by Mann & Millikan, pages 259–302.

† See Rayleigh, *Philosophical Magazine*, Vol. 16, pages 444–449, September, 1908.

‡ That is, $i = \tan^{-1}\mu$. This relation is known as Brewster's law. It may be easily shown from Fig. 206, remembering that the ratio of the sines of the angles i and r is equal to the index of refraction of the reflecting substance.

The oscillations of a beam of light which has been polarized by reflection are parallel to the reflecting surface. Thus, the oscillations of the polarized beams R and R, in Figs. 206 and 207, are perpendicular to the plane of the paper.*

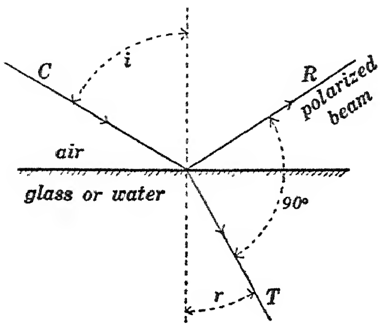


Fig. 206.

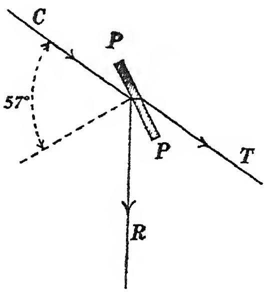


Fig. 207.

127. Reflection of plane polarized light from a glass plate. — Consider a beam of plane polarized light R, Fig. 208, which falls upon the glass plate A at the polarizing angle. If the glass plate A is turned about R as an axis, keeping the angle of incidence constant, the amount of light which is reflected varies from a maximum when the direction of oscillation of the incident beam is parallel to the surface of the glass plate A, to zero when the glass plate is turned one quarter of a revolution from this position. Figure 208 shows the two positions a and c of the glass plate A for which it reflects a maximum amount of the incident polarized beam R, and the two positions b and d for which it does not reflect any of the incident polarized beam R.

* It may be asked upon what evidence this statement is based; but it must be remembered that the notions of the elastic-solid theory of light are really unsound and that they are used in this chapter only for the sake of their concreteness. One of the most bitterly contested points in the elastic-solid theory refers precisely to this matter of direction of oscillation, and the bitterness of the contest was due to the lack of experimental evidence one way or the other. The whole matter is satisfactorily cleared up in the electromagnetic theory into a discussion of which we cannot enter here. A wave of light consists of a magnetic field and an electric field at right angles to each other and in the plane of the wave-front. The direction of the electric field is the direction of oscillation here referred to.

A small portion of the incident beam R is transmitted by the plate A in positions a and c, and all of the incident beam R is transmitted by A in the positions b and d. By using a number of glass plates instead of the single glass plate A in

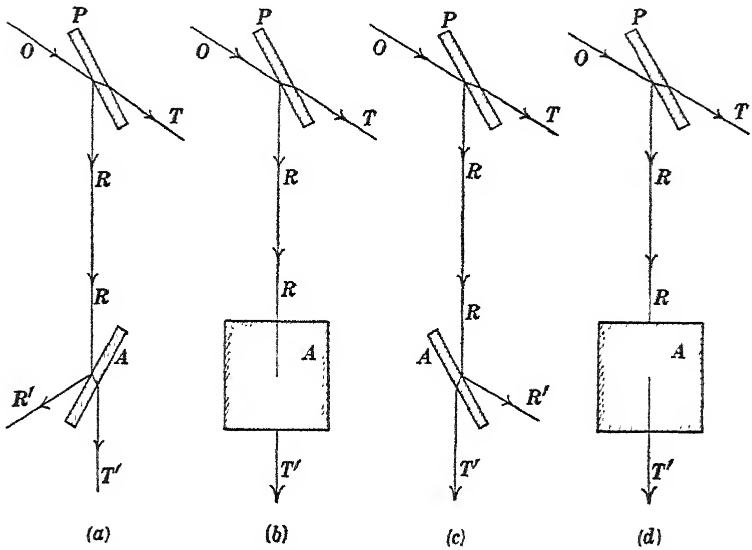


Fig. 208.

Fig. 208, the transmitted beam T' can be reduced sensibly to zero in positions a and c, Fig. 208. This arrangement is shown in Fig. 209.

An interesting experiment which shows the polarization of light by reflection is the following: Looking at a varnished picture so that light from a window is reflected from the varnish surface to the eye, the details of the picture are invisible because of the sheen due to the regularly reflected light. Looking through a tourmaline plate (or Nicol prism) which is held in the proper position, however, the regularly reflected light is cut off because it is polarized, and the details of the picture become distinctly visible.

128. Double refraction. — The theory of refraction which is discussed in Chapter III is limited to isotropic substances, and

it considers only the direction of the refracted ray, whereas a complete theory of refraction considers not only the direction of the refracted ray but its intensity and the character of its oscillations as to wave-length and polarization.* To give even a bare outline of the fundamental ideas which are involved in the complete physical theory of refraction is beyond the scope of this text, but that property of crystals which is known as double refraction may, however, be adequately described.

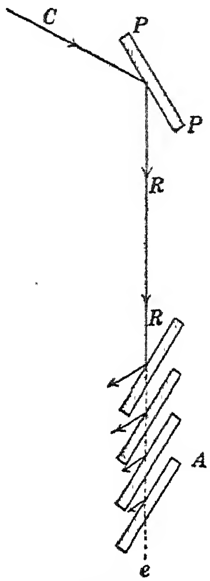


Fig. 209.

Many crystalline substances divide a beam of homogeneous light (one wave-length) into two beams by refraction. This phenomenon is called *double refraction.* The crystalline mineral, Iceland spar or calcite, separates the two refracted beams widely and therefore shows the effect very distinctly.

In order to get a clear idea of double refraction, let us consider a ray of light e, Fig. 210, which falls upon a glass plate AB and is refracted to the point p. If the point p were a luminous point, then pe would be a ray passing out from p, and an eye placed at e would see the point p at q. Under these conditions, the plate AB may be turned about the line fg as an axis without appearing to move the point q, the point p being stationary.

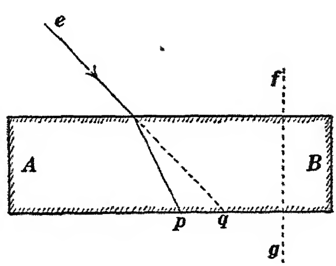


Fig. 210.

* This part of the physical theory of refraction is closely associated with the theory of reflection which is discussed in Drude's *Theory of Optics.* See foot-note to Art. 126. A very good discussion of the physical theory of refraction in crystalline substances is given in Drude's *Theory of Optics,* translated by Mann & Millikan, pages 308–357.

Huygens' theory of double refraction which is briefly discussed in this chapter is largely geometrical. For full discussion see Preston's *Theory of Light,* pages 324–347.

A beam of common light C, Fig. 211, falling upon a plate AB of Iceland spar, becomes two beams O and X in the crystal. Conversely a luminous point p, Fig. 211, sends out two particular rays o and x, parallel respectively to O and X, which become parallel rays r and r' in the air, so that an

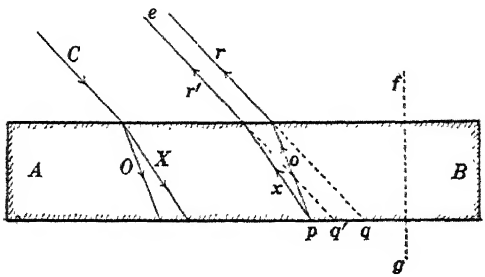


Fig. 211.

eye placed at e would see *two* images of the point p at q and q', respectively. If the plate of spar AB be turned about the line fg as an axis, while the point p remains stationary, then one of the images q remains stationary as if AB were a plate of glass, and the other image q' moves round q in a small circular path. The refracted ray o, or O, in the spar which corresponds to the stationary image q is called the *ordinary ray* inasmuch as it is refracted in the ordinary way as in glass; and the ray x, or X, in the crystal which corresponds to the moving image q' is called the *extraordinary ray* inasmuch as it is not refracted in the ordinary way as in glass. Some crystals (bi-axial crystals) divide a beam of common light into two beams neither of which follows the ordinary laws of refraction.

The rays r and r', Fig. 211, are completely polarized, and their planes of oscillation are at right angles. This may be shown by holding a tourmaline plate (or a Nicol prism) before the eye at e. As the tourmaline plate is turned, one after the other of the images q and q' becomes invisible.

Axis of symmetry of Iceland spar. Optic axis.—Any body may be turned one whole revolution about any axis and be in exactly its initial position so that to the eye it is just the same as

if the body had not been turned at all. Consider, on the other hand, a symmetrical body like a cube; there are certain axes about which a cube may be turned through one quarter, two quarters, or three quarters of a revolution and yet appear to the eye to be in exactly the same position as at first. This is an example of the kind of symmetry which is exhibited by crystals, and such an axis is called an axis of symmetry. A crystal of Iceland spar has one axis of symmetry about which it can be turned one third, or two thirds or three thirds of a revolution and appear as if it had not been turned at all.* Not only is the form of a crystal of Iceland spar symmetrical with respect to this axis, but the physical properties of the substance are also symmetrical with respect thereto. Thus, for example, the heat conductivity of Iceland spar is different in different directions but it is the same in all directions which are equally inclined to the axis of symmetry of the crystal. This axis of symmetry of the crystal is sometimes called the *optic axis* of the crystal.

129. Huygens' theory of double refraction in Iceland spar. — The phenomena of double refraction in Iceland spar were fully analyzed by Huygens, the discoverer of polarized light. Huygens assumed two secondary wavelets to enter a plate of Iceland spar from each point on its surface when an incident wave reached that point; one of these wavelets being a sphere and the other an ellipsoid of revolution. The envelope of the spherical wavelets determines the *ordinary* refracted wave as explained in Art. 34, and the envelope of the ellipsoidal wavelets determines the *extra-ordinary* refracted wave.

The spherical and ellipsoidal wavelets which enter a plate of Iceland spar at a point on its surface when an incident wave reaches that point, are most easily described by imagining a center of disturbance inside of the spar so that the wavelets may be a complete sphere and a complete ellipsoid, respectively. Let p, Fig. 212, be a center of disturbance in a piece of Iceland spar,

* The student should have access to a paste-board model of a crystal of Iceland spar in order to be able to understand this statement.

and let AB be the axis of symmetry of the crystal (optic axis). The circle represents the spherical wave which passes out from p, and the ellipse represents the ellipsoidal wave. The complete ellipsoid is generated by the rotation of the ellipse about AB as an axis, and the spherical and ellipsoidal wave surfaces touch each other at the poles A and B.

Straight lines drawn outwards from p in Fig. 212 are called *rays*. The disturbance which constitutes the spherical wave and the disturbance which constitutes the ellipsoidal wave may both be thought of as traveling outwards along these rays. The spherical wave is at each point perpendicular to the rays along which it is traveling; but the ellipsoidal wave is not at right angles to the rays at every point.

The velocity of the spherical wave in Iceland spar is the same in all directions, namely, 1/1.658 as great as the velocity of light in air. The velocity of the ellipsoidal wave in the direction of the axis AB is the same as the velocity of the spherical wave, and in all directions at right angles to AB the velocity of the ellipsoidal wave is 1/1.486 as great as the velocity of light in air. This is usually expressed by saying that the *ordinary index of refraction* of Iceland spar is 1.658, and that the *extraordinary index of refraction* of Iceland spar is 1.486. The axis AB in Fig. 212, or any line parallel to AB, is the optic axis of the crystal. Any plane which includes the optic axis is called a *principal plane.*

The oscillations of the spherical wave are everywhere perpendicular to the principal planes, that is to say, the movement of a spherical shell of the ether at the instant that the outwardly moving spherical wave reaches it may be imagined as a momentary rotatory twitch of the entire shell about AB as an axis, followed by a reverse twitch which brings the shell into its initial position.

The oscillations of the ellipsoidal wave are everywhere *in* the principal planes, that is, the movement of an ellipsoidal shell of the ether at the instant that the outwardly moving ellipsoidal wave reaches it may be imagined as a momentary twitch which

carries each part of the shell a short distance towards one pole of the ellipsoid along a meridian line and then back again. The points A and B are the poles of the ellipsoid in Fig. 212, and

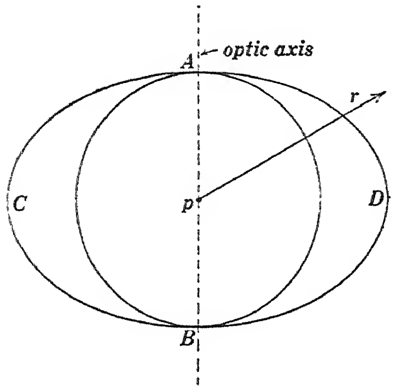


Fig 212. Wave surfaces in Iceland spar.*

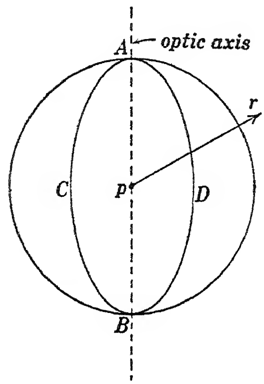


Fig. 213. Wave surfaces in quartz *

the meridian lines are the lines of intersection of the principal planes with the ellipsoid.

In Iceland spar, the diameter CD of the ellipsoidal wave is greater than its diameter AB parallel to the optic axis, as shown in Fig. 212. In quartz, however, the diameter CD of the ellipsoidal wave is less than its diameter parallel to the optic axis AB, as shown in Fig. 213.

Figure 214 shows Huygens' construction for the two refracted waves in Iceland spar. The figure represents the simple case in which the optic axis of the crystal lies in the plane which contains the incident ray i and which is perpendicular to the refracting surface, the *plane of incidence* as it is called.† WW represents an advancing wave in air, and $W'W'$ is the position this wave would have reached at *a given instant* if it had not encountered the piece of crystal. It is required to find the posi-

* Iceland spar is called a *negative* crystal and quartz is called a *positive* crystal in view of the difference shown in Figs. 212 and 213.

† The ordinary refracted ray o always lies in the plane of incidence. In the case which is represented in Fig. 214, the extraordinary refracted ray x also lies in the plane of incidence, but this is not generally the case.

tions of the two refracted waves at the given instant. When the wave WW reaches the point p, that point is a center of disturbance, and it sends out two wavelets into the crystal, a spherical wavelet and an ellipsoidal wavelet; and at the given instant the wavelets from p have had time to travel the distance d in air. Therefore the spherical wavelet in the crystal has a radius equal to $d/1.658$, and the ellipsoidal wavelet has a minor axis equal to the radius of the spherical wavelet and a major axis equal to $d/1.486$. The direction of the optic axis being given, both wavelets may be drawn as shown. Wavelets from other points in the refracting surface may be determined in a similar

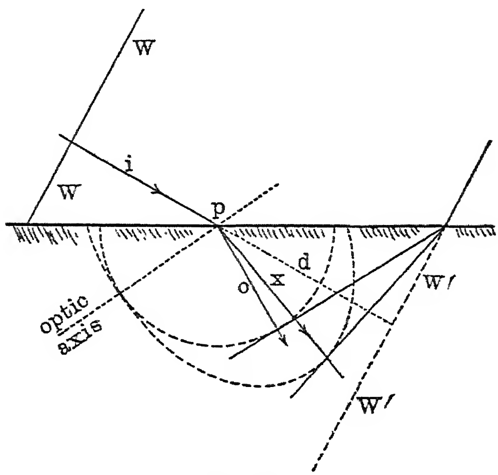


Fig. 214.

manner. The envelope of the spherical wavelets is the ordinary refracted wave, and the envelope of the ellipsoidal wavelets is the extraordinary refracted wave. The ordinary refracted ray o is the line drawn from p to the point where the spherical wavelet from p touches its envelope (ordinary refracted wave). This ray is at right angles to the ordinary refracted wave. The extraordinary refracted ray x is the line drawn from p to the point where the ellipsoidal wavelet from p touches its envelope (extraordinary refracted wave). This ray is generally not at right angles to the ex-

traordinary refracted wave. The direction of oscillation of the ordinary ray o is determined by the direction of oscillation of the spherical wavelets at the points where they touch their envelope; this direction is perpendicular to the plane of the paper in Fig. 214. The direction of oscillation of the extraordinary ray x is determined by the direction of oscillation of the ellipsoidal wavelets at the points where they touch their envelope; this direction is *in* the plane of the paper in Fig. 214. The directions of oscillation of the ordinary and extraordinary rays are most easily specified as follows: Imagine a principal plane (a plane containing the optic axis) drawn so as to include the ray o or x in Fig. 214. The oscillations of the ordinary ray are at right angles to this principal plane, and the oscillations of the extraordinary ray are *in* this principal plane.

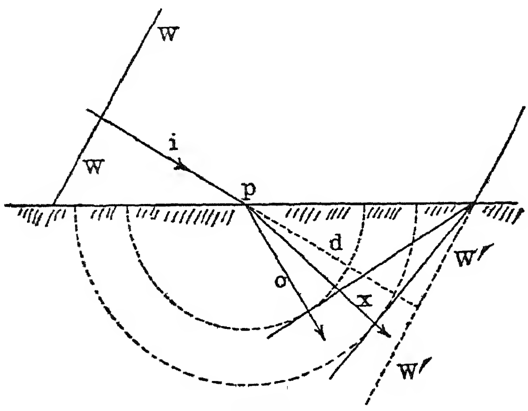


Fig. 215.

Figure 215 shows Huygens' construction for the case in which the optic axis is parallel to the refracting surface and at right angles to the incident ray i, or, in other words, the optic axis is perpendicular to the plane of incidence (the plane of the paper in the figure). The significance of the two dotted circles in Fig. 215 may be understood by referring to Fig. 212; these circles are the equatorial sections of the sphere and ellipsoid, respectively. The direction of oscillation of the ordinary ray is *in* the plane of the

paper and the direction of oscillation of the extraordinary ray is at right angles to the plane of the paper in Fig. 215.

Figure 216 shows Huygens' construction for an incident wave *WW* which is parallel to the refracting surface, the direction of

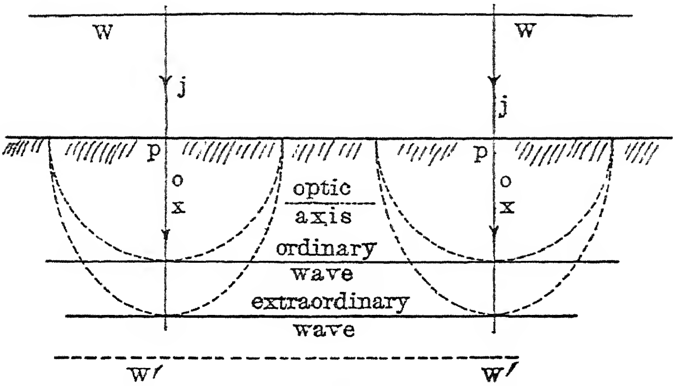


Fig. 216.

the optic axis being indicated by the short dotted line. Here the ordinary and extraordinary rays coincide, but there are two distinct refracted waves nevertheless.

130. The Nicol prism. — A beam of completely polarized light (plane polarized) may be obtained by reflection from a glass

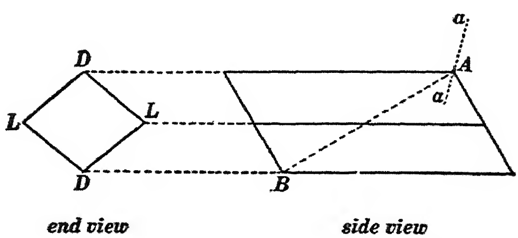


Fig. 217a. Fig. 217b.

plate as explained in Art. 126. A more convenient arrangement, however, for obtaining a beam of completely polarized light is the Nicol prism which is constructed as follows: A crystal of Iceland spar is reduced to the form shown in Fig. 217, by splitting

off layers of the crystal.* The line aa in Fig. 217 shows the direction of the optic axis. This rhomb of spar is sawed along dotted line AB perpendicular to the plane of the paper in Fig. 217. The sawed faces and the ends of the rhomb are polished, and the two blocks are cemented together again in their original positions with a thin layer of Canada balsam between them.

A beam of common light C, Fig. 218 is broken up by the prism into two beams o and x. The ordinary beam o is totally reflected

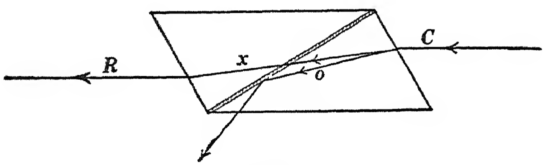


Fig. 218.

to one side by the layer of balsam, while the extraordinary beam passes on through the prism as shown. Therefore, the light R which emerges from the prism is completely polarized, and its direction of oscillation is *in* the plane of the paper in Fig. 218 (parallel to the shorter diagonal DD of the Nicol prism).

The reason for the total reflection of the ordinary ray is as follows: The index of refraction of the spar for the ordinary ray is 1.658 and the index of refraction of the spar for the extraordinary ray is between 1.486 and 1.658, according to its direction, or, say, 1.550. The index of refraction of the balsam is between 1.550 and 1.658. That is, the extraordinary ray goes from a rare to a dense medium in going from spar into balsam, and the ordinary ray would go from a dense to a rare medium in going from spar into balsam, the words dense and rare referring to large or small index of refraction, respectively. Therefore, the ordinary ray may be totally reflected if it strikes the balsam layer at sufficiently oblique incidence as explained in Art. 42. On the other hand, the extraordinary ray can only be partially reflected however oblique its incidence.†

* A crystal of Iceland spar has three cleavage planes, and the faces of the rhomb in Fig. 217 are cleavage planes of the crystal.

† Another device which is sometimes used for completely separating the rays o and x in Iceland spar and thereby producing a beam of plane polarized light, is Rochon's prism. See Preston's *Theory of light*, page 320.

131. Action of the Nicol prism on a beam of polarized light. — When a beam of ordinary light enters a Nicol prism as shown in Fig. 218, the beam is resolved into two polarized beams o and x; the beam x passes through the prism and the beam o is reflected to one side.

A beam of plane polarized light of which the oscillations are parallel to the diagonal DD, Figs. 217 and 219, becomes wholly

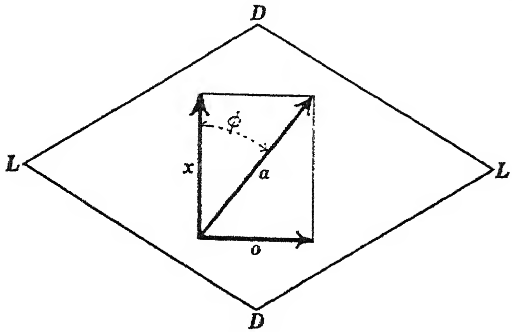


Fig. 219.

the extraordinary beam in the prism, and the whole * of the beam passes on through the prism.

A beam of plane polarized light of which the oscillations are parallel to the longer diagonal LL, Fig. 219, becomes wholly the ordinary beam in the prism, and it is totally reflected to one side by the layer of balsam.

Consider a beam of plane polarized light of which the oscillations are represented in direction and amplitude by the line a in Fig. 219. This beam is resolved by the crystalline material of the prism into two beams, ordinary and extraordinary, of which the amplitudes of oscillation are represented by the lines o and x, respectively. The extraordinary ray (which passes through the prism) has an amplitude x which is equal to $a \cos \phi$, and the intensity T of the transmitted beam is to the intensity I of the incident beam as $a^2 : a^2 \cos^2 \phi$, because the intensity of a beam of light is proportional to the square of its amplitude as explained on page 31. Therefore,

* A certian fractional part of x, Fig. 218, is reflected to one side by the layer of balsam.

$$T = I \cos^2 \phi \tag{13}$$

The intensity T of the beam which is transmitted by the Nicol prism varies therefore from a maximum $T = I$ when $\cos^2 \phi$ equals unity, to $T =$ zero when $\cos^2 \phi$ equals zero.

The intensity of the transmitted beam in Fig. 209 follows this same law, except that a larger portion of the light is absorbed in the glass due to imperfect transparency, ϕ being the angle through which A is rotated about the axis RR in Fig. 209.

132. The polariscope.—The polariscope consists of a Nicol prism P, Fig. 220, for producing a beam of plane polarized

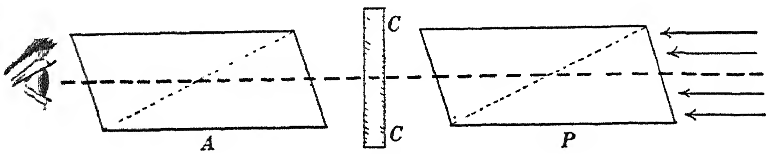


Fig. 220.

light, an arrangement for supporting a crystal plate CC to be examined, and a second Nicol prism A through which the plate CC is viewed. The Nicol prism P is called the *polarizer* and the prism A is called the *analyzer*. The analyzer A is mounted so as to turn freely about the axis of the instrument (the heavy dotted line in Fig. 220).*

When the analyzer A, Fig. 220, is turned so that the shorter diagonal of its face (DD, Fig. 219) is parallel to the shorter diagonal of the face of the polarizer P, then all of the light from P passes through A (crystal plate CC being removed), and the Nicols P and A are said to be *parallel*. When the analyzer A is turned so that the shorter diagonal of its face is at right angles to the shorter diagonal of the face of the polarizer P, then no light from P can pass through A (crystal plate CC being removed), and the Nicols P and A are said to be *crossed*.

* The arrangement shown in Fig. 209 constitutes a polariscope. In this case the object to be examined is placed between P and A, the eye is placed at e, and the pile of glass plates A is arranged to be turned about RR as an axis. The tourmaline tongs, Fig. 203*b*, is also a polariscope.

The arrangement of the polariscope for examining a plate of crystalline material CC in a convergent beam of polarized light is shown in Fig. 221*a*, in which DD is the focal plane of the

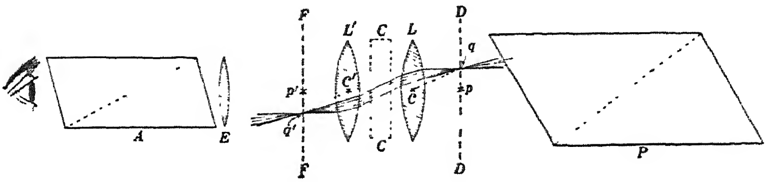


Fig. 221a.

lens L, and FF is the focal plane of the lens L'. Consider the polarized light from P *which passes through any given point* q *of the plane* DD. This light passes through the crystal plate as a pencil of rays parallel to the line drawn from q to the center of the lens L, and this light is concentrated at the point q' in the focal plane FF. The focal plane FF is viewed through an eye lens E and through an analyzing Nicol prism A. Under these conditions, the brightness and color of any point q' of the focal plane FF depends upon the action of the crystal plate CC on the pencil of rays parallel to the line $q'c'$, and therefore *the action of the crystal plate upon pencils passing through the plate in different directions is indicated by the* **distribution** *of light and color over the focal plane* FF, each point of the focal plane corresponding to a definite direction of pencil through the crystal plate. Figure 221*a* shows the two lenses L and L' as simple lenses, but in the practical form of instrument L and L' are compound lenses as shown in Fig. 221*b*.

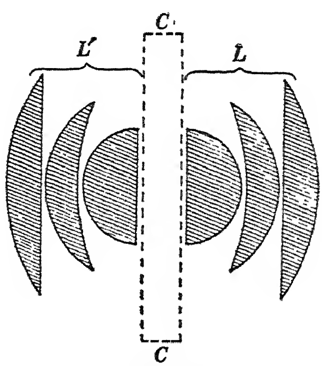


Fig. 221b.

133. Effect of a plate of a doubly refracting crystal upon a beam of plane polarized light. — In order to understand the character

of the oscillations of a beam of plane polarized light after it has passed through a crystal plate, it is necessary to consider the resultant of two oscillations at right angles to each other, the resultant of two oscillations being the movement of a point which performs both oscillations simultaneously. The following discussion refers to Figs. 222 to 230; to understand the discus-

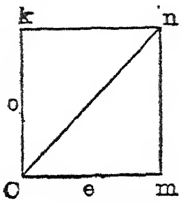


Fig. 222.

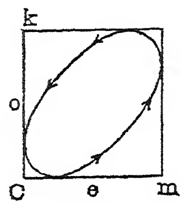


Fig. 223.

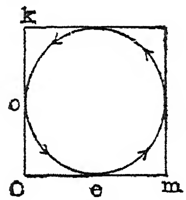


Fig. 224.

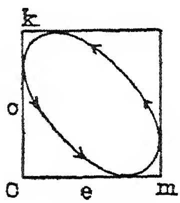


Fig. 225.

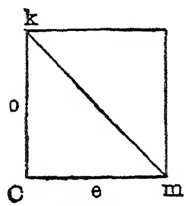


Fig. 226.

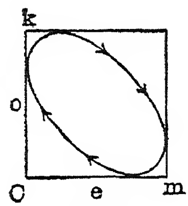


Fig. 227.

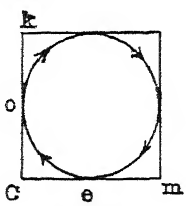


Fig. 228.

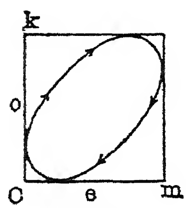


Fig. 229.

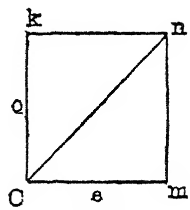


Fig. 230.

sion the reader should move a pencil point around the circles and ellipses repeatedly and watch the vertical and horizontal parts of the motion carefully.

Consider an oscillation back and forth along the line *e*, Fig. 222, and another of the same frequency along the line *o*. If the point which oscillates along *e* is at its turning point *m* at the same instant that the point which oscillates along *o* is at its turning point *k*, then the two oscillations are said to be in phase

with each other, and the resultant of the two oscillations is an oscillation along the line *Cn*.

If the point which oscillates along *e* has passed its turning point *m* and is, say, quarter way back towards *C* when the point *o* has reached its turning point *k*, then the oscillation *e* is about an eighth of a period ahead of the oscillation *o* in phase, and the resultant of the two oscillations is an elliptical oscillation as shown in Fig. 223.

If the point which oscillates along *e* has passed its turning point *m* and is, say, halfway back towards *C* when the point *o* has reached its turning point *k*, then the oscillation *e* is a quarter of a period ahead of the oscillation *o* in phase, and the resultant of the two oscillations is a circular oscillation as shown in Fig. 224.

When the oscillation *e* is three eighths of a period ahead of the oscillation *o* in phase, their resultant is the elliptical oscillation shown in Fig. 225. When the oscillation *e* is half a period ahead of the oscillation *o* in phase their resultant is the linear oscillation *km* shown in Fig. 226. The remaining figures 227, 228, 229, and 230 show the resultant of *e* and *o* when *e* is respectively $\frac{5}{8}$, $\frac{6}{8}$, $\frac{7}{8}$, and $\frac{8}{8}$ of a period ahead of *o* in phase.

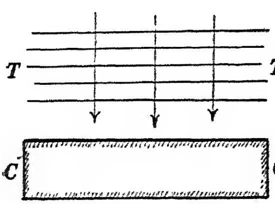


Fig. 231.

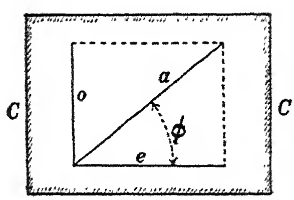


Fig. 232.

Consider a train of plane-polarized plane waves *TT*, Fig. 231, of wave-length λ, approaching the crystal plate *CC* as shown. Figure 232 is a top view of the crystal plate *CC* of Fig. 231, and the line *a* represents the direction and amplitude of the oscillations of the incident light *TT*. The incident beam is resolved by the crystal plate into two beams of which the oscillations

are represented by the lines e and o, Fig. 232. The oscillation e being *in* a principal plane of the crystal (a plane which includes the optic axis). *Let it be understood in the following discussion that the angle ϕ in Fig. 232 is equal to 45° so that o and e are equal in length.* The two beams corresponding to o and e travel through the crystal at different velocities, so that one beam is retarded with respect to the other. Let δ be the relative retardation produced by the crystal plate, that is, δ is the distance that beam o falls behind beam e; then, when the two beams of light emerge from the crystal plate, the oscillations of beam o will be δ/λ periods behind the oscillations of beam e in phase, and the resultant oscillation of the emergent beam will be similar to one of the resultant oscillations shown in Figs. 222 to 230, according to the value of δ/λ. Thus, if δ/λ is equal to $\frac{1}{4}$ (or equal to any whole number plus $\frac{1}{4}$), the emergent beam will be a right-handed circularly polarized beam of which the resultant oscillation is shown in Fig 224. If δ/λ is equal to $\frac{1}{2}$ (or equal to any whole number plus $\frac{1}{2}$), the emergent beam will be plane polarized, and its direction of oscillation will be the line km, Fig. 226. If δ/λ is equal to $\frac{3}{4}$ (or equal to any whole number plus $\frac{3}{4}$), the emergent beam will be a left-handed circularly polarized beam of which the resultant is shown in Fig. 228. If δ/λ is equal to zero or to any whole number, the emergent beam will be like the incident beam in every respect, as shown in Figs. 222 and 230.

These remarkable differences in the character of the oscillations of the emergent beam cannot be detected by the unaided eye, but when the crystal plate is placed in the polariscope, as shown in Fig. 220, the emergent beam is resolved by the analyzing Nicol prism producing observable effects which are described in Art. 134.

The quarter-wave plate. — When the crystal plate above described produces a relative retardation equal to one quarter of a wave-length ($\delta/\lambda = \frac{1}{4}$), then the plate is called a quarter-wave plate. The effect of a quarter-wave plate is to convert a plane-polarized

beam of light into a circularly polarized beam of light (when the angle ϕ in Fig. 232 is equal to 45°), or to convert a circularly polarized beam into a plane-polarized beam. Thus, the linear oscillation *Cn*, Fig. 222, is changed to the circular oscillation in Fig. 224 by a quarter-wave-length retardation of the oscillation *o*, the circular oscillation in Fig. 224 is changed to the linear oscillation *km*, Fig. 226, by an additional quarter-wave-length retardation of the oscillation *o*, the linear oscillation *km* in Fig. 226 is changed to the circular oscillation in Fig. 228 by another quarter-wave-length retardation of the oscillation *o*, etc.

When the angle ϕ in Fig. 232 is not equal to 45°, a plane-polarized beam *a*, Fig. 232, is converted by a quarter-wave plate into an elliptically polarized beam, as shown by the oscillation

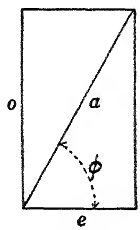


Fig. 233.

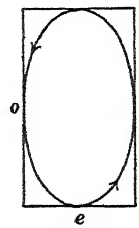


Fig. 234.

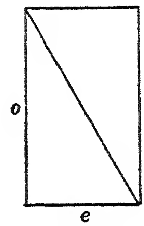


Fig. 235.

diagrams, Figs. 233 and 234; and if either axis of the elliptical oscillation of elliptically polarized light lies in a principal plane of the crystal plate (a plane which includes the optic axis), then the elliptically polarized light is converted into plane-polarized light by the quarter-wave plate, as shown by the oscillation diagrams, Figs. 234 and 235.

Plane-polarized light can be distinguished from common light and from circularly or elliptically polarized light by the fact that it is wholly cut off by a Nicol prism when the Nicol prism is turned into the proper position. Circularly or elliptically polarized light can be distinguished from ordinary light by the fact that circularly or elliptically polarized light may be converted into plane-polarized light by a quarter-wave plate. In the case of circularly polarized light the principal plane of the crystal plate

(quarter-wave plate) may be in any direction, and in the case of elliptically polarized light the principal plane of the crystal plate (quarter-wave plate) must coincide with one of the axes of the oscillation ellipse in order that plane-polarized light may be produced.*

134. **Appearance of a plate of a doubly refracting crystal in the polariscope.** — Consider a crystal plate CC' in a polariscope as shown in Fig. 220. The incident beam of parallel rays of plane-polarized light breaks up into beams o and e as indicated in Fig. 232; these two beams traverse the crystal plate at different velocities, thus producing retardation of one beam relative to the other, and the oscillations of the emergent beam are like one of the Figs. 222 to 230. The appearance of the crystal plate to the observer may be discussed on the basis of the oscillation curves, Figs. 222 to 230, or in a more simple way, as follows: Let us assume that monochromatic light (light of one wave-length) is used. The beam of light after emerging from the crystal plate may be thought of as being composed of the two distinct oscillations o and e, Fig. 232; that is, it is not necessary to consider the *resultant* of these two oscillations. Let the heavy dotted line in Fig. 236 be the direction of the shorter diagonal of the analyzer in Fig. 220. Then the analyzer transmits the two components o' and e' of o and e, respectively, as indicated in the figure, and the intensity of the beam which enters the observer's eye depends on whether the oscillations o' and e' are alike in phase or opposite in phase. These two oscillations o' and e' are alike in phase if the retardation δ is any whole number of wave-lengths, and they are opposite in phase if the retardation δ is an odd number of half wave-lengths. In the first case the intensity of the transmitted beam is a maximum, and in the second case the intensity of the transmitted beam is a minimum. This minimum is zero if o' and e' are equal in Fig. 236.

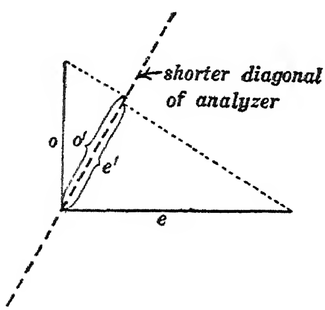


Fig. 236.

When white light is used in Fig. 220 instead of monochromatic light, then those particular wave-lengths in the transmitted beam are strengthened (in Fig. 236) for which the ratio δ/λ is a whole number, and those particular wave-lengths are weakened for which the ratio δ/λ is an odd number of halves. If the crystal plate is thick and if the difference in the velocities of the ordinary and extraordinary waves is considerable, then the retardation δ will be considerable and a great many different wave-lengths between red and violet will satisfy the above conditions, that is to say, a great many wave-lengths will be strengthened and intervening wave-lengths will be weakened, so that no color will be visible. If, however, the crystal plate is thin so that the retardation δ is small, then only a few wave-lengths, or, it may be, one single wave-

* Methods of detecting and studying polarized light are discussed in Preston's *Theory of Light*, pages 408–424.

length between red and violet will be intensified (or weakened) and brilliant color is produced.

Suppose the oscillations o' and e' are in phase with each other in Fig. 236. Then they are opposite to each other in phase in Fig. 237. In the one case a particular wave-length is strengthened, and in the other case the same wave-length is weakened by interference. Therefore, turning the analyzing Nicol through 90° changes the color from a certain tint for Fig. 236 to the complementary tint for Fig. 237.

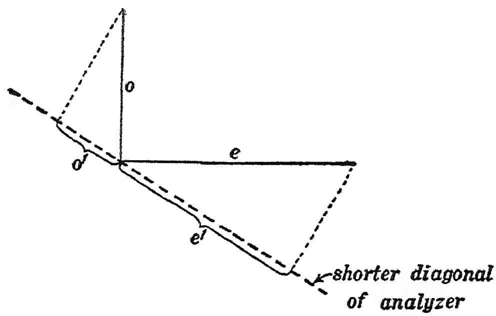


Fig. 237.

If the shorter diagonal of the analyzing Nicol is parallel to o (or to e), then no interference effects whatever are produced, because only one of the constituent beams o and e passes through the analyzer.

The entire discussion of this article applies to a plate of a bi-axial crystal such as mica, or to a plate of a uni-axial crystal such as Iceland spar. The specifications. however, apply to a plate of a uni-axial crystal, inasmuch as the term *optic axis* has been used.

135. Appearance of a plate of a doubly refracting crystal in a polariscope with a convergent beam of polarized light. — The following discussion applies to a plate of Iceland spar cut so that the optic axis is perpendicular to the two faces of the plate. Let *CC*, Fig. 221, represent the crystal plate in the polariscope, and let *CC*, Fig. 238, represent an enlarged view of the crystal plate. In the discussion of Fig. 221, it was shown that the light which illuminates a given point q' of the focal plane *FF* passes through the crystal plate as a beam of parallel rays more or less inclined to the axis of the polariscope, as shown in Fig. 221; and the distance $p'q'$ in Fig. 221 depends upon the inclination of the beam in the crystal plate. The beam in the crystal plate is resolved into ordinary and extraordinary parts with their directions of oscillation at right angles to each other, and the ordinary beam is more or less retarded by the crystal plate relatively to the extraordinary beam. Let δ be this relative retardation. The two beams are then focused by the lens L' at the point q', after which the beam spreads out again, passes through the eye lens E and through the analyzer A to the observer's eye. A certain component, only, of the oscillations of each beam passes through the analyzer, as shown in Figs. 236 and 237. Let us suppose that the shorter diagonal of the analyzer is in the position shown in

Fig. 237, then in the resultant beam from q' which enters the observer's eye those particular wave-lengths are weakened for which δ/λ is a whole number, and those particular wave-lengths are strengthened for which δ/λ is an odd number of halves.

All points like q' which are *at a given distance from the central point* p' in Fig. 221 correspond to beams which traverse the crystal plate at a given inclination, the

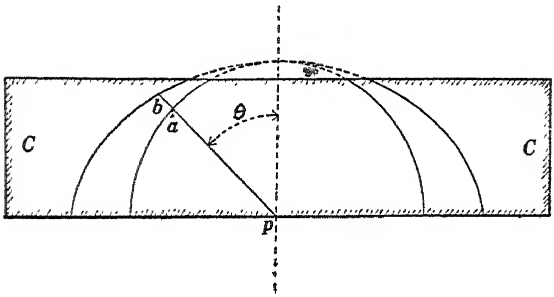


Fig. 238.

inclinaton θ as shown in Fig. 238; and the relative retardation δ of the ordinary and extraordinary parts is the same for all such beams. At increasing distances from the central point p' in Fig. 221 the relative retardation δ increases for two reasons, namely, (a) because the difference in velocities of the ordinary and extraordinary waves increases as indicated by the radii pa and pb in Fig. 238, and (b) because the distance traversed in the crystalline substance increases with increasing value of θ. Therefore, if monochromatic light is used, the central point p' in Fig. 221 will be surrounded by light and dark rings, or if white light is used the central point will be surrounded by colored rings, as shown in Figs. 239*a* and 239*b*.

Fig. 239a.

Fig. 239b.

Iceland spar is called *uni-axial* crystal. Sugar, mica, and many other crystalline substances are *biaxial*. Thus Fig. 240 shows the appearance of a plate of aragonite (a biaxial crystal) in a polariscope with convergent polarized light.*

* A good discussion of the optical properties of crystals is to be found in Preston's *Theory of Light*, pages 309–347. See also *A Text Book of Mineralogy* by E. S. Dana, John Wiley & Sons. *Physikalische Krystallographie* by P. Groth (Leipsig, 1895) and a book of the same title by T. Liebisch (Leipsig, 1896) are among the best treatises on the optical properties of crystals.

The dark cross in Fig. 239*a*, and the white cross in Fig. 239*b* are explained as follows: Figure 241 is a diagram of the field of view as seen by an observer looking through the polariscope of Fig. 221. The center of the field is at p', and the double-headed arrow A shows the direction of oscillation of the polarized light which comes from the polarizing Nicol P of the polariscope. Consider any point q' of the field of view. The ray which comes through the crystal plate and reaches this point q' is broken up by the crystal into ordinary and extraordinary rays of which the directions of oscillation are shown by the lines o and e, respectively, in Fig. 241, as explained

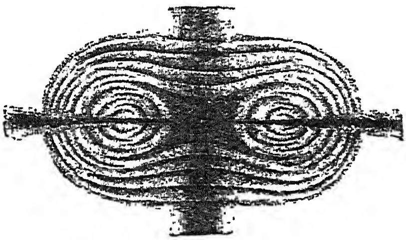


Fig. 240.

in Art. 129. When the point q' lies on the dotted line CC, Fig. 241, the ray which reaches q' comes through the crystal plate wholly as the ordinary ray, it emerges from the crystal plate with its character entirely unchanged by the crystal, and therefore it is not transmitted by the analyzer if the analyzer is in the "crossed position" and it is transmitted by the analyzer if the analyzer is in the "parallel position." When

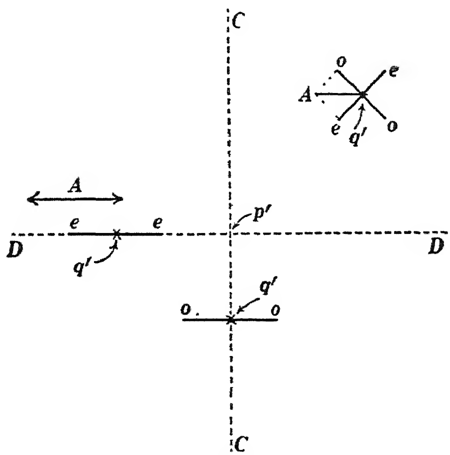


Fig. 241.

the point q' is on the dotted line DD, the ray which reaches q' comes through the crystal plate wholly as the extraordinary ray, it emerges from the crystal plate with its character entirely unchanged by the crystal, and therefore it is not transmitted by the analyzer if the analyzer is in the "crossed position" and it is transmitted by the

analyzer if the analyzer is in the "parallel position." Therefore with the analyzer in the "crossed position" the system of rings shows the dark cross as indicated in Fig. 239*a*, and if the analyzer is in the "parallel position" the system of rings shows the white cross as indicated in Fig. 239*b*.

136. Rotation of the plane of polarization. The saccharimeter. — Many substances, such as crystalline quartz and solutions of sugar, cause the direction of oscillation of a beam of polarized light to change continuously while the beam is in transit through the substance, so that the direction of oscillation of the emergent beam is different from the direction of oscillation of the incident beam. The angle α through which the direction of oscillation is turned varies with the wave-length of the light, and for a given wave-length it is proportional to the distance d traversed by the polarized beam in the substance; that is,

$$\alpha = kd$$

in which k is a proportionality factor which is called the *specific rotary power* of the substance.

In case of sugar solutions the angle α is proportional to the number of grams m of sugar per liter of solution, and to the distance d traversed by the polarized beam in the solution; that is,

$$\alpha = k'md \tag{15}$$

in which k' is a constant. For cane sugar the value of k' is 0.00665 for sodium light, when α is expressed in degrees, m in grams of sugar per liter of solution, and d in centimeters. *The saccharimeter* is a polariscope between the polarizer and analyzer of which is placed a long tube with glass ends. A solution of sugar is placed in this tube, and the analyzer is arranged to permit the measurement of the angle α, whence, knowing k' and d, the strength of the solution may be calculated from equation (15).*

* The saccharimeter of Laurent is described on pages 68–71 of Vol. III of Franklin, Crawford and MacNutt's *Practical Physics*. A very good discussion of various forms of saccharimeter is given by Otto Lummer on pages 1172–1188 of Müller-Pouillet's *Lehrbuch der Physik*, Vol. II, part 1, Braunschweig, 1897.

137. Polarization of sky-light. The light from the sky is polarized. The light which comes from the region of the sky near the sun is only very slightly polarized; the degree of polarization increases with increasing angular distance from the sun, reaches a maximum in the light from that region of the sky which is 90° from the sun, and then falls off with increasing angular distance. The direction of oscillation of polarized sky-light from a given point of the sky is at right angles to a plane passing through the sun, through the given point of the sky and through the observer's eye. The explanation of the polarization of sky-light, expressed in terms of the elastic-solid theory of light, is as follows: Consider an extremely minute particle of dust vibrating to and fro along a straight line. Such a particle can give off transverse waves (light waves) in directions at right angles to its direction of oscillation, but it cannot give off transverse waves in the direction of its line of oscillation. The dust particles in the atmosphere may be thought of as being set into oscillation by the waves of sun light, these oscillations being at right angles to the sun's rays. Imagine p, Fig. 242, to be a dust particle in a beam of sun light which is perpendicular to the plane of the paper. This particle will oscillate *in* the plane of the paper, and the waves which it gives out towards e (the eye of the observer) are plane polarized with their direction of oscillation *in* the plane of the paper.

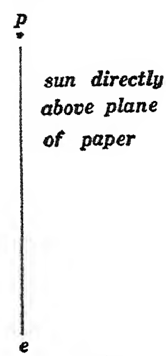


Fig. 242.

An interesting experiment is as follows: A tall glass jar is filled with water made very slightly cloudy by a few drops of dilute solution of rosin in alcohol, and a beam of light is passed downwards into the jar. The column of cloudy water gives off light sidewise in all directions, and this light is polarized as may be shown by looking at the column through a Nicol prism and slowly turning the prism. If a beam of plane-polarized light is sent downwards into the jar of cloudy water, the column of cloudy

water gives off light chiefly in directions at right angles to the direction of oscillation of the polarized beam.

An experiment which shows the rotation of the plane of polarization by a sugar solution is as follows: A tall glass jar is filled with a sugar solution made very slightly cloudy by a few drops of a solution of rosin in alcohol, and a beam of plane-polarized light is sent downwards into the jar. In this case the direction of oscillation of the polarized beam is rotated by the sugar solution, and the light which is given off sidewise by the cloudy liquid gives the column of liquid the appearance of a helix. In this case, the plane of polarization of the different wave-lengths of light are rotated differently by the sugar solution, and in fact the light which is given off sidewise by the cloudy liquid makes the column of liquid appear to be made up of a number of colored helices. This experiment must be performed in a very dark room, and the observer must be near to the column of sugar solution in order to see the effect distinctly.

SOUND.

The following treatise on sound is abbreviated to the utmost, and the discussion is limited to those things which are of practical importance. A most interesting book is *Sound* by John Tyndall, published by Longmans, Green & Co., in 1875. This book is almost entirely descriptive in character. One of the best simple treatises on sound is Poynting and Thomson's *Sound*, published by Charles Griffin and Co., in 1899. This book contains some of the more important discussions of the mathematical theory of sound. The most complete treatise on the theory of sound is Lord Rayleigh's *Theory of Sound* in two volumes. The second edition of this work was published by Macmillan and Company in 1894. The most complete treatise on sound from the physiological point of view is Helmholtz's *Tonempfindungen*. (Translated by Alexander J. Ellis, Longmans & Co.)

CHAPTER XII.

TONES AND NOISES. LOUDNESS, PITCH AND QUALITY.

138. Differences between the methods of optics and the methods of acoustics. — In the study of light we have been concerned chiefly with the phenomena of transmission and the phenomena associated with transmission such as reflection, refraction and diffraction. The more advanced study of optics is, indeed, concerned with the manner in which light waves are generated in luminous substances and the precise character of the disturbance which light waves produce in a body which receives them,* but these matters are beyond the scope of an elementary text. In the study of sound, on the other hand, we are concerned chiefly with the mode of generation of sound waves,† and with the effects produced by sound waves upon bodies which receive them. The phenomena associated with the transmission of sound, namely, the phenomena of reflection, refraction and diffraction are indeed as real as in the case of light, but these phenomena are not, as a rule, of great interest. The more important phenomena of this group are discussed in Chapter XVII.

139. Noises and musical tones. — It was pointed out in Chapter I that sound, in the physical sense of the word, consists of waves of disturbance in the air. These waves are produced by abrupt movements of bodies, and when they fall upon the ear a sensation of sound is produced. When the movement of a sounding body is periodic and regular, a *train* of sound waves is

* The study of spectrum analysis at the present time is concerned, to some extent, with the question as to the manner in which light waves are generated in luminous gases. The modern theory of refraction, including the theory of dispersion (see Drude's *Theory of Optics*, pages 382–399), has to do with the character of the disturbance produced in a body which receives light waves.

† See Arts. 4, 5, 7, 8, 10, and 12–25 for a preliminary discussion of sound waves.

produced; the sensation corresponding to such a wave train is called a *tone*.

Sound sensations which cannot be classified as tones or combinations of tones are called *noises*. For example, rattling noises are due to irregular successions of sharp clicks, each of which sends a single wave pulse to the ear. Hissing and roaring noises are due to complex and rapidly varying combinations of tones. In the case of hissing noises the tones, which may be few in number, are of very high pitch,* and in the case of roaring noises the tones are usually numerous and of lower pitch.

All manner of combinations of rattling, roaring and hissing noises occur, ranging from those combinations of musical tones which begin to be so complicated that a hearer cannot distinguish the various tones and recognize their relations to one other, to the extremely complex sounds from waterfalls, railway trains and busy streets.

Musical tones are generally accompanied by characteristic noises. Thus, the whispering noises of the breath and the sounds of the consonants used in articulation accompany the musical tones of a singer; and the faint noises produced by the fingers, keys and pedals always accompany piano music. Many noises, on the other hand, are accompanied by distinctly audible musical tones; thus, a light hammer-blow upon a floor or upon a piece of furniture produces a musical tone of short duration which is often prominent enough to be easily distinguishable.

140. Loudness. — It is a familiar fact that the sound emitted by a vibrating body, such as a guitar string, increases in loudness with increase of the amplitude of the vibrations (by amplitude is meant half the distance through which the middle of the string swings to and fro). When sound waves travel through the air, any given particle of the air oscillates to and fro through a certain amplitude, and the amplitude of the air oscillations increases with the amplitude of the given vibrating body which produces the

* See Art. 141.

sound waves.* The loudness of a tone depends upon the amplitude of the air oscillations.

No simple and satisfactory method has been devised for measuring the loudness of a sound, and in fact there is very little practical need of such measurement.

141. Pitch. — Consider a musical instrument such as the piano or harp. The short strings have a greater frequency of vibration than the longer strings, and that quality of a musical tone which depends upon the frequency of the vibrations is called *pitch*. The pitch of a tone is high or low according as the vibration frequency is great or small. Two vibrating bodies which give tones of the same pitch are said to vibrate in unison.

Determination of pitch.† — The direct determination of pitch is accomplished by counting the number of vibrations in a given time. The siren is sometimes used for this purpose. It consists of a circular metal disk mounted on a shaft which is geared to a revolution counter. The disk has one or more circular rows of equidistant holes, it rotates near to the wall of a chamber containing air under pressure,‡ and the holes in the disk come before apertures in the wall in rapid succession. The puffs of air thus produced blend into a tone, the pitch (number of vibrations per second) of which is known from the observed speed of the disk and the known number of holes in the row. The disk is sometimes driven by an electric motor and sometimes by the action of the puffs of air. In the latter case the holes in the disk are inclined like the vanes of a wind-mill.

Standards of pitch. — A vibrating body which has an invariable frequency of vibration may have its pitch accurately determined once for all, and the pitch of any sound may then be determined

* A vibrating string may oscillate through a wide amplitude and emit a much weaker sound than is emitted by a board or plate which vibrates through a much smaller amplitude, so that the amplitude of the vibrating body *alone* does not determine the amplitude of the air oscillations.

† See Poynting and Thomson's *Sound*, pages 36–47.

‡ In the simplest form of siren, a nozzle is held near the rotating disk, and air issues from the nozzle in a series of puffs as the holes pass in front of the nozzle.

by comparison with this standard. Standards of pitch are usually in the form of tuning-forks. Tuners of musical instruments always compare a tone to be determined with the tone of a standard fork by the unaided ear. A little practice enables such a comparison to be made with accuracy when the ratio of the frequencies of the two tones is very nearly in a simple ratio such as 1 : 2, 2 : 3, 3 : 4, etc. When the difference in pitch is very small this comparison can be made with great accuracy by counting the beats (see Art. 162).

Pitch limits of audibility. — When a vibrating body has a frequency less than about 30 complete vibrations per second, it does not produce a musical tone, but the ear perceives the separate impulses as a fluttering noise. When a vibrating body makes more than from twenty to forty thousand complete vibrations per second (according to the person), no sensation of sound at all is produced. The variation in the upper pitch limit of audibility with different individuals is strikingly illustrated by the fact that sometimes one person can hear a flood of sound due to the chirping of insects when another person can hear nothing at all.

Young persons can usually hear sounds of higher pitch than old persons. In fact, the impairment of hearing with advancing age shows itself generally by a lowering of the upper pitch limit of audibility, and a test which is frequently used by the physician is to determine the upper pitch limit of audibility by means of an adjustable whistle which was devised by Galton and which is called *Galton's whistle*. This whistle, which is very small in size, is made of a metal tube in which a piston is operated by means of a micrometer screw, and the pitch of the tone produced by the whistle (number of vibrations per second) is determined* once for all for various readings of the micrometer screw. In testing the upper limit of audibility, the whistle is blown by an intermittent blast of air and the vibrating air column in the whistle is gradually shortened by moving the piston until the sound of the whistle is no longer heard by the subject who is being tested.

* See paper by M. T. Edelmann, *Annalen der Physik*, July, 1900.

142. Tone quality or timbre. — It is a familiar fact that one can easily distinguish musical tones produced by different musical instruments independently of differences in loudness and pitch. Thus, a note of the same pitch and loudness produced by a singer

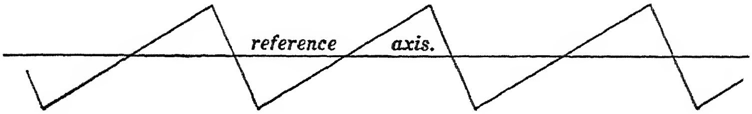


Fig. 243.

and by a piano are so entirely different that there is no difficulty whatever in distinguishing the one from the other. This difference in quality depends upon the character of the oscillations of the vibrating body (or upon the character of the oscillations of the air particles in the sound waves which are produced by the vibrating body). Thus, a point on a violin string usually moves slowly in

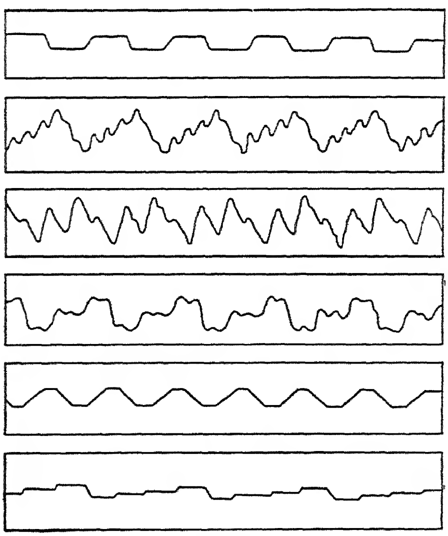


Fig. 244.

one direction and more rapidly in the opposite direction, like the up and down motion which would have to be given to a pencil point to trace the zig-zag line in Fig. 243 with the paper moving uniformly to the left; but the character of the motion of a point on a

violin string may be varied in a remarkable way by changes in the position, velocity and pressure of the bow. Thus, Fig. 244 * shows photographic tracings of the to-and-fro motion of a point on a violin string, and the sound produced by the string is in each case strikingly different from that produced in the other cases. Figure 21 shows the character † of the oscillations which correspond to various vowel sounds when sung by a baritone voice.

143. Simple and compound vibrations. — When a particle moves to and fro along a straight line performing simple harmonic motion,‡ its vibrations are called *simple vibrations*. When the to-and-fro motion of a particle is periodic, but not simply harmonic, its vibrations are called *compound vibrations.*§

Examples. — The vibrations of a pendulum bob, and the vibrations of the prong of a tuning fork are simple vibrations. The vibrations of a reed which is slowly lifted and quickly dropped by the successive cogs of a rotating wheel are compound vibrations.

Graphical representation of simple and compound vibrations. — Consider a point p, Fig. 245, vibrating up and down along the line AB, and imagine the paper to move with uniform velocity to the right, then the point p will trace a curved line cc. If the vibrations of p are simple, the curve cc will be a curve of sines. If the vibrations of p are compound, the curve cc will

* Taken from a paper by Krigar-Mensel and Raps, *Wiedemann's Annalen*, Vol. 44, page 623.

† It must not be thought from this illustration that a given vowel sound is produced by a characteristic type of oscillation. This matter is fully discussed in Article 158.

‡ Simple harmonic motion is the projection on a fixed straight line of uniform motion in a circle.

§ The importance of simple harmonic motion as a fundamental type of vibratory motion is due to the fact that any mechanical system, such as a heavy weight on the end of a spring, in which the entire mass is concentrated in one part of the system and the elasticity in another part of the system, performs simple harmonic motion. A mechanical system like a stretched string or an air column, in which every part of the system has an appreciable mass and an appreciable degree of elasticity, can vibrate so that any given particle of the system may perform any type of oscillatory motion whatever, and in some cases the oscillatory motion is not even periodic, as in the case of compound vibrations of plates and rods.

be a periodic curve, that is, each section of it will be exactly similar to every other section, but it will not be a curve of sines.

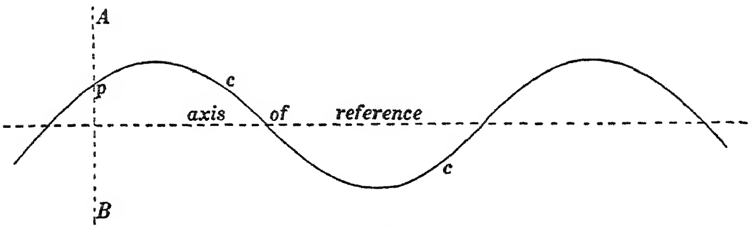


Fig. 245.

The curve shown in Fig. 245 is a curve of sines, and it represents a simple vibration. The curve shown in Fig. 246 is a periodic curve but it is not a curve of sines, and it represents a compound vibration. The various curves in Fig. 244 represent compound vibrations.

Definitions.—The number of complete vibrations per second of a vibrating body is called the frequency of the vibrations. The duration of one complete vibration is called the period of the vibrations, and one half the distance through which the particle moves to and fro is called the amplitude of the vibrations.

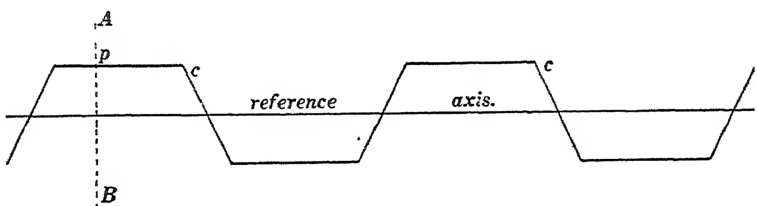


Fig. 246.

144. Superposition of simple vibrations. Fourier's theorem.—A particle may perform simultaneously two or more distinct vibratory movements; in such a case the vibrations are said to be superposed. Thus, if the hand be moved slowly up and down and if at the same time one finger be moved quickly up and down, the moving finger would trace a curve similar to the second or fourth curves in Fig. 244. In this example one vibra-

tion is assumed to be of low frequency and the other of high frequency in order that the movements may not be too greatly confused; as a matter of fact, however, any number of vibratory movements, whatever their amplitudes and frequencies, may be performed by a particle simultaneously.

Fourier's theorem. — Any periodic vibration of frequency n, however complicated, may be matched by superposing simple vibrations of which the frequencies are n, $2n$, $3n$, $4n$, and so on, provided the respective amplitudes are properly chosen. That is, any given compound vibration of frequency n may be thought of as composed of a series of superposed simple vibrations of which the frequencies are n, $2n$, $3n$, $4n$, and so on.

Application of Fourier's theorem to the explanation of tone quality or timbre. — Aside from accompanying noises which frequently characterize musical tones, the difference in quality between two musical tones depends upon the character of the oscillations. If the oscillations are simply harmonic, the tone is called a *simple tone*. If the oscillations are compound, the tone is called a *compound tone* or *clang*. According to Fourier's theorem, a compound tone is the blending together of a series of simple tones of which the frequencies are n, $2n$, $3n$, $4n$, and so on. The tone of lowest frequency n is called the *fundamental* and the other tones are called *overtones*. The overtones in the note which is produced by a piano string can be heard easily and distinctly by a musically trained ear. A method for enabling any one to hear the overtones of a compound tone or clang is described in Art. 157.

145. The compounding of simple vibrations which are not in the same direction. Lissajou's figures. — The compounding or superposing of two simple vibrations which are at right angles to each other and of the same frequency, is described in Art. 133, and the differences in the resultant motion of the particle due to the differences of phase between the two component vibrations, are described in detail and shown in Figs. 222 to 230. Figure 247 shows the curves traced by a point which vibrates up and down at one frequency and to right and left at another frequency; the ratios of the two frequencies being indicated at the left in the figure. The various shapes of curves for a given ratio of frequencies are due to differences in phase of the component vibrations. These curves are called *Lissajou's figures* from their discoverer.*

* See Poynting and Thomson's *Sound*, pages 47 and 71–76. The *vibration microscope* of Helmholtz is an instrument for the accurate comparison of vibration

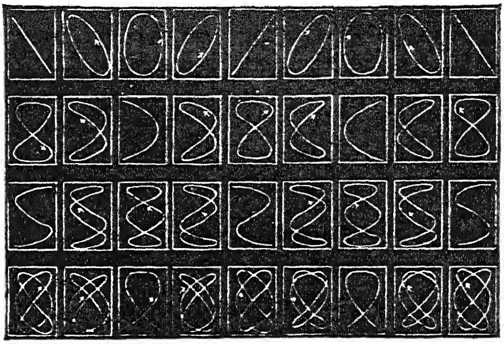


Fig. 247.

146. Simple and compound wave-trains. Fourier's theorem.— When a wave-train passes through a medium, each particle of the medium oscillates. When each particle of the medium performs simple harmonic motion during the passage of a wave-train, the wave-train is called a *simple wave-train*. A simple wave-train is represented graphically by a curve of sines.

When, during the passage of a wave-train, the particles of the medium perform periodic movements which are not simply harmonic, the wave-train is called a *compound wave-train*. A compound wave-train is represented graphically by a periodic curve which is not a curve of sines. Thus, the sine curve in Fig. 245 may be thought of as representing a simple wave-train, and the periodic curve in Fig. 246 may be thought of as representing a compound wave-train.

Fourier's theorem.—The periodic curve in Fig. 246 may be matched by superposing a series of sine curves of which the wave-lengths are λ, $\lambda/2$, $\lambda/3$, $\lambda/4$, etc., where λ is the wave-length of the curve in Fig. 246.* Therefore, a compound wave-train may be thought of as made up of a series of simple wave-trains of which the wave-lengths are as above specified. It is for this reason that a wave-train which is not represented by a curve of sines is called a compound wave-train.

frequencies, and it depends upon the production of Lissajou's figures; see Poynting and Thomson's *Sound*, pages 77–79.

* This statement of Fourier's theorem is substantially equivalent to the statement given in Art. 144.

A body which performs simple vibrations sends out a simple wave-train of sound waves, and a body which performs compound vibrations sends out a compound wave-train of sound waves.

147. The physical significance of Fourier's theorem in its application to sound. — The fibers of the *basilar membrane* in the internal ear are the elements which respond to musical tones. These fibers are more or less like pendulums and they perform approximately simple harmonic motion when they are disturbed in any way. *A compound wave-train striking the ear affects those particular fibers of the basilar membrane which are in unison with the respective simple wave-trains which enter into the composition of the compound wave-train.** It is for this reason that the simple components of a compound wave-train are important and significant; and the simple components of a compound vibration are important because they produce simple wave-trains. This matter is discussed more fully in Chapters XIV and XV.

It must be remembered that the terms *simple vibration* and *simple wave-train* do not refer to simplicity in the ordinary sense; thus, the extremely simple mode of oscillation of a stretched string which is mentioned in Art. 22 and which is more fully discussed in Art. 148, is certainly much simpler than what is hereafter called a "simple mode of oscillation of a string," but the sound waves which are produced by the oscillating string in Fig. 248 have a complex action on the ear (affecting a certain group of fibers of the basilar membrane); whereas the sound waves which are produced by a "simple mode of oscillation," in the special sense in which this term is used in the theory of sound, produce a simple effect upon the ear, inasmuch as such waves excite only one † element of the basilar membrane.

* This matter is explained in Chapters XIV and XV.

† No attempt is here made to give the complete theory of the action of the ear; in fact, a simple tone affects a narrow group of elements in the ear, and the audible beats which are produced when two tones of slightly different pitch are sounded together, depend upon the simultaneous effects of both tones on the same elements in the ear. These effects depend upon the fact that the vibrations of the fibers of the basilar membrane are damped. See Art. 155.

CHAPTER XIII.

FREE VIBRATIONS OF ELASTIC BODIES OR SYSTEMS.

148. Vibrations of plucked strings. — When a stretched string is pulled to one side and released, the string vibrates in a manner which is represented in Fig. 248. In this figure A_1B_1 represents the configuration of the string at the instant of release, and A_2B_2, A_3B_3, etc., represent successive instantaneous configurations at intervals separated by eighths of a period (time of one complete oscillation). The points WW (*waves of starting*) travel towards the ends of the string. The portion of the string between W and W is straight and it moves sidewise at uniform velocity v as indicated by the small arrows. When the waves of starting WW reach the ends of the string they are reflected, and they travel back towards the middle of the string as shown in A_4B_4, and so on.*

Any given point of the string in Fig. 248 remains stationary until the wave of starting W in A_2B_2 reaches it; the given point then moves at uniform velocity v until the reflected wave of arrest $W'W'$, A_4B_4, reaches it, after which instant the point remains stationary for a time, and then it moves at constant velocity v in the reverse direction, and so on, repeatedly.

The motion of a guitar string which is plucked near one end is represented in Fig. 249. The uniformly moving straight portion of the string is equally inclined to the two stationary parts of the string.

It is evident from the above discussion that a plucked string may vibrate in an endless variety of ways, depending upon the initial deformation. Thus, if the string is deformed as shown in Fig. 250, by placing the fingers at the points a and b, then

* Compare this with the discussion of the oscillation of water in a short canal in Art. 22.

the middle point n of the string will remain stationary and each *half* of the string will vibrate in the manner represented in Fig.

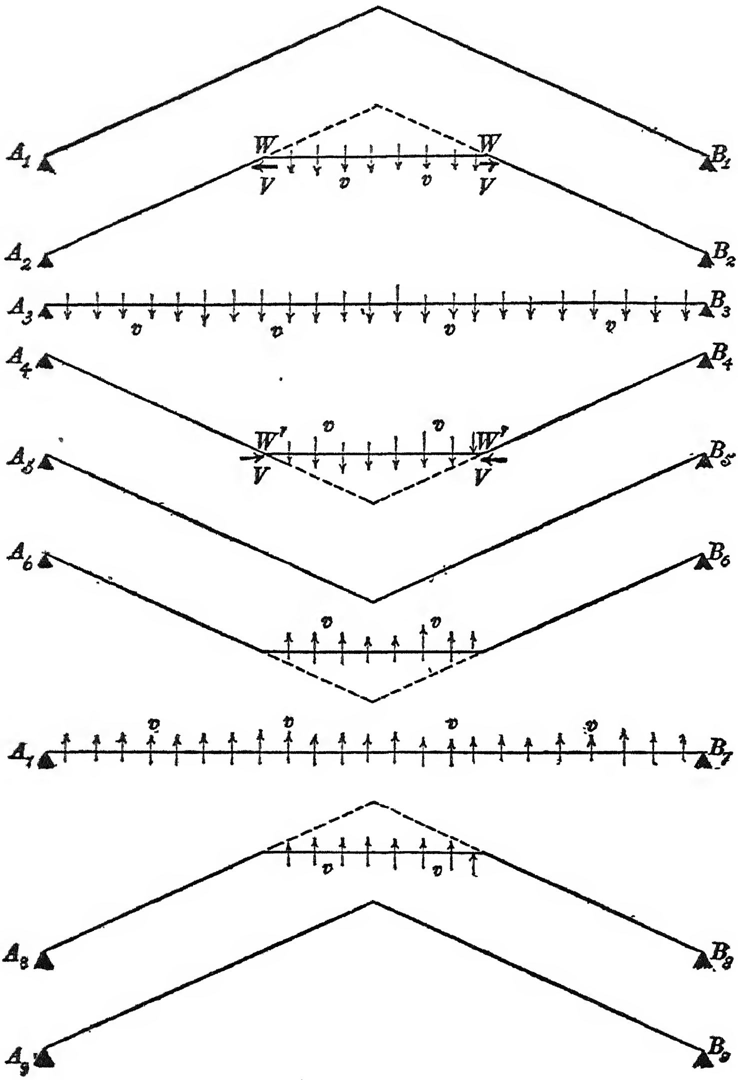


Fig. 248.

248. The fine dotted lines in Fig. 250 show an early stage of the oscillation.

Piano strings, which are actuated by hammer blows, vibrate somewhat differently from guitar strings which are actuated by plucking.*

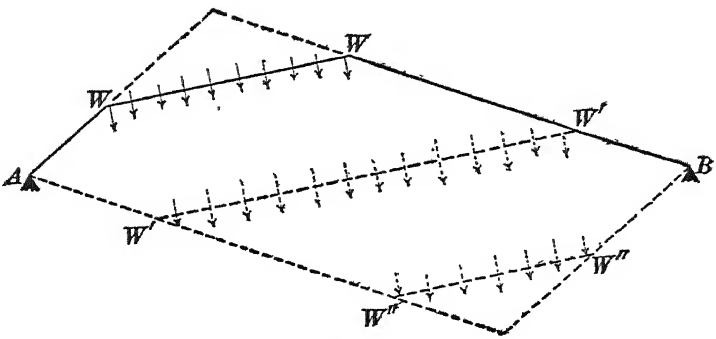


Fig. 249.

149. Vibrations of air columns (particular case). — Consider a long tube closed at both ends, and imagine the air in the tube to be slightly compressed in one end and slightly rarefied in the other end by a gate-valve *G* as shown in the upper part of Fig. 251. When the gate-valve is suddenly opened, the air in the middle portion of the tube suddenly falls to normal pressure and

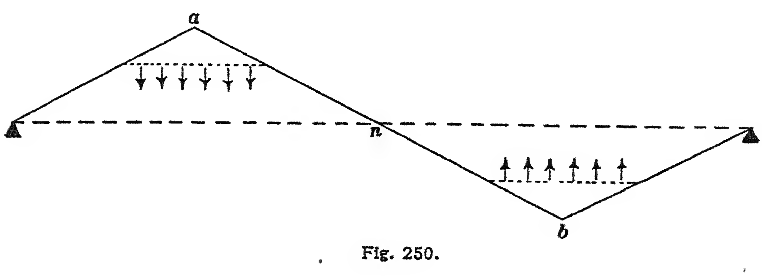


Fig. 250.

is set into uniform motion as shown in the lower part of Fig. 251. This condition of uniform motion and normal pressure is established by two waves of starting *WW* which travel towards the ends of the tube at the ordinary velocity of sound, and the air in the tube performs one complete oscillation during the time re-

* A very good mathematical discussion of the vibration of strings is to be found in Byerly's *Fourier's Theory and Spherical Harmonics*, Ginn & Co., 1893.

quired for a sound wave to travel over twice the length of the tube. The oscillation of the air in the tube shown in Fig. 251 is

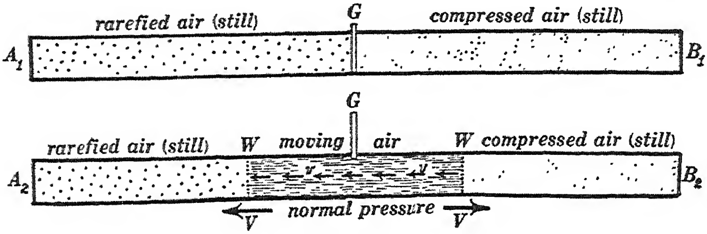


Fig. 251.

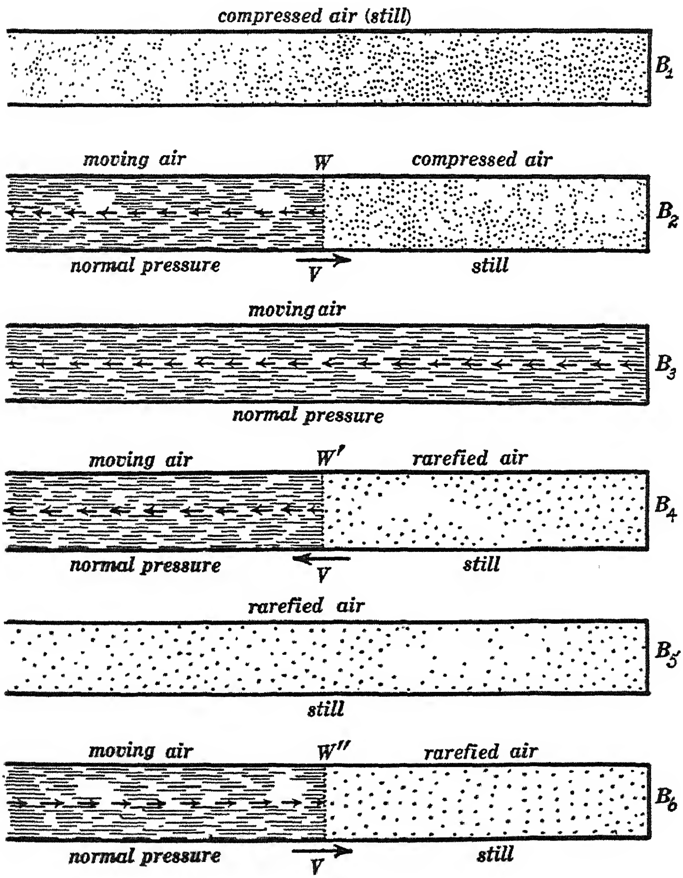


Fig. 252.

exactly similar to the oscillation of the water in a short canal as described in Art. 22, and it is precisely analogous to the oscillation of a string which is plucked at its center as shown in Fig. 248.

After the gate-valve G is opened in Fig. 251, the air at the middle of the tube is always at normal atmospheric pressure, and the air in each half of the tube would oscillate the same way if the tube were cut in two at the middle and left open to the air. Thus, Fig. 252 represents six successive stages of the oscillation of the air in a tube (open at one end and closed at the other) in which air is initially compressed. One complete oscillation takes place in the time required for a sound wave to travel over four times the length of the tube, as may be seen by following the details of Fig. 252 carefully (Fig. 252 shows only five eighths of a complete oscillation).

150. Simple modes of vibration. — In the above described oscillations of strings and air columns *each particle of the string, or air, performs periodic motion which is very far from being simple harmonic motion.* It is desirable, however, as explained in Art. 147, to consider only those modes of oscillations of a string, or air column, in which each particle of the string, or each particle of the air *does perform simple harmonic motion.* Such a mode of oscillation of a string or air column is called a *simple mode.*

It is evident, from the discussion in Arts. 22, 148 and 149, that the oscillation of a string or air column involves wave motion along the string or air column, and the discussion of what are called simple modes of oscillation may be carried out with the greatest ease by considering the passage of a simple wave-train along a string or air column and its reflection from the ends of the string or air column.

Transverse vibrations of strings (simple modes). — Consider an indefinitely long stretched string AB, Fig. 253, fixed to a rigid support at one end B. Imagine a simple wave-train of transverse waves, of wave-length λ, to approach the fixed end B. This wave-train will be reflected at B, the reflected train and

advancing train will together form a *stationary wave-train*, as explained in Art. 24, and the nodes of this stationary wave-train will be at a distance $\lambda/2$ from each other as shown in Fig. 253, the fixed end B of the string being also a node.

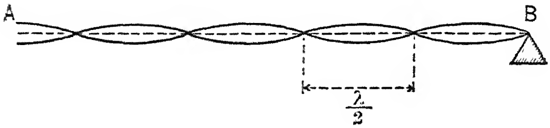


Fig. 253.

This stationary wave-train once established, a rigid support might be placed under the string at any one of the nodes, and the string between this new support and B would continue its vibratory motion unchanged, except, of course, that its motion would be slowly stopped by friction. Therefore, the length l of a vibrating string may be any multiple of $\lambda/2$,* where λ is the wave-length of the two oppositely moving wave-trains whose superposition constitutes the actual oscillatory motion of the string, that is,

$$l = \frac{n\lambda}{2} \tag{i}$$

in which n is any whole number. Let V be the velocity with which a wave-train travels along the stretched string, let τ be the period of one oscillation of the string, and let f be the number of oscillations per second (the frequency). Then, we have

$$\lambda = V\tau \tag{ii}$$

as explained in Art. 23. Substituting this value of λ in equation (i) and solving for τ, we have

$$\tau = \frac{2l}{nV} \tag{iii}$$

or since $f = 1/\tau$, we have

$$f = \frac{nV}{2l} \tag{iv}$$

*This is merely a brief way of saying that the half-wave-length $\lambda/2$ may be any aliquot part of l, for the length of the string is given in any particular case.

This equation expresses the frequency of oscillation of a string (vibrating in a simple mode) in terms of the velocity of transmission V* of waves along the string, and the length l of the string, n being any whole number.

When n is unity, the whole string is one vibrating segment, the string vibrates in what is called its *fundamental mode*, and gives what is called its *fundamental tone*. When $n = 2$, the string vibrates in two segments, and gives what is called its *second*† *overtone*. When $n = 3$, the string vibrates in three segments and gives what is called its *third overtone*, etc.

A string may be made to vibrate approximately ‡ in a fundamental mode by drawing a violin bow across it near one end and touching it with the finger at a, as shown in Fig. 254. This figure

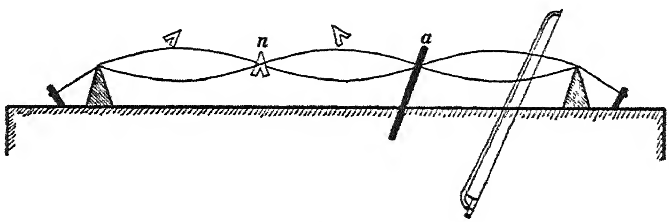


Fig. 254.

shows the string vibrating in three segments [$n = 3$ in equation (iv)]. In this case a light paper rider remains on the string at the node n, whereas such a rider is thrown off the string at other points as indicated in the figure.

When a string is plucked, or struck with a hammer, or stroked with a violin bow, it performs simultaneously its various simple modes of oscillation, and gives its fundamental tone together with its various overtones. The relative intensities of the fundamental and various overtones depend upon the position of the point where

* See page 13.

† This is really the first overtone, considering that the fundamental is not, properly speaking, an overtone; but it is convenient to designate the order of the overtones by the corresponding values of n.

‡ In Fig. 254 the string vibrates simultaneously in all those simple modes for which a node exists at the point a. In the particular case represented in Fig. 254 the 3d, 6th, 9th, 12th, etc., modes are represented in the vibrations of the string.

the string is plucked, or struck, or bowed. Thus a guitar or mandolin string gives a quality of tone which varies quite perceptibly with the location of the point where the string is plucked. In the piano, the best quality of tone is produced when each hammer strikes its string at a point about one seventh of the length of the string from one end, except in case of the very short strings where the hammers must strike near the middle of the strings to give tones of the requisite loudness.

Longitudinal vibrations of air columns (simple modes). — Consider an indefinitely long tube AB, Fig. 255, closed at one end

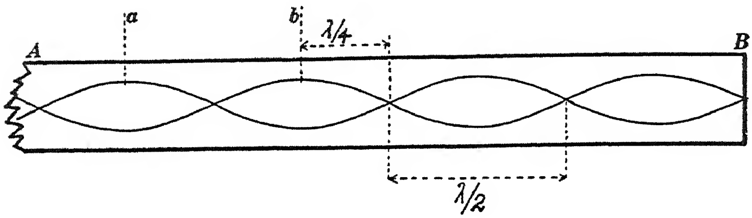


Fig. 255.

B. Imagine a simple wave-train of longitudinal waves, of wave-length λ, to approach the closed end B, in the tube. This wave-train will be reflected at B, the reflected train and advancing train will together form a stationary wave-train as explained in Art. 24, and the nodes of this stationary wave-train will be at a distance $\lambda/2$ from each other as shown in Fig. 255, the closed end B of the tube being also a node. Once this stationary wave-train is established, an air-tight gate-valve might be placed in the tube at any one of the nodes, and the air between this gate and the closed end B would continue its vibratory motion unchanged, except, of course, that its motion would be slowly stopped by friction. Therefore, the length l of a vibrating air column (closed at both ends) may be any multiple of $\lambda/2$,* where λ is the wave-length of the two oppositely moving wave-trains whose superposition constitutes the actual oscillatory motion of the air, that is,

* This is, of course, a brief way of saying that the half-wave-length $\lambda/2$ may be any aliquot part of l, for the length of the tube is always given in any particular case.

$$l = \frac{n\lambda}{2}\left(\text{tube}\begin{Bmatrix}\text{closed}\\\text{open}\end{Bmatrix}\text{at both ends}\right) \tag{v}$$

in which n is any whole number. Substituting for λ the value $V\tau$, where V is the velocity of sound and τ is the period of one oscillation of the air column, and solving for λ we have

$$\tau = \frac{2l}{nV}\left(\text{tube}\begin{Bmatrix}\text{closed}\\\text{open}\end{Bmatrix}\text{at both ends}\right) \tag{vi}$$

or, since $f = 1/\tau$, we have

$$f = \frac{nV}{2l}\left(\text{tube}\begin{Bmatrix}\text{closed}\\\text{open}\end{Bmatrix}\text{at both ends}\right) \tag{vii}$$

This equation expresses the frequency of oscillation of an air column (closed at both ends or open at both ends) in terms of the velocity of sound V and the length l of the column, n being any whole number.

The air pressure at any antinode of the stationary wave-train in Fig. 255 is invariable and equal to atmospheric pressure, but the air at an antinode oscillates back and forth (*parallel to the axis of the tube*, of course). Therefore, the tube may be cut off and left open at any antinode, or at any two antinodes, such as a and b, Fig. 255, *and the vibratory motion of the air in the detached portion ab (open at both ends), or in the detached portion bB (closed at one end) would continue unchanged*, except that the motion would die away because of friction and because of the emission of sound waves from the open end or ends. Therefore, the length l of a vibrating air column which is open at both ends may be any multiple of $\lambda/2$,* so that equations (v), (vi) and (vii) apply to a tube open at both ends as well as to a tube closed at both ends.

It follows also from the above statements, that the length l of a vibrating air column which is closed at one end may be any

* The open end of a tube is not exactly at an antinode, or, in other words, the distance from the open end to the first node is not exactly equal to $\lambda/4$. Therefore, the overtones of an air column with one or both ends open are not exactly harmonic, even if the walls of the containing tube are rigid.

odd multiple of $\lambda/4$, where λ is the wave-length of the two oppositely moving wave-trains whose superposition constitutes the actual oscillatory motion of the air; that is,

$$l = \frac{n'\lambda}{4} \text{ (tube closed at one end)} \tag{viii}$$

in which n' is any odd number. Substituting for λ the value $V\tau$, where V is the velocity of sound and τ is the period of one oscillation of the air column, and solving for τ, we have

$$\tau = \frac{4l}{n'V} \text{ (tube closed at one end)} \tag{ix}$$

or, since $f = \frac{1}{\tau}$, we have

$$f = \frac{n'V}{4l} \text{ (tube closed at one end)} \tag{x}$$

This equation expresses the frequency of oscillation of an air column (closed at one end) in terms of the velocity of sound V and the length l of the column, n' being any odd number.

A very beautiful experiment showing the vibration of an air column in various simple modes is the following: An ordinary whistle mouthpiece is fixed to the end of a long glass tube. A weak blast of air causes the whistle to sound a low-pitch tone, and with increasing air pressure, the air column in the tube breaks up into a greater and greater number of vibrating segments, giving a higher and higher tone. These vibrating segments may be rendered visible by placing lycopodium powder in the tube and maintaining a sufficiently constant air pressure to give an invariable tone on the whistle. The air, as it surges back and forth (parallel to the length of the tube) in the vibrating segments, sweeps the lycopodium powder into small heaps at the nodes. It is best in this experiment to have the end of the tube closed to prevent violent air currents, and to use dry air in order that the lycopodium may not become moist and adhere to the walls of the tube.

151. **The organ pipe** is a device, very similar to the bark whistles known to children, in which a column of air is set into vibration and caused to give a musical tone. Sectional views of

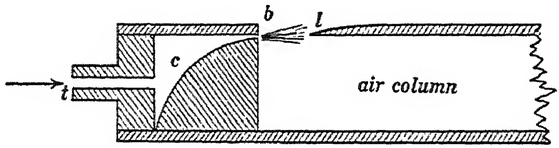


Fig. 256.

complete organ pipes are shown in Figs. 257 and 258, and Fig. 256 shows the details of construction. Air under moderate pressure enters at the tube t flows through the chamber c, issues as an air blast b, and strikes against the sharp lip l. The quavering of this air blast starts the air column vibrating feebly, these vibrations react upon the air blast and cause it to play from

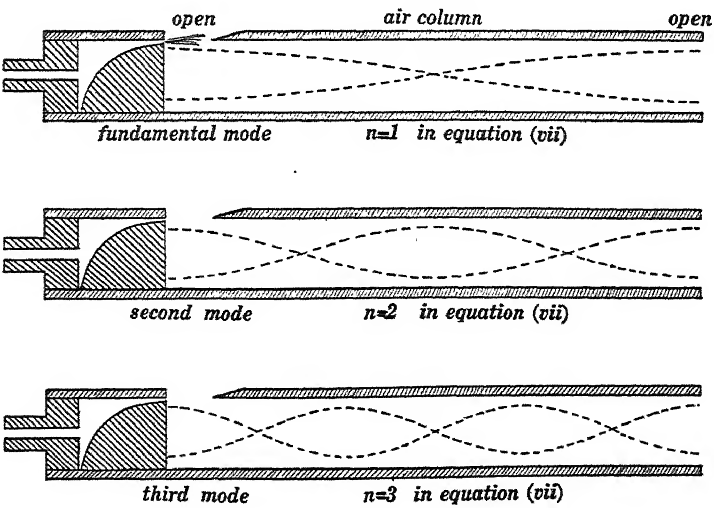


Fig. 257.

one side to the other of the lip l, and this effect reinforces the vibrations so that they quickly become energetic and are sustained as long as the air blast continues.

The dotted curves in Fig. 257 are intended to show the char-

acter of the oscillations of the air column in an organ pipe which is open at both ends, and the dotted curves in Fig. 258 are intended to show the character of the oscillations of the air column in an organ pipe which is closed at one end.

Figures 257 and 258 represent simple modes of oscillation of organ pipes. As a matter of fact, however, an organ pipe always performs its various simple modes of oscillation simultaneously, and the musical tone produced by an organ pipe is therefore com-

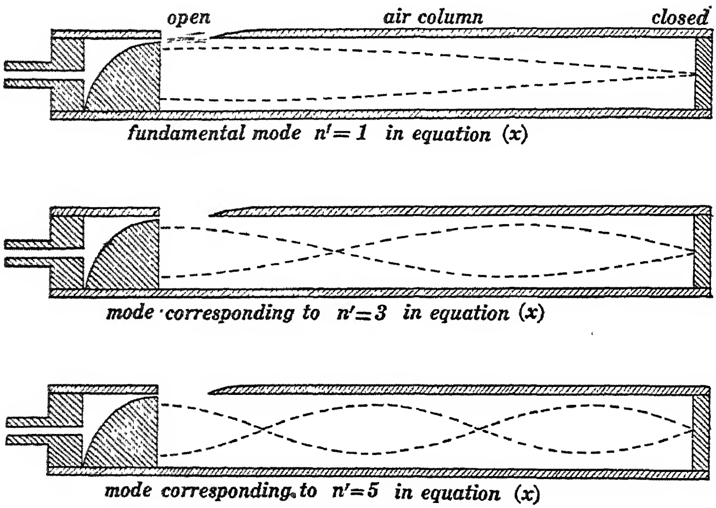


Fig. 258.

posed of a series of simple tones blended together. With broad short pipes the overtones are usually very weak, and the sound approaches what is called a simple tone in character. With long narrow pipes, the overtones are more pronounced; and when a long narrow organ pipe is blown strongly a single one of its overtones is usually produced, the others remaining inaudible.

In the case of a long slender organ pipe with rigid walls the overtones are very nearly harmonic, that is, their frequencies are as, 2, 3, 4, 5, 6, etc., the fundamental being taken equal to unity. In the case of short organ pipes, and especially in case of organ pipes with thin yielding walls, the overtones are distinctly non-

harmonic, that is, their frequencies are not as the successive whole numbers.

The clarinet, the flute, the cornet and the bugle are examples of musical instruments in which tones are produced by the vibrations of air columns. In the clarinet and the flute, the length (and thereby the pitch) of the vibrating air column is altered by uncovering openings in the side of the tube. In the cornet, the length of the vibrating air column is altered by including or excluding extra lengths of tube by means of valve connections. In the bugle, the length of the vibrating air column is fixed, and the various notes are obtained by causing the air column to vibrate in one or another simple mode; therefore the only tones which can be produced on a bugle are the tones whose vibration frequencies are proportional to the numbers 1, 2, 3, 4, 5, 6, 7, 8, etc. In fact, the tones 3, 4, 5 and 6 are the ones ordinarily employed in the bugle.

152. Vibrations of rods.—The longitudinal vibrations of rods are exactly similar to the longitudinal vibrations of air columns and need not be discussed. When a rod is struck on one side with a hammer, it is caused to vibrate with a sidewise or transverse motion. One can always hear a number of distinct tones in the sound produced by a vibrating rod when the vibrations have been produced by a hammer blow, and each of these tones is due to a *simple mode* of vibration of the rod; a simple mode of oscillation being a type of oscillation in which every particle of the rod performs simple harmonic motion of a certain frequency.

In the case of vibrating strings and air columns, the vibration frequencies which correspond to the various simple modes of oscillation are approximately in proportion to the successive whole numbers, that is to say, the tones constitute what is called a harmonic series. In the case of rods, however, the vibration frequencies which correspond to the various simple modes of oscillation do not form a simple series of numbers. Thus, Fig. 259 represents three successive simple modes of oscillation of a

long steel rod.* The frequencies of the successive simple modes are given in terms of the frequency of the fundamental mode which is taken arbitrarily as 1000. The fundamental mode has

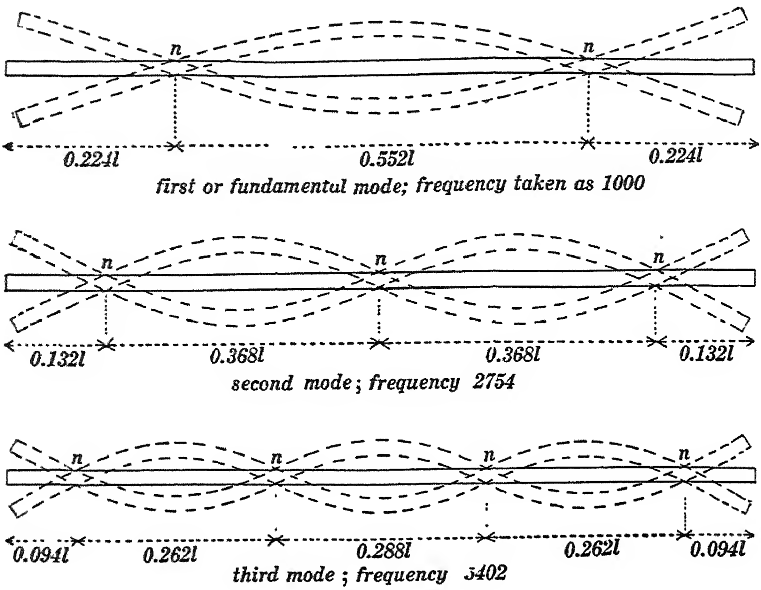


Fig. 259.

two nodes, the second mode has three nodes, and the third mode has four nodes. The locations of the nodes are indicated in terms of the length l of the rod. In order to cause a rod to vibrate according to one of the sketches in Fig. 259, the rod would have to be supported at one or the other set of nodes and excited by a violin bow. The positions of the nodes and the relative frequencies of the various modes depend to some extent upon the ratio of length to diameter of rod.

The tuning fork is a stiff rod bent into the form shown in Figure 260. The character of its fundamental mode of vibration is shown by the dotted lines. The two nodes *nn* are near together, and the intervening segment, together with the metal

* This figure is based on a theoretical paper by Seebeck which was published in 1848. See Winkelmann's *Handbuch der Physik.*

post P, move up and down through a small amplitude and cause the sounding board upon which the fork is mounted to vibrate in unison with the fork. The second simple mode of vibration of a fork usually gives a tone two or three octaves above the fundamental, and it dies out quickly after the fork is set into vibration by a hammer blow, leaving the fundamental alone. The tuning fork, therefore, gives a simple tone.

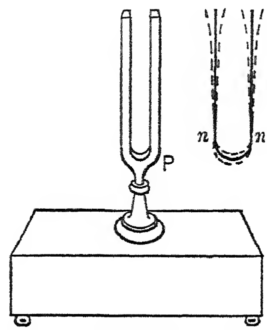


Fig. 260.

153. Vibrations of plates. — When a plate, such as a circular saw of steel, is struck on one side with a hammer, it is set into vibration, and one can always hear a number of distinct tones in the sound produced. Each of these tones corresponds to a simple mode of vibration of the plate, a simple mode of oscillation being a type of oscillation in which every particle of the plate performs simple harmonic motion of a certain frequency. The vibration frequencies which correspond to the various simple modes of oscillation of a plate do not form a simple series of whole numbers.

An elastic plate may be made to perform a simple mode of oscillation by supporting it in a clamp as shown in the upper part of Fig. 261, touching the fingers lightly against the plate in the positions (found by trial) corresponding to the nodes, and drawing a violin bow across the edge of the plate. When sand is strewn upon the plate, it is thrown away from the vibrating segments and heaped along the nodal lines. The lower part of Fig. 261 shows four sand figures obtained in this way. These figures were first studied by the Italian physicist Chladni and described in his treatise on acoustics in 1787.

The bell may be looked upon as a cup-shaped plate. When a bell is struck, a number of distinct tones may be heard. These tones correspond to simple modes of vibration of the bell, and

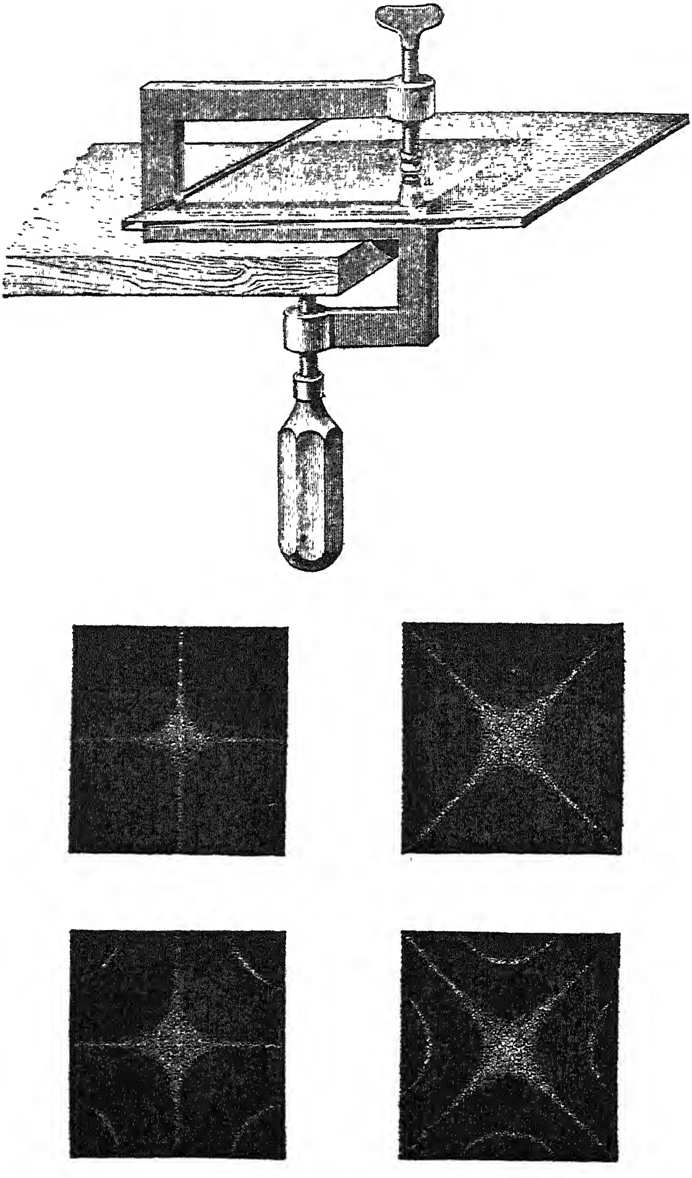


Fig. 261.

their freqencies are not usually in proportion to the whole numbers 1, 2, 3, 4, 5, etc. Bell founders have, however, learned to produce bells of such shape as to make the more prominent tones harmonic; such a bell gives a rich and pleasing tone. Figure 262 shows the more prominent tones of a Russian bell in the

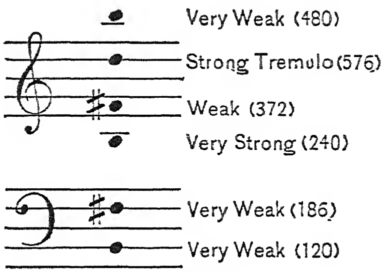


Fig. 262.

library of Cornell University. The numbers represent the number of vibrations per second. The tone marked "tremulo" consists of *two tones* differing very slightly in pitch, but of nearly the same loudness, and tremulous character is produced by the beats of the tones.

CHAPTER XIV.

FORCED VIBRATIONS AND RESONANCE.

154. Free vibrations and forced vibrations. — The vibrations which a body performs when it is disturbed in any way and left to itself are called its *free* or *proper* vibrations. When a simple wave-train of sound waves of any wave-length strikes a body, the body is made to vibrate in unison, or in the same rhythm, with the impinging waves.* Such vibrations are called *forced* or *impressed* vibrations. A compound wave-train causes a body to perform simultaneously the various simple vibrations which correspond to the various simple wave-trains which enter into the composition of the compound wave-train.

155. Damping — Vibrations are said to be *damped* when they die out quickly. This damping effect is due in part to the dissipation of energy in the body in the form of heat, as it is repeatedly distorted, and in part to the giving up of energy to the surrounding air. Thus, the vibrations of a light body which exposes considerable surface to the air, a diaphragm, for example, die out quickly: whereas, the vibrations of a heavy and highly elastic body, like a tuning fork, are but slightly damped. A heavy tuning fork performs several thousand perceptible vibrations when struck. The column of air in an organ pipe performs several hundred perceptible variations after the exciting cause ceases. A drum-head performs very few perceptible vibrations when struck.

156. Resonance. — Very perceptible vibrations are impressed upon a light diaphragm by sound wave-trains of any wave-length

* This statement refers to the ultimate character of the motion and not to the motion which takes place during the time that the steady state of vibration is being established. During the time that the steady state of vibration is being established, the body behaves as if its free vibrations and the ultimate forced vibrations existed simultaneously, and, as the free vibrations die out, the forced vibrations are left alone.

(frequency), but *the vibrations which are impressed upon a heavy body, of which the damping is slight, are much more violent when the impressed frequency approaches the frequency of the free vibrations of the body*. Thus, the sound of a tuning fork (removed from its sounding board) is perceptibly reinforced when it is held near the open end of a tube of any length; but if the length of the air column is adjusted, by pouring water into the tube for example, the sound becomes louder as the proper frequency of the air column approaches that of the fork; and it reaches a very distinct maximum of loudness when the impressed vibrations become proper to the air column.

When the vibrations of a body are subject to very slight damping, the impressed vibrations at proper frequency are very energetic as compared with the impressed vibrations of improper frequency. Thus, a massive tuning fork, mounted upon its sounding board, is thrown into very energetic vibration by a tone of its proper frequency sustained for four or five seconds, but it is scarcely affected at all by a tone of which the frequency differs very slightly from the proper frequency of the fork. *This pronounced maximum violence of impressed vibration at proper frequency is called* **resonance**, *and the vibrating body or air column is called a* **resonator.**

When impressed vibrations are proper to a body, the impulse of each successive wave adds to the existing motion, and the vibrations increase in violence until the energy given to the vibrating body by the impinging waves is all dissipated by damping. It is for this reason that the impressed vibrations become quite energetic when the damping is small. When, however, improper vibrations are impressed upon a body, the periodic forces with which the waves act upon the body must take the place, more or less, of the *internal elastic forces* which ordinarily cause the body to vibrate; consequently the vibrations cannot become very energetic.

157. Analysis of compound tones by means of resonators. —A resonator of which the proper frequency coincides with the fre-

quency of one of the component tones of a compound tone or clang, responds to the component tone by resonance and increases its loudness. Thus, if the frequencies of the fundamental and successive overtones of a string are 100, 200, 300, 400, etc., vibrations per second, a resonator of which the proper frequency is 400 vibrations per second will be set into energetic vibration by the sound of the string, and this particular overtone will be increased in loudness.

Any component tone of a clang may be easily detected, or brought to notice, by intermittently strengthening it by means of a resonator tuned to unison with it. A convenient resonator for this purpose is shown in Fig. 263*a*. It consists of a hollow glass ball with an open mouth at *a* and an open tip *b*. The open tip *b* is inserted into the ear, and the sound waves act upon the inclosed air through the opening *a*. In order to analyze a clang, one after another of a series of such resonators must be applied to the ear. Fig. 263*b* shows an adjustable resonator of

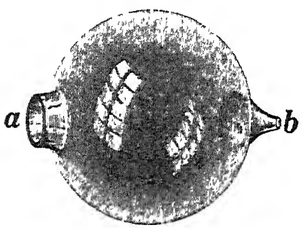


Fig. 263a.

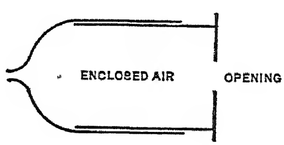


Fig. 263b.

Koenig. It consists of two brass cups, one sliding into the other, and the outer cup ends in a small nipple which is inserted into the observer's ear.

158. Vowel sounds. — Every vowel sound is characterized by one or two *tones of* **definite** *pitch.* The characteristic tones of some of the vowels, as determined by Helmholtz,* are shown in the following table, and Fig. 264 gives these characteristic tones in terms of the ordinary musical notation.

* See *Sensations of Tone* (a translation of Helmholtz's *Tonempfindungen* by A. J. Ellis) pages 153–173.

Fig. 264.

Characteristic tones of vowel sounds.

Vowel.	Tone.	Vibration Frequency *
ṳ as in *rude*.	*f*	173
ō as in *no*.	*c''*	517
a̤ as in *paw*.	*g''*	775
ä as in *part*.	*db'''*	1,096
ā as in *pay*.	*f* and *bb'''*	346 and 1,843
ĕ as in *pet*.	*c''''*	2,068
ē as in *see*.	*f* and *d''''*	173 and 2,322

In producing a given vowel sound, the mouth cavity is shaped so that the contained air has a proper frequency of vibration which corresponds to the characteristic tone of the vowel to be produced. The sound from the vocal chords is very complex (it contains tones of almost every pitch), and that particular tone which is in unison with the free or proper vibrations of the air in the mouth cavity, is greatly strengthened by resonance, thus producing the desired vowel sound.

In ordinary speech, the sound produced by the vocal chords is very harsh and suffices to excite the resonance of the mouth cavity for the production of any vowel sound. The smooth tone of a singer, however, may not contain the characteristic tone of the vowel, so that this tone cannot be strengthened by resonance. In this case, those overtones of the note which is sung, which are nearest the characteristic tone of the vowel for which the mouth cavity is set, are slightly strengthened, and in this way the vowel sound is produced, although rather incompletely. It is a well-known fact that spoken words are much more easily understood

* Complete vibrations per second.

than words which are sung, and the difference in distinctness is largely due to the imperfect character of vowels when sung.

Overtones near a given pitch are more widely separated in a note of high pitch than in a note of low pitch, so that the mouth cavity, shaped to give the characteristic tone of a vowel, is less likely to produce the desired effect with high notes than with low notes. Thus, the words of a soprano singer are less distinct than the words of a bass singer of the same schooling.

The proper tones of the mouth cavity when it is shaped for the production of the vowels ō, a̤ and ā̈ may be easily heard by thumping against the cheek when the mouth is prepared to sound these vowels. A very striking experiment is the following: The lungs are filled with purified hydrogen, and a series of words like *rude*, *no*, *paw*, *part*, *pay*, *pet*, *see*, are spoken very deliberately and carefully. The mouth cavity is automatically shaped to produce the various characterizing tones, but the presence of hydrogen instead of air in the mouth raises the pitch of the mouth cavity nearly two octaves and the result is that the attempt to produce the vowels ṳ, ō, a̤, ā̈, ā, etc., fails utterly and in a most laughable manner.

159. The phonograph. — A thin diaphragm carries a light tool which scratches a minute groove in a smooth rotating cylinder made of a hard wax-like compound. A sound striking the diaphragm impresses vibrations upon it and causes the tool to cut a groove of varying depth. A record of the sound is thus made upon the cylinder. To reproduce the sound, a round-ended tool which is attached to the diaphragm is adjusted to follow the groove, and the varying depth of the groove lifts the tool up and down causing the diaphragm to vibrate as before and the sound is reproduced.

CHAPTER XV.

THE EAR AND HEARING.

160. The human ear. — It was pointed out by Helmholtz that our perception of the various simple tones in a clang must depend upon the existence of a series of organs (the end organs of the auditory nerves) in the ear, each of which has a proper vibration frequency and is sensitive (by resonance) to simple tones nearly in unison with it. Thus, a compound tone excites those particular elements in the ear whose vibration frequencies correspond to the various simple tones of the clang. This action may be illustrated by means of the piano as follows: A musical tone of characteristic quality, a vowel sound, for example, is sung loudly against the sounding board of a piano of which the dampers are raised so as to leave the strings free. Those strings which are capable of vibrating in unison with the various simple tones of the clang are set into vibration by resonance, and one can hear a continuation by the piano of the vowel sound after the singing ceases. Imagine each string of a piano to be connected to a nerve fiber, and we have an apparatus which would perceive sounds as they are actually perceived by the ear.

The elements in the ear which respond to tones by resonance are the shreds of the *basilar membrane* which are stretched across a long slender cavity called the *cochlea*. This cavity is coiled upon itself like a small shell,* hence its name.

161. Persistence of sound sensations. — When the stimulation of a nerve ceases, the accompanying sensation continues for a length of time which depends upon the intensity of the stimulation and which varies greatly with the different nerves. Sensations of light persist much longer than sensations of sound. Thus,

* See *Sensations of Tone* (Ellis' translation of Helmholtz's *Tonempfindungen*), pages 188–226.

an intermittent light gives a sensibly continuous sensation when the flashes follow one another at the rate of thirty or forty per second, whereas an intermittent sound, for example, the sound of a tuning fork of high pitch which is shut off from the ear periodically has been found by Mayer to give a continuous sensation when the frequency of intermittence reaches 135 per second.

162. Consonance and dissonance. — An intermittent or fluctuating tone produces an unpleasant sensation which is called *discord* or *dissonance*. A steady tone, or one which fluctuates so rapidly as to give a steady sensation, produces a pleasing effect which is called *concord* or *consonance*. These terms discord (or dissonance) and concord (or consonance) are used ordinarily to describe the effects produced by two or more simultaneous tones, that is, they are used to describe the relations of tones in music. Nevertheless, the above definitions are physically correct as will appear in the following articles and in Chapter XVI.

Consider a tone which is intermittently shut off from the ear, the intermittence beginning at low frequency and increasing to greater and greater frequency. The dissonance at first increases with increasing frequency of intermittence, reaches a maximum, then falls off and disappears entirely when the frequency of intermittence becomes so great as to give a continuous sensation of tone. The frequency of intermittence for which the dissonance is a maximum, and the frequency of intermittence for which the sensation becomes continuous, depend upon the vibration fre-

Vibration Frequency of Intermittent Tone.	Frequency of Intermittence.	
	When Tone Becomes Smooth.	When Discord is a Maximum.
64	16	6.4
128	26	10.4
256	47	18.8
384	60	24.0
512	78	31.2
640	90	36.0
768	109	43.6
1,024	135	54.0

quency of the tone which is used, as shown in the above table, which is from experiments made by Mayer.

The fluctuations which produce dissonance in music are the fluctuations due to beats, which are described in the following paragraph. When two tones are in unison they give a smooth sensation. If the vibration frequency of one tone is slowly increased, beats occur with greater and greater frequency, the resulting intermittent sound produces a more and more discordant sensation which soon reaches a maximum, after which the discord decreases and finally disappears when the beats reach a sufficiently high frequency.

163. Beats and combination tones.* — Consider two simple tones of which the vibration frequencies, f and f', are nearly the same. At a certain instant the wave-trains which constitute these two tones will be in like phase as they enter the ear, giving a maximum loudness of sensation. When the tone of higher pitch has gained half a vibration (or half a wave-length) over the other, the wave-trains will be opposite in phase as they enter the ear, giving a minimum loudness of sensation. When the higher tone has gained a whole vibration over the other the waves will again enter the ear in like phase, and the loudness will again be a maximum, and so on. These periodic changes of loudness of two tones of approximately the same frequency constitute what are called *beats*. The number of beats occurring in one second is equal to $f-f'$.

Very prominent beats may be produced by sounding two similar organ pipes simultaneously, the open end of one of the pipes being partly covered to make it give a tone of slightly lower pitch than the other pipe.

Combination tones. — The principle of superposition, namely, that two trains of waves may pass through the same region at the same time without affecting each other (see page 18), or that an elastic system may perform a series of simple harmonic movements simultaneously without the various vibratory

* See pages 140 and 141.

movements being affected by each other (see Art. 144), is not strictly true. The failure of this principle of superposition is due to what may be termed the incomplete elasticity of ordinary substances, that is, to the fact that the elastic forces brought into play by the sum of the two distortions is not exactly equal to the sum of the elastic forces brought into play by the two distortions separately. The principle of superposition fails most decidedly when an imperfectly elastic substance is vibrating very violently.

Two weak tones do not sensibly affect each other when they are transmitted simultaneously through the same region of air or when they act simultaneously upon any elastic or approximately elastic system, such as the transmitting mechanism of the ear. As the tones grow louder, however, certain other tones produced by their mutual action begin to be heard. These accompanying tones are called *secondary tones* or *combination tones*. The imperfectly elastic chain of bones in the ear has most to do with the formation of combination tones. Everyone, perhaps, has noticed the harsh rattle in the ear which accompanies the loud sound of, a dinner bell. This is due to the incomplete elasticity of the tissues which connect the tiny bones together. Indeed, it is a sort of rattling effect of the bones on each other, and it is the same thing essentially as a combination tone, only greatly exaggerated.

Difference tones. — The most prominent combination tone is that of which the frequency is equal to the difference of the frequencies of the two primary tones. This is called the *difference tone of the first order*. This difference tone forms difference tones with each of the primary tones and these are called *difference tones of the second order*, and so on.

Summation tones. — Very much less prominent than the difference tones is the combination tone of which the frequency is equal to the sum of the frequencies of the two primary tones. This is called the *summation tone of the first order*. Summation tones of the first order form summation tones with each primary tone and these are called *summation tones of the second order*, and so on.

Example. — The easiest method, perhaps, to produce an audible combination tone is to use two large steel bars of which the vibration frequencies are, say, 500 and 600 vibrations per second,

respectively. When these bars are properly suspended (at the nodes of the fundamental mode, see Fig. 259), and struck in quick succession with a heavy hammer, a very distinct tone is heard of which the pitch is 100 vibrations per second.

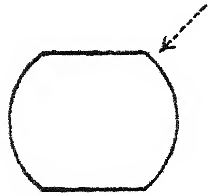


Fig. 265.

A single steel bar of which the section is of the shape shown in Fig. 265, gives a tone of low pitch when struck on one of the flat faces, and a tone of high pitch when struck in a direction at right angles to this. A hammer blow in the direction of the dotted arrow causes the bar to give both tones simultaneously, and the first-order difference tone may be heard distinctly throughout a large room.

CHAPTER XVI.

THE PHYSICAL THEORY OF MUSIC.*

164. Pitch intervals and their measurement. — Consider two musical tones of different pitch (different frequencies of vibration). The pitch interval between these tones is usually expressed in terms of the ratio of the vibration frequencies, and any two musical tones which have the same ratio of vibration frequencies are said to have the same pitch interval between them. Thus the notes, *c* and *g* on the piano have the same frequency ratio 2:3 in every octave of the entire piano scale; in one case it may be 200:300, in another case, 400:600, and so on; and the pitch interval between *c* and *g* is the same in all octaves.

Pitch intervals are ordinarily expressed in terms of frequency ratios. In the discussion of the tempered scale, however, it is more convenient to express pitch intervals in terms of the logarithms of their frequency ratios. Consider, for example, a number of tones of which the vibration frequencies are n, an, a^2n, a^3n, etc. The frequency ratio of any two successive tones in this series is a and therefore the pitch intervals are all equal. Let p represent the value of this pitch interval. The frequency ratio of the first and third tones is a^2, and their pitch interval should be expressed as $2p$; the frequency ratio of the first and fourth tone is a^3, and their pitch interval should be expressed as $3p$, and so on. In order that we may add together the two numbers which express two pitch intervals to get the number which expresses the sum of the two pitch intervals, we must use for p the logarithm of the frequency ratio as may be seen from the following tabular arrangement.

Frequency ratios,	a	a^2	a^3	a^4	etc.
Logarithms of frequency ratios,	l	$2l$	$3l$	$4l$	etc.
Pitch intervals,	p	$2p$	$3p$	$4p$	etc.

165. Consonance and dissonance of compound tones. — The consonance and dissonance of simple tones have been discussed in

* The physical theory of music has been developed mainly by Helmholtz, and it is given in his great work entitled *Tonempfindungen* (translated by Alexander J. Ellis, title of translation, *Sensations of Tone*). A very interesting discussion of harmony in music is Helmholtz's Popular Lecture entitled "*On the Physiological Causes of Harmony in Music*," translated by A. J. Ellis, *Helmholtz's Popular Lectures*, First series, Longmans, Green & Company, 1873.

Art. 162. Tones ordinarily used in music are, however, compound, and musical consonance and dissonance depends to a great extent upon this fact. The fundamental tone usually predominates in a compound tone, and the overtones 2, 3, 4, 5, 6 and 8 usually occur, decreasing in loudness in the order given. *The following discussion of consonance and dissonance is limited to the influence of these six overtones.*

When two compound tones *A* and *B* are in unison, their respective overtones are in unison also; the combined sound of the two tones is therefore entirely free from roughness due to beats; and the two tones are *completely consonant.*

When two compound tones *A* and *B* are *not* in unison, then, even if their pitch difference is so great that their fundamental tones do not produce audible beats, some of the overtones of *A* will generally be near enough to some of the overtones of *B* to produce audible beats and give distinct dissonance. If the pitch of *B* is slowly raised or lowered starting from unison with *A*, this dissonance passes through a very marked minimum value every time one (or more) of the overtones of *A* comes into unison with one (or more) of the overtones of *B*. The two compound tones *A* and *B* are said to be *approximately consonant* when their dissonance thus reaches a minimum value. The following example will make this clear. Let the compound tone *B* be a very little higher in pitch than *A*. Then the fundamentals and every pair of overtones produce beats, and the dissonance is great. As the pitch of *B* is raised, this dissonance falls off as each pair of jarring overtones becomes more widely separated in pitch. This falling off in dissonance continues until the fifth overtone of *B* comes into the neighborhood of the sixth overtone of *A*; the beats of this pair of overtones then cause a distinct rise in the dissonance which is followed by a rapid fall as the two overtones come into exact unison. As the tone *B* continues to rise in pitch there is a rapid rise in the dissonance as the two overtones move apart in pitch, and so on.

The ordinates of the curve, Fig. 266, show the values of the

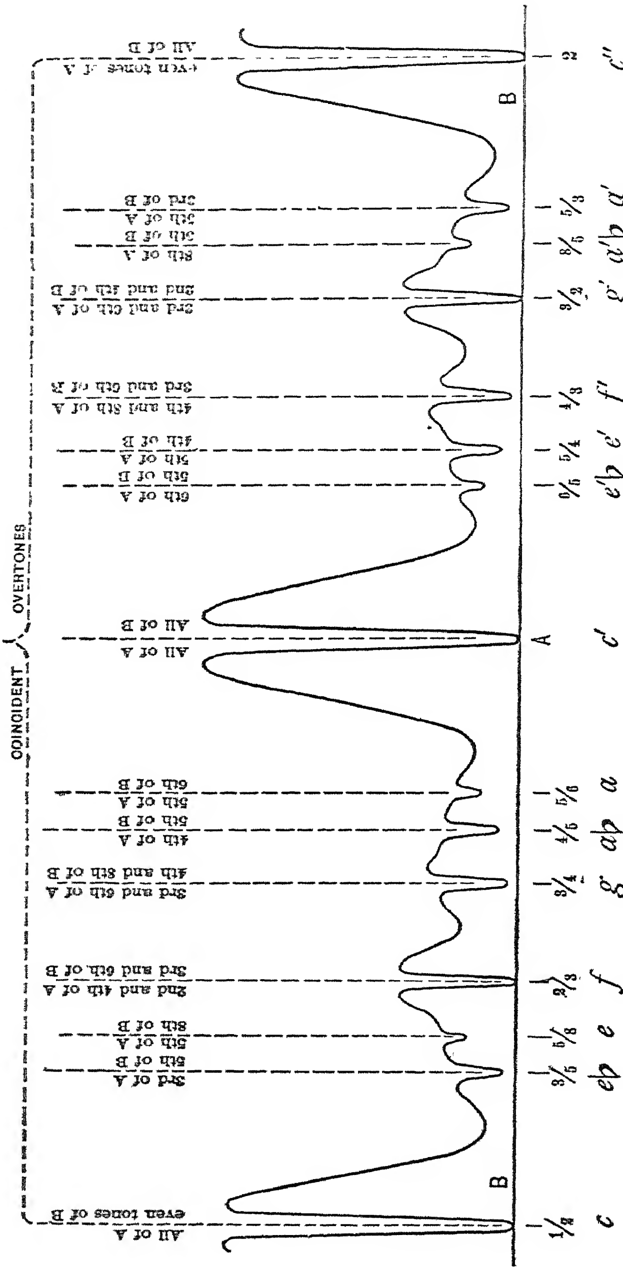


Fig. 266.

dissonance of two violin tones A and B in so far as the overtones 2, 3, 4, 5, 6 and 8 are concerned. The tone A is kept at a constant pitch (see middle of figure), and the tone B starting with unison with A is at first slowly raised in pitch until its vibration frequency is twice as great as the vibration frequency of A; and then beginning again with unison with A, the tone B is slowly lowered in pitch until its vibration frequency is half as great as the vibration frequency of A. The fractions below the line BAB show the values of the frequency ratio B/A for which the various minima of dissonance occur. The numerical evaluation of dissonance is an approximation.*

Combination tones have some action in the production of dissonance when two tones are sounded together. The dissonance due to combination tones and dissonance due to overtones have minimum values for the same frequency ratios.

166. Consonant pitch-intervals.—Two compound tones are said to be completely consonant when they are in unison, and they are said to be approximately consonant when their dissonance is at a minimum due to the coincidence of two or more overtones. When two compound tones are approximately consonant, the pitch interval between them is called a *consonant interval.* Thus, Fig. 266 shows the various consonant intervals, and the following table exhibits the various consonant intervals in the order of the completeness of their consonance. The names of the consonant intervals as given in this table are explained later.

TABLE OF CONSONANT INTERVALS.

1 : 1 Unison.	3 : 5 Major sixth.
1 : 2 Octave.	4 : 5 Major third.
2 : 3 Fifth.	5 : 6 Minor third.
3 : 4 Fourth.	5 : 8 Minor sixth.

The bounding of consonant intervals.—Those overtones and combination tones which determine a consonant interval by their coincidences, and which produce the greater part of the dissonance

* See Helmholtz's *Sensations of Tone* (Ellis' translation), pages 272–299.

when the interval is slightly out of tune, are said to *bound* the interval. For example, the fourth and eighth overtones of A and the third and sixth overtones of B bound the interval $c'f'$ as shown in Fig. 266, and the larger part of the dissonance of two tones which have nearly but not exactly the frequency ratio 4 3, is due to the jarring action of these particular overtones.

The great increase of dissonance due to a slight error of tuning of a consonant interval is the basis of our remarkably acute sense of the accuracy of tuning of these intervals. This acute sense of pitch in connection with consonant intervals has a great deal to do with the effectiveness of consonant intervals in music; for there can be no refinement of musical expression without an acute sense to seize upon it, and it is the ultimate dependence of this acute sense upon the presence of prominent overtones which explains the peculiar musical value of such tones as those of the violin and of the human voice.

167. Variation of the character of consonance with the quality of the tones. — A consonant interval is the more striking in character in proportion as it it is more sharply bounded (see Art. 166). Thus, the fifth (2 : 3) is a very sharply bounded consonant interval (see Fig. 266), whereas the minor sixth (5 : 8) is not sharply bounded, and the fifth is therefore much more striking in character than a minor sixth. The sharpness with which the various consonant intervals are bounded depends upon the relative loudness of overtones, and therefore the character of a given consonant interval varies with the quality of the tones used.

This is exemplified in a striking way by clarinet tones, which have only odd harmonics. The consonance of two such tones is most striking when it depends upon coincidence of odd harmonics of both tones, in fact, a consonant interval which depends upon or would depend upon the coincidence of even harmonics is bounded only by combination tones in the case of two clarinets. Thus, with two clarinet tones the major sixth (3 : 5) is more striking in character than the fourth (3 : 4), and, perhaps, even as sharply defined as the fifth (2 : 3); while the minor third (5 : 6)

and the major third (4 : 5) are very poorly defined. A major third (4 : 5) formed by a violin tone and a clarinet tone is very much more striking when the clarinet tone is the lower than when the violin tone is the lower, because in the first case, the fifth overtone of the clarinet tone coincides with a fourth overtone of the violin tone, and in the latter case there is no fourth overtone of the clarinet tone to coincide with the fifth overtone of the violin tone. A minor third (5 : 6) is much more striking when the violin tone is the lower. The violin tone has both even and odd harmonics.

In the case of pure tones such as tones of tuning forks and broad organ pipes, combination tones only serve to bound the consonant intervals. With such tones the octave (1 : 2) is very sharply defined, the fifth (2 : 3) less sharply, and the remaining consonant intervals are scarcely bounded at all. Helmholtz, indeed, has found that the sound of two tuning forks is smooth or consonant whatever the pitch interval, provided the tones are not loud enough to bring out the combination tones strongly.

168. The major and minor triads. — Two or more simultaneous tones which form a more or less consonant combination constitute what is called a *chord.* A chord which consists of three tones is called a *triad.* Any tone of a chord may be accompanied by its octave, or may be replaced by its octave, without greatly altering the character of the chord. This is evident when we consider that no new overtones are introduced into the sound by the octave.

The major triad and its modifications. — The three tones of which the vibration frequencies are as 4 : 5 : 6 constitute what is called the major triad. By replacing the first tone (4) by its octave (8), we obtain a modification of this triad; and by replacing the first and second tones (4) and (5) by their octaves (8) and (10), we obtain another modification. The three forms of the major triad are, therefore,

4	5	6		
	5	6	8	
		6	8	10

The minor triad and its modifications.—The three tones of which the vibration frequencies are as 10 : 12 : 15 constitute what is called the minor triad. The interval between the first two tones is a minor third (5 : 6), between the last two tones is a major third (4 : 5), and between the first and last the interval is a fifth (4 : 6); so that the minor triad contains the same consonant intervals as the major triad (4 : 5 : 6). The modifications of the minor triad are

10	12	15		
	12	15	20	
		15	20	24

The primary forms of the major and minor triads, namely, (4 : 5 : 6) and (10 : 12 : 15), are those in which the three tones are separated by the smallest pitch intervals.

Difference in character of major and minor triads.—The major and minor triads contain the same consonant intervals, and the coincident overtones are identical in the two cases. The combination tones, however, are very different. The following schedule shows the combination tones of the first and second orders.

The major triad.

Primary tones,				4	5	6
First difference tones,	1	2				
Second difference tones,		2	3	4	5	

The minor triad.

Primary tones,							10	12		15
First difference tones,	2	3	5							
Second difference tones,				7	8	9	10	12	13	

This schedule shows that the difference tones of the major triad are exact duplications, or duplications in the lower octaves, of the primary tones. That is, no foreign tones are introduced into the major triad by the combination tones. On the other hand, some of the difference tones of the second order, namely, 7, 8, 9 and 13, which occur in the minor triad, are dissonant, and give to this triad a character very different from that of the major triad.

169. Musical scales. — The successive tones in a melody and the simultaneous tones in harmony are chosen with reference chiefly to their consonance. A collection of the notes which are available for melody or harmony is called a *musical scale.*

The major scale. — Consider a given tone c'. The tones which can be used with c' with more or less consonance are those designated by $e'\flat$, e', f', g', $a'\flat$ and a' in Fig. 266. Ignoring the tones $e'\flat$ and $a'\flat$, which have low degrees of consonance with c', we have the following series of musical tones, each of which has some degree of consonance with c':

Tones,	c'	e'	f'	g'	a'	c''	I.
Vibration frequencies,	1	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	2	

The tone c', with reference to which this series of tones is selected, is called the *tonic* of the series. The tone g', having, next to the octave, the most complete consonance with c', is called the *dominant;* and the tone f', which is next in order of consonance, is called the *subdominant* of the series.

For purposes of harmony, it is desirable to be able to build major triads (4 : 5 : 6) upon the *tonic*, upon the *dominant*, and upon the *subdominant* of a series of musical tones. Two of these major triads may be built up with the tones in series I, namely, c', e', g' (4 : 5 : 6) and f', a', c'' (4 : 5 : 6). To build a major triad upon g', two additional tones, say b' and d'', are required such that $g' : b' : d'' = 4 : 5 : 6$. Therefore the vibration frequencies of b' and d'' are $\frac{15}{8}$ and $\frac{9}{4}$ respectively. Taking a tone d' an octave below d'', we have the series:

Tones,	c'	d'	e'	f'	g'	a'	b'	c''	II.
Vibration frequencies,	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2	

This is the ordinary musical scale, called the *major scale.*

The minor scale. — Choosing, with the help of Fig. 266, the tones *below* c', which are most nearly consonant with c', we have the series:

Tones,	c	$e\flat$	f	g	$a\flat$	c'
Vibration frequencies,	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	1

or, in order that this series may be more easily compared with I, we may choose all of these tones an octave higher, whence we obtain the following series of musical tones:

Tones,	c'	$e'\flat$	f'	g'	$a'\flat$	c''	III.
Vibration frequencies,	1	$\frac{6}{5}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{8}{5}$	2	

This series III is more melodious when sounded in the order of descending pitch than when sounded in the reverse order, for the reason that the tones of the series are more nearly consonant with c'' than with c', and whichever of these tones is sounded first is made correspondingly prominent. The series I (and also II) is more melodious when sounded in the order of ascending pitch.

The series III includes the two minor triads (10, 12, 15) c', $e'\flat$, g', and f', $a'\flat$, c''. To build a minor triad upon g', two additional tones, say $b'\flat$ and d'', are required, such that $g' : b'\flat : d'' = 10 : 12 : 15$, so that the vibration frequency of $b'\flat$ is $\frac{9}{5}$ and the vibration frequency of d'' is $\frac{9}{4}$. Taking a tone d' an octave below d'', we have the series:

Tones,	c'	d'	$e'\flat$	f'	g'	$a'\flat$	$b'\flat$	c''	IV.
Vibration frequencies,	1	$\frac{9}{8}$	$\frac{6}{5}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{8}{5}$	$\frac{9}{5}$	2	

This series of tones is called the *descending minor* scale. For purposes of melody this scale is changed to the following for ascending movements:

Tones,	c'	d'	$e'\flat$	f'	g'	a'	b'	c''	V.
Vibration frequencies,	1	$\frac{9}{8}$	$\frac{6}{5}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2	

This is called the *ascending minor* scale.*

Discussion of major and minor scales. — The scales II and IV are better suited to the requirements of harmony than is scale V.

* Scale IV is called the descending *melodic* minor scale, scale V is called the ascending *melodic* minor scale; a third minor scale which is more largely used than either of these in modern music is the *harmonic* minor scale which is as follows:

c' d' $e'\flat$ f' g' $a'\flat$ b' c''

The use of this scale results in part from employment in music of consonant effects depending upon the seventh harmonic (that is, the harmonic overtone which vibrates seven times as fast as the fundamental).

The scale II is suited to harmony in which major triads predominate. It is for this reason called the major scale. The scale IV is suited to harmony in which minor triads predominate. It is for this reason called the minor scale.

The following schedules exhibit all of the major and minor triads which can be formed of the tones of major and minor scales.

The major scale.

major triad major triad major triad

$f \quad a \quad c' \quad e' \quad g' \quad b' \quad d''$

minor triad minor triad

The minor scale.

major triad major triad

$f \quad a\flat \quad c' \quad e'\flat \quad g' \quad b'\flat \quad d'$

minor triad minor triad minor triad

The tonic triad is shown in each case by the bold-faced type. In the major scale the tonic triad, the dominant triad, and the subdominant triad are major triads. In the minor scale these triads are minor triads.

The naming of consonant intervals.—The pitch intervals between the tonic and the third and sixth tones of the major scale are called the *major third* and *major sixth* respectively. The pitch intervals between the tonic and the third and sixth tones of the minor scale (IV) are called the *minor third* and *minor sixth* respectively. The intervals between the tonic and the fourth and fifth tones of either scale are called the *fourth* and *fifth* respectively.

Note.—The tones which are consonant with the tonic c' are called *related tones of the first order.* The tones d' and b' of the major scale and d' and $b'\flat$ of the minor scale which are consonant with g' are called *related tones of the second order* (that is, second order as related to the tonic c').

170. Forms of musical expression.—Expression of any kind depends upon the use of forms which may be readily seized upon

and sharply distinguished by the senses (see Art. 166). The elementary forms of musical expressions are *tempo*, *rhythm*, *melody*, *harmony* and *modulation*. The rapidity of succession of the tones in music is called the *tempo*; the manner in which successive tones are set off in accented groups is called *rhythm*; a sequence of tones is called a *melody*; and certain combinations of simultaneous tones constitute *harmony*.

171. Modulation. — Two chords are said to be related when they have one or more tones in common. Thus the two major triads

$$400 : 500 : 600 \quad \text{and} \quad 600 : 750 : 900$$

are related because they have the common tone 600. A sequence of chords, each related to the one preceding it, is called a *modulation*.

Examples. — The following schedules, in which the tones are set off in major and minor triads, show the possible modulations in the major and minor scales (see Art. 169).

Major scale.

major triad major triad major triad

$$f \quad a \quad c' \quad e' \quad g' \quad b' \quad d''$$

minor triad minor triad

Minor scale.

major triad major triad

$$f \quad a\flat \quad c' \quad e'\flat \quad g' \quad b'\flat \quad d''$$

minor triad minor triad minor triad

More extended modulations than those exhibited in this schedule require the use of tones related, in the third order,* to the tonic c'. Let us consider, for example, an extension in both directions of the modulation of the major scale. We have:

$$\overbrace{w \quad x \quad f} \quad a \quad c' \quad e' \quad g' \quad b' \quad \overbrace{d'' \quad y \quad z}$$

The brackets represent major triads. The tones y and z are not related to c', but they are related to g' exactly as b' and

* See note at end of Art. 169.

d'' respectively are related to c'. Thus the extension of this modulation upwards leads to a set of tones having a new tonic, namely, g'. In like manner the tones w and x are related to f exactly as f and a respectively are related to c', so that an extension of this modulation downwards leads to a set of tones having a new tonic, namely, f. Such an extended modulation in one direction or the other is called a *change of key*.

172. The tempered scale. — A great number of distinct tones is required for extended modulation, and it would be impracticable for a player to use a piano or organ having a separate key (and string or organ pipe) for each tone. This difficulty is overcome, at the expense of accuracy of tuning of the various consonant intervals, by the use of what is called the *tempered scale*. This consists of twelve tones (thirteen, counting both end tones) in each octave, the pitch intervals between successive tones being equal. The octave is thus divided into twelve equal pitch intervals. The logarithm of the frequency ratio of each of these intervals is therefore one twelfth of the logarithm of 2, which is the frequency ratio of the octave (compare Art. 164). The accompanying table shows the logarithms of the frequency ratios of each tone of the major scale to the tonic, and also the logarithms of the frequency ratios of each tone of the tempered scale to the tonic.

This table shows that the first, third, fifth, sixth, eighth, tenth, twelfth, and thirteenth tones of the tempered scale are very nearly in unison with the successive tones of the untempered major scale. These tones may, in fact, be used for the tones of a major scale; and since the intervals of the tempered scale are all equal, it is clear that *any tone of the tempered scale may be chosen as a tonic, and that the third, fifth, sixth, eighth, tenth, twelfth and thirteenth tones, counting from the chosen one, constitute a major scale.* All such major scales are equally well in tune. Unlimited modulations may be carried out on this scale inasmuch as any tone reached in a modulation has a group of tones related to it as a tonic. A minor scale may also be made up from the tempered scale. The tempered scale is now universally used.

COMPARISON OF TONES OF TEMPERED SCALE AND MAJOR SCALE.

Number of Tone in Tempered Scale.	1	2	3	4	5	6	7	8	9	10	11	12	13
Log. of ratio of tone of tempered scale to tonic.	0	0,0251	0,0502	0,0753	0,1003	0,1254	0,1505	0,1756	0,2007	0,2258	0,2509	0,2759	0,3010
Nearest tone in major scale.	*c*		*d*		*e*	*f*		*g*		*a*		*b*	*c'*
Log. of ratio of tone of major scale to tonic.	0		0,0511		0,0979	0,1250		0,1761		0,2219		0,2730	0,3010
Difference of logs.	0		—0,0009		+0,0024	+0,0004		—0,0005		+0,0039		+0,0029	0

CHAPTER XVII.

MISCELLANEOUS PHENOMENA DEPENDING UPON THE REFLECTION, REFRACTION AND DIFFRACTION OF SOUND. ARCHITECTURAL ACOUSTICS.

173. Echo.—This familiar phenomenon is produced by the reflection of sound. The echo from the side of a large building is very clear and distinct, and the smooth face of a cliff or a well-defined forest front may produce an echo sufficiently distinct to repeat words. An echo grows less distinct the more irregular the reflecting surface, and it becomes a confused roar when the reflecting surface is very irregular. With multiple reflections, as in the case of the two walls of a cañon, a sharp loud sound, such as the report of a gun, becomes a prolonged rumble like thunder.

Reflection often produces the effect of apparent change of direction of a sound, when the direct waves from the source are masked or diverted so that the hearer perceives only the reflected wave-trains. This effect is sometimes strikingly produced at a street corner where the sound from an approaching trolley car is reflected from the wall of an opposite building.

174. Influence of the refraction of sound upon hearing at a distance.* — Phenomena due to regular refraction, as sound waves pass from one medium into another in which the velocity is different, do not occur to ordinary observation. The following phenomena, however, which are due essentially to refraction, are of common occurrence.

The velocity of the wind is usually less near the ground than higher up, and the upper portion of a sound wave *W*, Fig. 267, proceeding against the wind, is retarded. The direction of pro-

* A most interesting discussion of the transmission of sound through the atmosphere may be found on pages 257–323 of Tyndall's book, *On Sound*.

gression of the wave is thus turned upwards and the sound tends to leave the region near the ground. When the wave travels with the wind, the tendency is to concentrate the sound near the ground. This is the explanation of the familiar fact that it is much more difficult to make one's self heard against than with the wind.

Sound travels faster in hot than in cold air. When the air near the ground is warmer than it is higher up, the upper por-

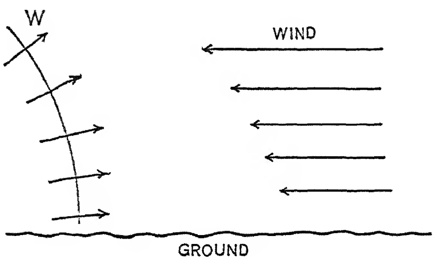


Fig. 267.

tion of a sound wave is retarded and the sound tends to leave the ground. When the air near the ground is relatively cool. the tendency is for the sound to be concentrated near the ground. The greater distinctness of distant sounds by night than by day is due largely to this cause.

*Influence of diffraction upon the sense of direction of a sound.** — Our sense of the location (direction) of a source of sound seems to depend in part upon diffraction. The approaching waves reach one ear without much obstruction while the other ear is more or less shaded by the head. This shading action is greater the shorter the wave-length, so that a compound tone produces different sensations in the two ears. Differences of sensation brought about in this manner are significant of direction and have come to be perceived as such.

175. Influence of motion on wave-length and upon apparent direction of wave progression. Doppler's principle and astronom-

* See Lord Rayleigh, *Philosophical Magazine*, Vol. 13, pages 214–232, February, 1907.

ical aberration. — If a person moves in the direction in which a train of sound-waves is traveling, the frequency with which the waves fall upon the ear will be altered and a change of pitch will result.* Movement of the observer in the direction of progression of a wave-train lowers the pitch, and movement of the observer in the direction opposite to the direction of progression of the wave-train raises the pitch. This effect depends only upon the relative motion of the sounding body and the hearer, and the above described effects are produced when the hearer is stationary and the sounding body is moving. Thus, the whistle of an approaching locomotive is raised in pitch and that of a receding locomotive is lowered in pitch. This effect is especially noticeable to a person on a rapidly moving train who listens to the whistle or bell of a passing locomotive moving in the opposite direction on an adjoining track. This effect of motion upon the apparent wave-length or frequency of a wave-train is called *Doppler's principle*, and it is applicable to light as well as to sound. Thus, the motion of a luminous body towards or away from an observer produces a change in the wave-length of the light given out by the body; and when the characteristic grouping of the lines of a star's spectrum leaves no doubt as to the identity of the group of lines with corresponding lines in the solar spectrum, then a slight displacement of the entire group of stellar lines towards the violet end or red end of the spectrum is ascribed to a motion of the star towards or away from the earth, respectively.

The rapid motion of the earth in its orbit affects the apparent direction of rays of light from a star very much as the motion of a steam-boat affects the apparent direction of the wind. The result is that all stars are seen forwards (in the direction of the earth's motion) of their true position. This effect is called astronomical aberration.†

* See problems 2 and 3.

† No theory of light has yet been devised which is entirely consistent with all of the optical phenomena which are due to the rapid orbital motion of the earth.

176. Acoustics of the auditorium.* — The simplest auditorium is a level stretch of smooth, hard ground with a single hearer. In this case the sound of the speaker's voice spreads as a hemispherical wave and it decreases in loudness approximately as the inverse square of the distance. If the ground is covered by a large audience, the lower edges of the hemispherical sound waves are to a great extent absorbed, and thus greatly reduced in intensity; and the sound reaches the distant auditors largely by bending down from above, that is, by diffraction.

The first and most obvious improvement is to raise the speaker above the level of the audience, or to arrange the audience upon rising tiers of seats so that each auditor may receive sound by direct radiation, as it were, from the speaker. A second improvement is to place a reflecting wall behind the speaker so as to reflect more of the sound of the speaker's voice towards the audience. A third improvement is to arrange a roof so that the sound which radiates upwards may be reflected towards the audience and utilized. The closed auditorium thus arrived at is approximately perfect in so far as it directs practically the whole of the speaker's voice towards the audience, thus giving the maximum of attainable loudness for a given effort by the speaker. The closed auditorium, however, gives rise to undesirable effects, confusion and distortion, which may be to some extent eliminated by proper design.

Confusion. — The closed auditorium gives rise to an overlapping of the successive sounds in speech or music. This is due to the prolongation of each sound by repeated reflections from the walls and ceiling of the room. If the room is very large these successive reflections of a given sound may be separately audible as a *multiple echo.* If the room is small, or if a large room has its reflecting surfaces broken up by pillars and alcoves, the successive reflections blend into a more or less continuous roar. This effect is called *reverberation.* In some auditoriums this

* The material in this and the succeeding articles is adapted from Professor W. C. Sabine's papers on Architectural Acoustics, *American Architect*, 1900.

reverberation lasts as long as five or six seconds after a loud sound, and when we consider that there are from fifteen to twenty separate articulate sounds produced by a speaker per second, counting both vowel and consonant sounds, we can understand what a serious matter excessive reverberation may be. In good theaters for the hearing of speech, the reverberation has a duration of from one to two seconds after a loud musical note, and in good music halls the reverberation has a duration of from two to three seconds.

Distortion. — Consider the various simple tones which enter into the composition of a compound tone such as a vowel sound. The wave-trains corresponding to these simple tones are reflected from the various walls of a room, and there are certain points in the room where one or more of the tones are strengthened while others may be weakened by interference. Thus, the relative loudness of the various simple tone components of the complex sound is changed and the character of the sound is altered. Furthermore, the air in a room has certain proper frequencies of vibration, the same as the air in an organ pipe; that is, the air in a room has certain proper tones, and any tone which is in unison with a proper tone of a room is more or less strengthened by resonance. Distortion by interference is most prominent in a room with flat reflecting walls, and distortion by resonance is most prominent in small rooms or in a room having alcoves. Distortion is seldom, if ever, a serious matter.

177. The elements of auditorium design. — In the design of an auditorium provision must be made first of all for directing the sound of the speaker's voice, or the sound of an orchestra, towards the audience. Each auditor should be seated so as to receive sound by direct radiation from the source, and the reflecting surfaces, such as the ceiling and the wall back of the speaker, should be located so that the reflected sound may not reach the auditor more than, say, a twentieth of a second later than the sound which is radiated directly to the auditor; that is, the path of the reflected sound should not be more than about seventeen meters

longer than the path of the directly radiated sound, otherwise the successive articulate sounds of a speaker will overlap perceptibly. Furthermore, the reflecting surfaces, to be effective, must be large in comparison with the wave-lengths of the prominently useful tones in speech and in music.

The walls of a large auditorium from which reflected sounds reach the auditors more than a fifteenth or a twentieth of a secand after the directly radiated sound, should be broken up by pillars and alcoves so as to avoid sharp echo and the consequent confusing repetition of the articulate elements of speech.

Reverberation. — For brevity, let us call those reflecting surfaces which send sound to the auditors with less delay than a twentieth of a second, *primary* reflecting surfaces; and those surfaces which reflect sound to the auditors with more delay than a twentieth of a second, *secondary* reflecting surfaces. All primary reflecting surfaces should be hard and smooth, so as to reflect as much as possible of the incident sound, and all secondary reflecting surfaces should be soft, covered with thick felt for example, so as to reflect as little as possible of the incident sound. The duration of reverberation could, of course, be reduced to the greatest possible extent by making all reflecting surfaces soft, but a serious sacrifice of loudness would be involved. The control of reverberation is therefore dependent upon the proper treatment of the secondary reflecting surfaces.

178. The equation of decaying sound in a room. — The experimental determination of the absorbing power of various wall surfaces, and the use of such experimental data in the calculation of the duration of reverberation as a guide in the design of an auditorium, depends upon the equation which expresses the intensity i (sound energy per unit volume of the room) of a decaying sound at an instant t seconds after the source of the sound has ceased. This equation is:

$$i = I\epsilon^{\frac{-86a}{V}\cdot t} \tag{i}$$

in which ϵ is the Naperian base and V is the volume of the room

in cubic meters. The significance of the quantity a is as follows: The rate at which sound energy is lost in the room is proportional to the intensity of the sound at each instant: the rate at which sound energy would be lost through an open window is also proportional to the intensity of the sound at each instant. Therefore, the effect of walls and objects in a room in causing a loss of sound is *equivalent to a certain definite area, a square meters, of open window.* The walls and objects in a room cause a loss of sound energy in three more or less distinct ways: (1) As the air particles near a wall oscillate to and fro parallel to the wall, sound energy is converted into heat by the friction. (2) When the air particles move to and fro perpendicularly to a porous wall, friction occurs in the pores of the wall and energy loss results. (3) A wall partakes more or less of the motion of the contiguous air and if the material of the wall is imperfectly elastic, energy losses occur.

Derivation of equation (i). — It is required to find the number of units of sound energy which pass out of a room per second through a window of area a. Let V be the volume of the room and i the sound energy per unit volume at time t seconds

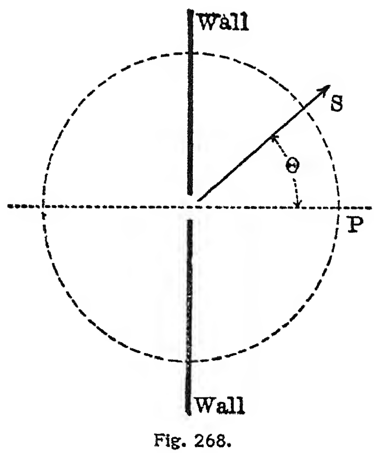


Fig. 268.

after the cessation of the sound source. It is assumed that the sound energy is uniformly distributed throughout the room and that the sound in the neighborhood of a point is traveling, on the average, equally in every direction.

The problem under consideration is simplified if we consider the flow of energy in

one direction through a hole in a wall built across the room so as to divide the room into halves. Describe a sphere of radius r with its center at the hole. The area of the sphere is $4\pi r^2$. We may imagine the sound energy per unit volume i, to be divided into $4\pi r^2$ equal parts, each of which parts is traveling at the normal velocity of sound (344 meters per second at ordinary room temperature) towards a separate unit of area of the sphere. Draw a diameter to the sphere perpendicularly to the wall. Consider the zone of the spherical surface which lies between θ and $\theta + d\theta$, where θ is the angle shown in Fig. 268. The area of this zone is $2\pi r^2 \sin\theta \cdot d\theta$. Therefore the fractional part, $2\pi r^2 \sin\theta \cdot d\theta / 4\pi r^2$, of the total sound energy near the hole is streaming towards the various elements of this zone and the component, normal to the wall, of the velocity of these streams of energy is $344 \cos\theta$. Therefore

$$\frac{2\pi r^2 \sin\theta \cdot d\theta}{4\pi r^2} \times i \times 344 \cos\theta \times a$$

is the rate at which energy is streaming through the hole towards the various elements of the zone. The total rate at which energy is streaming through the hole from left to right is therefore

$$E = 172ai \int_0^{\pi/2} \sin\theta \cos\theta \cdot d\theta$$

or

$$E = 86ai \tag{ii}$$

This rate of flow of sound energy out of a room through a window of area a is equal to the rate, $d(Vi)/dt$, at which the total sound energy Vi in the room is decreasing, the room being assumed for the purpose of the immediate discussion to lose sound energy only through the window, therefore

$$\frac{d(Vi)}{dt} = -86ai \tag{iii}$$

whence

$$i = I\varepsilon^{-\frac{86a}{V}\cdot t}$$

in which ε is the Naperian base, and I is the value of i when t equals zero.

In an actual room the loss of sound energy due to absorption by the walls and objects in the room is sensibly *proportional at each instant to the intensity* i *of the sound* at that instant. Therefore equation (iii) expresses the rate of loss of total sound energy at each instant in an actual room, a being the open window area which is equivalent to the walls and objects in the room in so far as loss of sound energy is concerned, and equation (i) expresses the value of the decaying sound i at time t.

179. Absorbing power of various surfaces and objects. — The accompanying table gives the absorbing power of various surfaces and objects as determined by Sabine. These absorbing powers were determined by the methods explained in Art. 182.

To find the open-window-equivalent of a given surface, multiply the area of the surface by p.

TABLE.

Absorbing power of surfaces.

Character of Surface.	Equivalent Area (p) of Open Window for Each Square Meter of Surface
Wood sheathing.	0.061
Plaster on wood lath.	0.034
Plaster on wire lath.	0.033
Window glass, single thickness.	0.027
Plaster on tile.	0.025
Brick set in cement.	0.025
Carpet rugs.	0.20 to 0.29
Cretonne cloth.	0.15
Shelia curtains.	0.23
Linoleum, loose on floor.	0.12
Hair felt, 2.5 cm. thick, 8 cm. from wall.	0.78
Audience, compact.	0.96
Audience, compact, per person.	0.44
Isolated man (average).	0.48
Isolated woman (average).	0.54
Upholstered chairs, hair and leather.	0.30
Hair cushions, per seat.	0.21
Elastic-felt cushions, per seat.	0.20
Plain wood settees, per seat.	0.008

To find the open-window-equivalent of a number of similar objects, multiply the number of objects by the open-window-equivalent of one of the objects.

To find the open-window-equivalent of all the surfaces and objects in a room, add the open-window-equivalents of each. This gives the value of a in equations (i), (ii) and (iii).

180. Calculation of duration of reverberation.—The degree of reverberation in a room is expressed by Sabine as the time t_1 required for a musical tone to decay from any initial loudness to a loudness one millionth as great. Let I be the initial loudness, then after t_1 seconds the actual loudness i will have decayed to the value

$$i = \frac{I}{1{,}000{,}000}$$

Substituting this value of i in equation (i) and solving for t_1 we have:

$$t_1 = 0.161 \frac{V}{a} \qquad \text{(iv)}$$

Example. — The New Music Hall in Boston has 1,040 square meters of plaster on lath which is equivalent to 34 square meters of open window, 1,830 square meters of plaster on tile which is equivalent to 45 square meters of open window, 22 square meters of window glass which is equivalent to 0.6 square meter of open window, 625 square meters of wood which is equivalent to 38 square meters of open window, and the full audience including the orchestra is 2,659 persons which is equivalent to 1,169 square meters of open window. Therefore, the value of a for this room with a full audience is 1,286 square meters of open window. The volume of the room is 16,200 cubic meters. From these data equation (iv) gives 2.08 seconds as the value of t_1.

Remark 1. — This example serves to show the predominating influence of the audience upon the duration of reverberation. If the seats are hard and smooth the value of a for the empty room would be about a fifth as great as in the above example, and the value of t_1 would be about ten seconds. If the seats have cushions, the value of a for the empty room would be about half as great as in the above example and the value of t_1 would be about four seconds.

Remark 2. — Consider two rooms A and B built on the same plan and of the same materials, room A being, say, three times as large in every dimension. Then the total absorbing power a of room A will be nine times as great as the absorbing power of room B, while the volume of A will be twenty-seven times as great as B. Therefore, the value of t_1 will be three times as great in room A as in room B. That is, reverberation is more and more pronounced the larger the room, and the importance of properly designing large rooms is correspondingly great.

181. Growth of sound in a room. — When a source of sound, an organ pipe, for example, starts in a room, the intensity i of sound in the room increases until the sound is absorbed by walls and objects as fast as it is generated at the source. Let g be the rate at which the sound is generated and $-d(Vi)/dt$ the rate at

which the sound is lost by absorption, then, when the sound has reached its full degree of loudness, we have

$$g = -\frac{d(Vi)}{dt}$$

Substituting this value of $d(Vi)/dt$ in equation (iii) and solving for i we have:

$$i_u = \frac{g}{86a} \tag{v}$$

in which i_u is the ultimate loudness or intensity reached by the sound in a room of which the total absorbing power is equivalent to a square meters of open window, and g is the rate at which sound energy is given off by the sounding body.

Remark. — A number n of similar organ pipes similarly blown give off sound energy n times as fast as one pipe, and therefore, the ultimate loudness of the sound produced in a given room is n times as great for the n pipes as it is for a single pipe.

182. Determination of absorbing power. *First method.* — A standard source of sound, for example, an organ pipe supplied with air at constant pressure, is set up in a room having hard, smooth walls and provided with a number of adjustable windows. The material of which the absorbing power is to be determined is spread upon the walls or floor and the time that the sound remains audible after cessation of the source is observed. The material is then removed and the windows are opened until the sound remains audible for the same length of time as before. The total area of opening of the windows is then the measure of the absorbing power of the material.*

Second method. Determination of total absorbing power of a room. — The time t'' for the sound of initial intensity I produced by one organ pipe to decay to minimum audible intensity i' is observed, and the time t''' for the sound of initial intensity nI produced by n similar organ pipes to decay to minimum audible intensity i' is observed. We then have from equation (i) the two equations:

$$i' = I\varepsilon^{-\frac{86at''}{V}} \tag{a}$$

and

$$i' = nI\varepsilon^{-\frac{86at'''}{V}} \tag{b}$$

Dividing equation (b) by equation (a) we have

$$1 = n\varepsilon^{-\frac{86a}{V}(t'''-t'')}$$

* Some allowance should, of course, be made for the absorbing power of the bare wall or floor which is covered by the material.

or

$$o = \log n - \frac{86a}{V}(t''' - t'') \log \varepsilon$$

whence

$$a = \frac{V \log n}{86(t''' - t'') \log \varepsilon} \tag{vi}$$

Third method. Determination of absorbing power of the different kinds of wall surfaces and objects. — Suppose, for example, that there are four kinds of absorbing surface to be determined. Choose four different rooms, A, B, C and D, in which these four surfaces occur in markedly different proportions.

Let

$$A', A'', A''' \text{ and } A^{iv}$$

$$B', B'', B''' \text{ and } B^{iv}$$

$$C', C'', C''' \text{ and } C^{iv}$$

$$D', D'', D''' \text{ and } D^{iv}$$

be the respective measured areas of the four classes of reflecting surface in the respective rooms; let p', p'', p''' and p^{iv} be the absorbing power of each kind of surface, and let a', a'' a''' and a^{iv} be the total absorbing powers of the respective rooms. Then

$$a' = A'p' + A''p'' + A'''p''' + A^{iv}p^{iv}$$

$$a'' = B'p' + B''p'' + B'''p''' + B^{iv}p^{iv}$$

$$a''' = C'p' + C''p'' + C'''p''' + C^{iv}p^{iv}$$

$$a^{iv} = D'p' + D''p'' + D'''p''' + D^{iv}p^{iv}$$

The four total absorbing powers a', a'', a''' and a^{iv} may be determined as explained above in the second method. Then these four equations permit the calculation of p', p'', p''' and p^{iv}.

APPENDIX A.

CARDINAL POINTS AND CARDINAL PLANES OF LENS SYSTEMS.

1. Centered system of thin lenses. — A number of lenses used together constitute a *lens system.* When the centers of curvature of the various spherical surfaces of the lenses lie on one straight line, the system is called a *centered system.* The following articles treat of the action of a system of thin lenses on the assumption that the various imperfections (see Chapter VI) are negligible, and the discussion is limited to the consideration of pencils of rays which are but very slightly inclined to the axis of the system. The following figures, however, show rays making considerable angles with the axis for the sake of clearness.

Inasmuch as the individual thin lenses are assumed to be free from any imperfections, the following propositions are evidently true. The following discussion of cardinal points and cardinal planes is based upon these propositions.

(*a*) A narrow pencil of rays from any luminous point O in or near the axis of a lens system, is sensibly homocentric after passing through the system, and the emergent pencil is concentrated at or appears to have come from a point O'. Two such points are called *conjugate points.*

(*b*) A group of luminous points (an object) near the axis and in a plane perpendicular to the axis, has, as its image (with definite magnification, positive or negative) a similar group of points near the axis in another plane perpendicular to the axis, Two such planes are called *conjugate planes.*

Cor. 1. — Any incident ray passing through a point O must, upon emergence, pass through O', the conjugate to O. An incident ray and the corresponding emergent ray are called *conjugate rays.*

Cor. 2. — Two incident rays intersecting at O, intersect, upon emergence, at O'. A particular case is where one incident ray is coincident with the axis of the system. Such an incident ray emerges without change of direction, and therefore the points where any two conjugate rays intersect the axis constitute a pair of conjugate points.

Cor. 3. — An incident ray which passes through the points O and p, must, upon emergence, pass through O' and p', the conjugates to O and p.

Note 1. — If an emergent ray be reversed, for example, by normal reflection from a mirror, it will retrace its path. If, therefore, the ray r' is conjugate to the ray r, then r is conjugate to r'; and if the point O' is conjugate to O, then O is conjugate to O'.

Note 2. — Any specification which completely determines the position upon emergence of any given incident ray, is a complete specification of the lens system.

2. Specification of a lens system. — *A lens system is completely specified, in so far as its optical properties are concerned, when the positions of two pairs of conjugate planes are given together with the magnification associated with each pair.* The magnification is the ratio of diameter of object to image, and it is considered to be negative when the image is inverted.

Proof.—Let aa' and bb', Fig. 1, be two given pairs of conjugate planes. Let m be the magnification associated with aa' and let m' be the magnification associated with bb'. Figure 1 is constructed for $m = -1.5$ and $m' = +1.5$, that is to say, any point in plane a is m times as far from the axis as its conjugate in plane a', and any point in plane b is m times as far from the axis as its conjugate in b'. Let

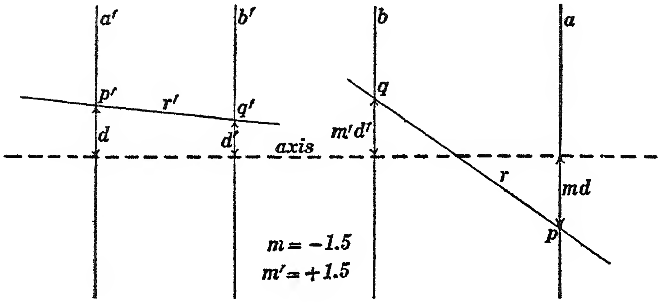


Fig. 1.

r' be any given incident ray, cutting the planes a' and b' at the points p' and q' as shown. The emergent ray r must pass through the points p and q which are conjugate to p' and q', respectively. The emergent ray is thus determined, and the action of the lens system upon the incident ray r' is therefore completely established by the given data.

3. Focal points of a lens system.*—Consider an incident ray r', Fig. 2, coming from the left parallel to the axis. Its conjugate ray r is as shown. If the

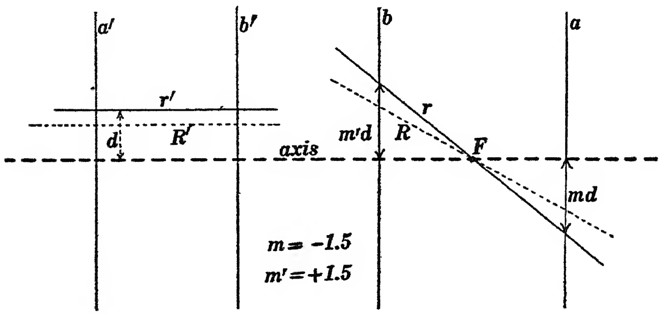


Fig. 2.

distance d changes, the distances md, $m'd$ change in the same ratio, as shown by the dotted rays R and R', and a certain point F remains fixed (in Fig. 2 the point F is midway between the planes a and b because the magnifications m and m' are equal and opposite.) Therefore, all rays from the left, parallel to the axis, pass through the point F, or seem to have come from F after emergence. This point F is called the *right focal point* of the system. Figure 3 shows the construction for the

* What are usually called *principal foci* are here referred to.

left focal point, F' (in Fig. 3 the point F' is midway between the planes a' and b' because the magnifications m and m' are equal and opposite).

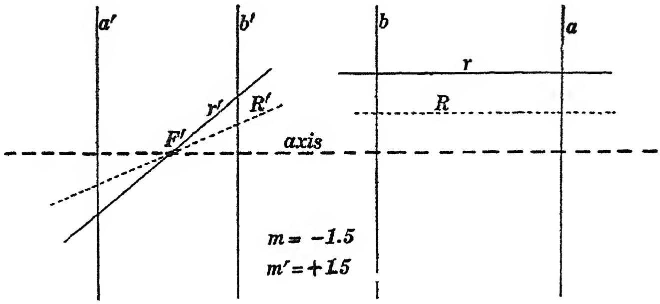


Fig. 3.

4. Principal planes of a lens system. — Those two conjugate planes for which the magnification is *plus one* (+ 1) are called the *principal planes* of the system. The following discussion shows that there is always such a pair of planes, and shows their location relative to the given conjugate planes aa' and bb'.

Let r, Fig. 4, be an incident ray from the right and let r' be its conjugate. Let R', colinear with r, be an incident ray from the left and let R be its conjugate. The rays r and R intersect at o, and the rays r' and R', which are conjugate to r and R respectively, intersect at o'. Therefore o and o' are conjugate points, and P and P' are conjugate planes; and since o and o' are at the same distance from the axis, it follows that the magnification for the conjugate planes P and P' is unity, so that P and P' are the required principal planes of the system. The distance P to F is called the *right focal length* of the system, and the distance P' to F' is called the *left focal length* of the system. The system represented in Figs. 1, 2, 3, 4, 6 and 7 is a converging system because the right focal point F is to the right of the principal plane P, and the left focal point

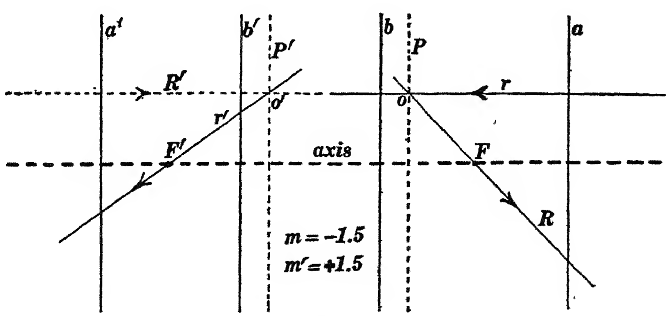


Fig. 4.

F' is to the left of the principal plane P', that is to say, both focal lengths are positive. The two focal lengths of the lens system shown in Fig. 4 are unequal.

The two focal lengths of a lens system have a ratio equal to the ratio of the refractive indices of the media in which the respective focal points lie. In most lens systems used in practice, the two focal lengths are equal since both focal points are in air.

Examples. — The plane LL in Fig. 114, page 108, is the right principal plane of the lens system shown in that figure, and the distance from LL to F is the right focal length of the system. The plane LL is determined, as in Fig. 4, by the intersection of the emergent ray R with the line of the incident ray R' (extended if necessary). The point A in Fig. 115 shows the position of the right principal plane of the lens system for red light, and the point B shows the position of the right principal plane for violet light.

Figure 5 shows to scale the actual positions of the principal planes of a symmetrical bi-convex lens (not of infinitesimal thickness), the glass of which has a refractive index of 1.5. The figure also shows the geometrical construction for

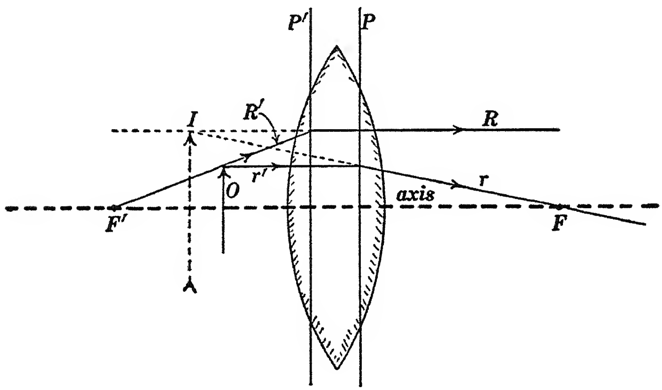


Fig. 5.

determining the position of the image of a given object as follows: Draw the ray r' parallel to the axis from O to the principal plane P, and thence draw the ray r through the focal point F. Draw the ray R' from F' through O to P', and thence draw the ray R parallel to the axis. The point of intersection of the emergent rays r and R is the conjugate of O and it determines the position of the image. It is instructive to compare this geometrical construction with that which is given in Fig. 76, page 74, for a simple thin lens.

5. The inverse principal planes of a lens system. — Those two conjugate planes for which the magnification is *minus one* (— 1) are called the *inverse principal planes* of the system. The following discussion shows that there is always such a pair of planes, and shows their location relative to the direct principal planes P and P' and the focal points F and F' of the system. Let the principal planes P and P' and the focal points F and F' of the system be given as shown in Fig. 6. These elements completely specify the system. Draw the ray R from the right parallel to the axis of the system, and draw its conjugate ray R' through the left focal point F', as shown. Choose the point q' on the ray R' at the distance d below the axis. The conjugate to q' is, of course, on the ray R, and therefore at the

same distance d above the axis. Therefore, the plane Q' passing through q' is one of the inverse principal planes. To determine the other inverse principal plane, draw the incident ray r' from the left and its conjugate ray r passing through the right focal point of the system. The point q is conjugate to the point q' and therefore the plane Q is the other inverse principal plane.

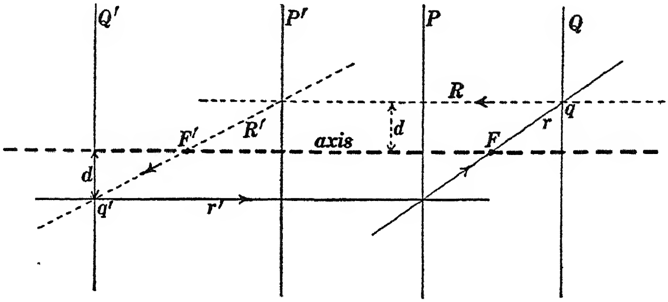


Fig. 6.

From Fig. 6 it is evident that the distance PQ is equal to $2f$ and that the distance $P'Q'$, is equal to $2f'$, where f and f' are the right and left focal lengths of the system, respectively.

6. Nodal points of a lens system. — *There are two conjugate points in the axis such that any incident ray passing through one of these points is, upon emergence, parallel to its direction upon incidence.* These two conjugate points are called the *nodal points* of the system. To understand the geometrical significance of the nodal

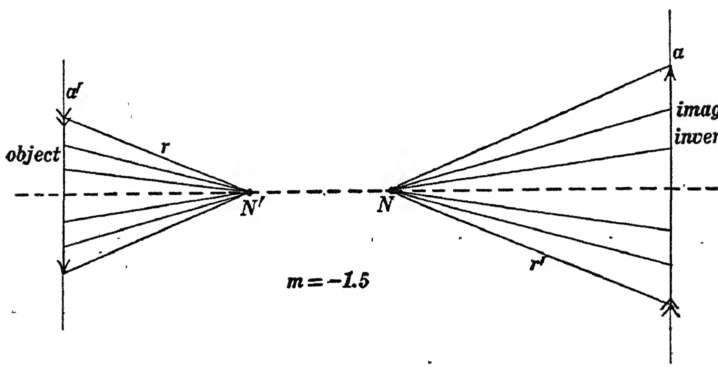


Fig. 7.

points, consider an object and its image as shown in Figs. 7 and 8. The various rays from the object which pass through the nodal point N' radiate, upon emergence, from the nodal point N, the direction of each emergent ray being parallel to the direction of that particular ray upon incidence, so that the object as seen from N' is of the same angular magnitude as the image as seen from N, or in other words, the

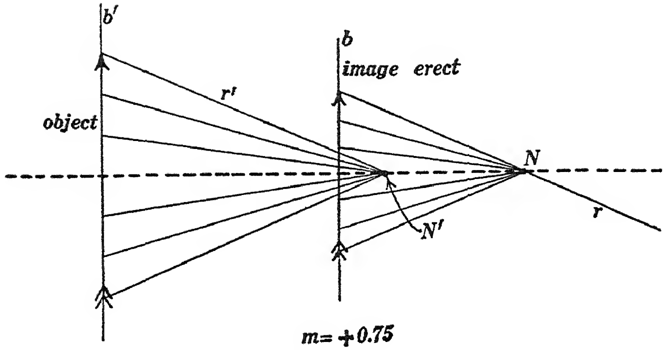


Fig. 8.

diameter of object and the diameter of image are directly proportional to their distances from the respective nodal points.

Consider an object in one of the principal planes P' and its image in the principal plane P. The image is the same size as the object and erect, and therefore the nodal points must be at the same distance and in the same direction from the respective principal planes (see Fig. 9). Consider an object in the inverse principal

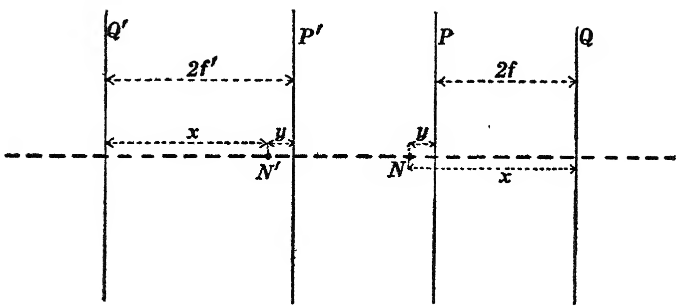


Fig. 9.

plane Q and its image in the inverse principal plane Q'. Inasmuch as the image is inverted and of the same size as the object, the two nodal points N' and N must be at the same distance but in opposite directions from the respective inverse principal planes Q' and Q (see Fig. 9).

The values of x and y in Fig. 9 satisfy the equations:

$$x + y = 2f'$$

$$x - y = 2f$$

whence

$$x = f + f'$$

$$y = f' - f$$

It remains to show that the points N' and N are conjugate to each other. Draw the two parallel lines r' and r in Fig. 10. Since the distances $N'Q'$ and NQ are equal, and since the distances $N'P'$ and NP are equal it is evident that the two points p and p' are at the same distance above the axis, and that the two

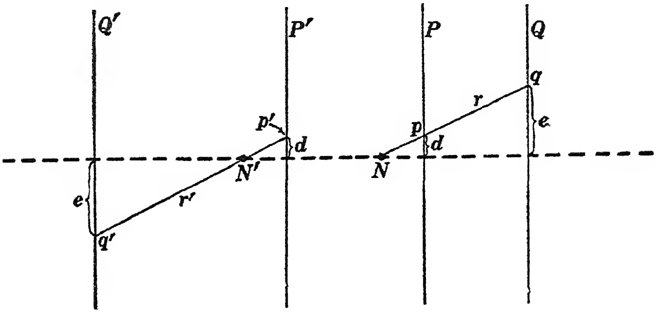


Fig. 10.

points q and q' are at equal distances on opposite sides of the axis. Therefore q and q' are conjugate points and p and p' are conjugate points, so that r and r' are conjugate rays, and therefore the points N and N', where r and r' cut the axis, are conjugate points.

When the right and left focal lengths of a lens system are equal as shown in Fig. 11, then the nodal points N' and N lie in the principal planes.

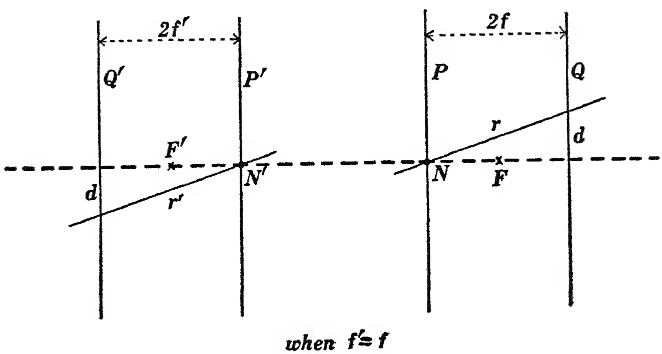


Fig. 11.

7. Focal length of a lens system (doublet) expressed in terms of focal lengths of individual lenses and their distance apart.—The discussion of this matter is greatly facilitated by considering the curvature of a wave surface at different points as it passes through the system. The curvature of a spherical surface may be expressed as $1/r$, where r is the radius of the sphere. Thus, a sphere of 100 centimeters radius has a curvature of 0.01 per centimeter. A wave like WW, Fig.

12, may be thought of as having a *positive curvature* and a wave like WW, Fig. 13, may be thought of as having a *negative curvature*. The curvature of WW in Fig. 12 is continually increasing in value because its radius is decreasing, and the curvature of WW in Fig. 13 is decreasing in value because its radius is increasing.

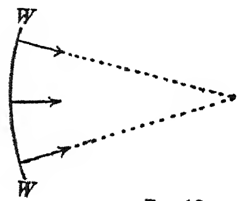


Fig. 12.

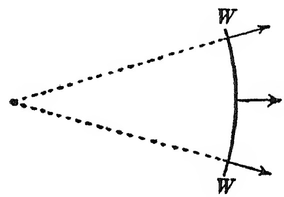


Fig. 13.

The effect of a simple converging lens is to *add* a definite amount $1/f$ to the curvature of the wave, where f is the focal length of the simple lens. In the case of a diverging lens, the focal length is considered negative, and the effect of such a lens is to subtract $1/f$ [or to add $(-1/f)$] from the curvature of a wave. These propositions may be established by a careful consideration of the substance of Art. 49.

Example.—A wave reaches a simple converging lens from a point distant a centimeters from the lens so that the curvature of the wave when it reaches the lens is $-1/a$ (see Fig. 69, page 70). The lens adds to the curvature by the amount $1/f$ giving a positive curvature of $1/f - 1/a$, which is, of course, equal to $1/b$, where b is the distance shown in Fig. 69. Therefore, we have

$$\frac{1}{f} - \frac{1}{a} = \frac{1}{b} \quad \text{or} \quad \frac{1}{f} = \frac{1}{a} + \frac{1}{b}$$

which is equation (5) on page 68.

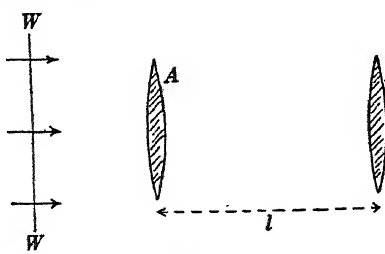


Fig. 14.

Consider two simple lenses A and B, Fig. 14, of which the respective focal lengths are f_1 and f_2, l being the distance of the lenses apart. Consider a plane wave WW (of zero curvature). After passing the first lens A, the curvature of this wave is $1/f_1$. When this spherical wave reaches the lens B, its radius of curvature has decreased to $f_1 - l$, and therefore its curvature is $1/(f_1 - l)$. After passing through the lens B, the curvature of the wave is

$$\frac{1}{f_1 - l} + \frac{1}{f_2}, \quad \text{or} \quad \frac{f_1 + f_2 - l}{(f_1 - l)f_2}, \quad \text{or} \quad \frac{1}{\dfrac{(f_1 - l)f_2}{f_1 + f_2 - l}}$$

The denominator of the last expression is therefore the distance from B to the point F where the wave WW is brought to a focus. From Fig. 15 it is evident that the distance PB is the same fractional part of BF that AB is of BF_a, so that

$$PB : \frac{(f_1 - l)f_2}{f_1 + f_2 - l} :: l : f_1 - l$$

whence

$$PB = \frac{f_2 l}{f_1 + f_2 - l}$$

and, since the focal length f_c, of the lens system AB, is equal to $PB + BF$, we have

$$f_c = \frac{f_1 f_2}{f_1 + f_2 - l}$$

or

$$\frac{1}{f_c} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{l}{f_1 f_2} \qquad \text{(i)}$$

The achromatization of the Ramsden and Huygens eye-piece doublets.—The two lenses of the Ramsden and Huygens eye-pieces are placed at a distance apart equal to one half the sum of their individual focal lengths, as stated on page 116 $[l = \frac{1}{2}(f_1 + f_2)]$. Under these conditions, if both lenses are made of the same kind

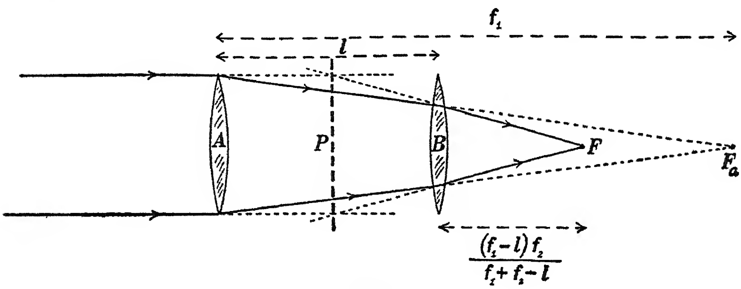


Fig. 15.

of glass, the focal length f_c of the combination is the same for all wave-lengths (colors), although the location of the principal plane P, Fig. 15 (and, of course, the location of the focal point F), is *not* the same for all colors. The Ramsden and Huygens eye-pieces are therefore achromatized for focal length but not achromatized for the focal point. The constancy of f_c for all colors may be shown as follows: A careful consideration of equation (i) on page 69 gives

$$d\left(\frac{1}{f}\right) = \frac{1}{f}\left(\frac{d\mu}{\mu - 1}\right) \quad \text{or} \quad dp = p\left(\frac{d\mu}{\mu - 1}\right)$$

as the expression for the variation of the "power" $(p = 1/f)$ of any simple lens due to a given change (from one color to any other given color) of the index of refraction μ. But the factor $d\mu/(\mu - 1)$ is the same in value for both lenses of a Ramsden or Huygens doublet for any given pair of colors, because the lenses are of the same kind of glass. Therefore, writing equation (i) above in the form

$$p_c = p_1 + p_2 - p_1 p_2 l \tag{ii}$$

and differentiating with respect to p_1 and p_2, using the relation

$$dp = p\left(\frac{d\mu}{\mu - 1}\right),$$

we have:

$$dp_c = p_1\left(\frac{d\mu}{\mu - 1}\right) + p_2\left(\frac{d\mu}{u - 1}\right) - 2lp_1p_2\left(\frac{d\mu}{\mu - 1}\right)$$

and the condition $dp_c = 0$ gives

$$p_1 + p_2 = 2lp_1p_2$$

or

$$l = \tfrac{1}{2}(f_1 + f_2)$$

APPENDIX B.

RADIATION.*

8. Radiant heat and its effects. — The various homogeneous components (simple wave-trains) of the radiation from a hot or luminous body have this common property, namely, they generate heat in a body which absorbs them. Therefore, every portion of the radiation from a hot body is properly called *radiant heat.* Radiant heat of which the wave-length lies between 39 and 75 millionths of a centimeter affects the optic nerves and gives rise to sensations of light (see page 159). Radiant heat of certain wave-lengths has a marked chemical effect which is exemplified by the reduction of carbon dioxide in the growth of plants, by the bleaching action of bright sunlight, and by the action of light upon the photographic plate. The intensity of this chemical action varies greatly with the wave-length. In the case of photography, the green, blue and violet rays are most active, while the extreme red rays are almost wholly inactive.

9. Normal radiation at a given temperature. — Some insight into the character of the radiation from a hot body can be obtained by considering first the very special case in which the given hot body is entirely surrounded by other bodies at the same temperature. The radiation from the given body under these conditions is called the *normal radiation* for the given temperature.

Discussion of normal radiation. — Consider a number of bodies in an enclosed space, a closed vault, for example. These bodies exchange heat until they settle to a uniform temperature, thus reaching a state of *thermal equilibrium.* Ordinarily we think of

* A very good résumé of the theory of radiation is given by C. E. Mendenhall and F. A. Saunders, *Astrophysical Journal*, Vol. XIII, page 25, January, 1901. An excellent discussion of this subject is given by L. Graetz, in Winkelmann's *Handbuch der Physik*, Vol. III, pages 362–430.

heat as being radiated from a body only when the body is hotter than its surroundings; as a matter of fact, however, bodies continue to exchange heat by radiation even after they have settled to thermal equilibrium, but it is then a balanced exchange; each body receives heat as fast as it gives off heat (Prevosts' principle of exchanges). The character of the radiation in a region in thermal equilibrium is independent of the physical nature of the bodies which are present, in fact the radiation in such a region is perfectly definite in composition (by composition is meant the relative intensities of the various wave-lengths) at a given temperature, and it is called the normal radiation for that temperature.

The radiation *coming from a body* in a region in thermal equilibrium is not only equal in quantity to the radiation which is *received by the body*, but it is also identical in composition. The radiation from the body and the radiation to the body are both the normal radiation for the given temperature.

Two aggregates of radiation are said to be *complementary* to each other when, if taken together, they constitute normal radiation for a given temperature.

An ideal black body is a body which gives out normal radiation even when it is **not** *surrounded by other bodies at the same temperature* (see Art. 20).

10. Equation of normal radiation. — Consider the radiation from a black body of which the absolute temperature is T. This radiation is an outward streaming of energy from the body, a certain amount of energy streams each second across a square centimeter of area at a given point in the neighborhood of the body. Let $E.d\lambda$ be the portion of this energy (per second per square centimeter) which is carried by all wave-trains of which the wave-lengths lie between λ and $\lambda + d\lambda$. Then

$$E = \frac{C}{\lambda^5(\epsilon^{c/\lambda T} - 1)} \qquad \text{(i)*}$$

* This equation is due to Planck. See a series of papers by Planck in *Drude's Annalen* for 1901. The method followed by Planck in leading up to this equation is

in which ϵ is the Naperian base, and C and c are constants. The value of C* depends upon the size of the radiating body and its distance away from the observer. The value of c is 1.46 when λ is expressed in fractions of a centimeter and T in centigrade degrees.

11. Stefan's law. — *The total radiation (including all wave-lengths) from an ideally black body is proportional to the fourth power of its absolute temperature.* This law was formulated by Stefan in 1879. It is involved in equation (i), however, as may be shown by integrating E between the limits $\lambda = 0$ and $\lambda = \infty$. Boltzmann † has shown that Stefan's law can be derived from the second law of thermodynamics.

12. Wien's displacement law. — Let λ_m be the wave-length in the spectrum corresponding to the maximum value of E. Wien showed in 1894 that the product $\lambda_m T$ is a constant, or in other words, the wave-length corresponding to maximum energy in the spectrum is inversely proportional to the absolute temperature of the radiating black body. Wien's law is also involved in equation (i).

13. Examples. — The meaning of equation (i) may be understood from Fig. 16, in which the ordinates of the full-line curves represent the values of E as calculated by equation (i), the ordinates of the dotted curves represent the values of E as observed by means of the spectro-bolometer (see Art. 25) by Lummer and

essentially the same as the method employed in the kinetic theory of gases for establishing Maxwell's equation for the distribution of velocities among the gas-molecules, with the addition that in Planck's discussion the molecules are assumed to be vibrating electrical doublets and the laws of electromagnetic induction are made use of in establishing a relation between the wave motion and molecular motion in a region in equilibrium.

* If the black body is in the form of a sphere, then at the surface of the sphere the value of C is $c^4 \times 0.8192 \dfrac{\text{erg}}{\text{cm.}^2\text{ sec. degree centigrade}}$. That is to say, $C/\lambda^5(\epsilon^{c/\lambda T} - 1)$, when multiplied by $d\lambda$ (expressed in centimeters), gives ergs per square centimeter per second at the surface of the black body.

† *Wiedemann's Annalen*, Vol. XXII, page 291, 1884. Boltzmann's argument is given on page 373 of Vol. III, Winkelmann's *Handbuch der Physik*.

Pringsheim,* and the abscissas represent wave-lengths in millionths of a meter (μ). The sharp depressions in the dotted curves are due to the absorption of certain wave-lengths by water vapor and carbon dioxide in the air.

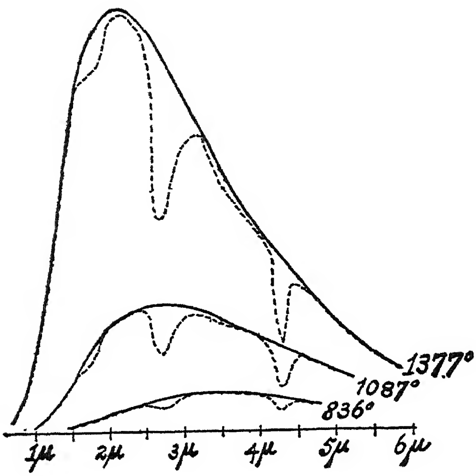


Fig. 16.

The ordinates of the curves in Fig. 17 represent the distribution of energy in the spectra of approximately black bodies at 0° C., at 100° C. and at 178° C., and also in the spectrum of the sun. The long black band represents the spectrum, and the small white gap in it, *RV*, represents the visible part of the spectrum. The abscissas represent wave-lengths in millionths of a meter (μ). The depressions in the solar curve are due to selective absorption by the sun's photosphere and by the earth's atmosphere. One may assume a smooth curve enveloping the observed curve of distribution of energy in the solar spectrum in Fig. 17, to represent approximately the curve of distribution of energy in the spectrum of a black body at the sun's temperature. The wave-length corresponding to the maximum of this smooth curve would be about $\lambda = 0.5\,\mu$. The wave-length corresponding to the

* See Winkelmann's *Handbuch der Physik*, Vol. III, page 391. Lummer and Pringsheim used the *artificial black body* (see Art. 20) as a radiator.

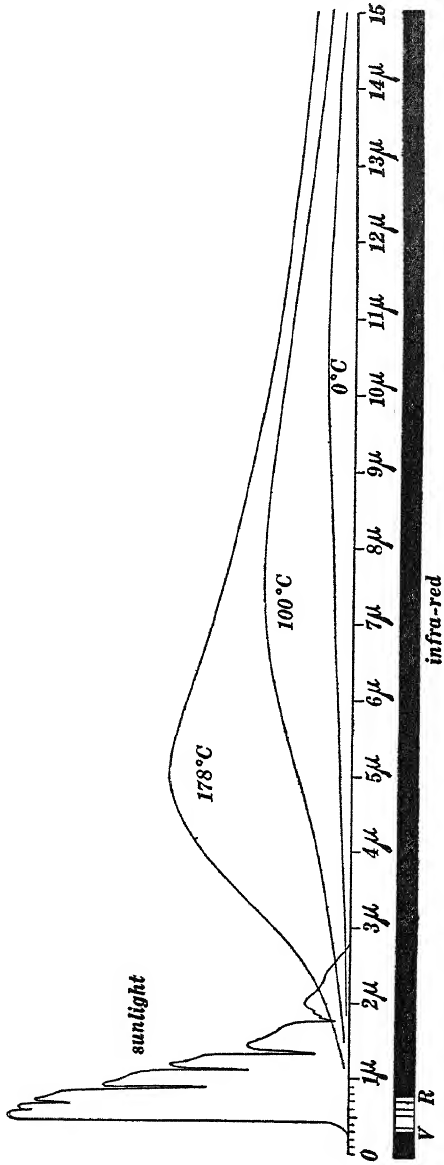

sunlight
178°C
100°C
0°C
0
1μ
2μ
3μ
4μ
5μ
6μ
7μ
8μ
9μ
10μ
11μ
12μ
13μ
14μ
15
V
R
infra-red

Fig. 17.

maximum intensity in the spectrum of an approximately black body at 178° C. (= 451° absolute) is about 5 μ. Therefore, according to Wien's law, the temperature of the sun is about 4500° C

14. Optical pyrometry.* — The laws of black-body radiation furnish a basis for the determination of temperature, as follows:

(*a*) *On the basis of Stefan's law.* — Inasmuch as the total radiation from a black body is proportional to the fourth power of its absolute temperature, the temperature of a radiating black body can be calculated from the observed intensity of its total radiation making due allowance for the size of the body and for the distance of the point at which the intensity of the radiation is measured.

(*b*) *On the basis of Wien's law.* — If the wave-length corresponding to maximum value of E is determined in the spectrum of the radiation from a black body, the temperature of the black body may be calculated as exemplified in the discussion of Fig. 17.

(*c*) *By measuring the ratio of the values of E at two particular places in the spectrum (two values of λ).* — This ratio is independent of the constant C, and it depends only upon the invariable value of the constant c and the temperature of the source (a black body), so that the temperature can be calculated when the ratio has been measured. This is essentially the basis of the practical form of pyrometer known as the Wanner pyrometer.†

All these optical methods for determining high temperatures are subject to error when the radiating body does not conform to the laws of the ideal black body, and indeed the only body which does conform to these laws is the artificial black body which is described in Art. 20. The value of the above methods for measuring temperature is that they can be used for temperatures much

* A very good discussion of Optical Pyrometry is given by Waidner and Burgess, *Bulletin of the Bureau of Standards*, Vol. I, pages 189–254, February 1905.

† If one considers that the constant C in Planck's equation is known when the size and distance of the black radiating body are known, it is evident that the temperature T may be calculated from the observed value of E for one wave-length only. See Winkelmann's *Handbuch der Physik*, Vol. III, page 151. The Wanner Pyrometer is fully described by Waidner and Burgess, *Bulletin of the Bureau of Standards*, Vol. I, pages 226–232.

higher than can be measured by any other means, and *any indirect temperature indicator can be calibrated by one of the above methods, using the artificial black body*. The most widely used optical pyrometer is that of Morse or Holborn and Kurlbaum in which a small carbon-filament glow-lamp is seen projected upon the hot body whose temperature is to be measured. The current through the glow-lamp is increased until the filament is no longer visible (because of absence of contrast between filament and background), and the temperature is then inferred from the observed value of the electric current. The slight differences of color between filament and background are eliminated by using a red glass. The true temperatures corresponding to the various currents are determined once for all by any reliable method of calibration, for example, by one of the above optical methods.

15. Reflection, transmission, absorption and emission of radiation. — When an aggregate of radiation falls upon a body, some of the radiation is turned back or *reflected*, some passes on through the body and is said to be *transmitted*, and some is taken up by the body and is said to be *absorbed*. A primary source of radiation is said to *emit* the radiation it gives off. Thus, a lamp is the original, or primary, source of the light which it gives off, and this light is said to be emitted by the lamp.

The radiation coming from a body is, in general, made up of three parts, namely, (*a*) the radiation emitted by the body (*b*) the radiation reflected by the body from other sources, and (*c*) the radiation transmitted by the body from sources behind it. In a region in thermal equilibrium the total radiation coming from a body is normal in composition, and therefore, in a region in thermal equilibrium, there is a fixed and simple relationship between emitting power, reflecting power and transmitting power of a substance, inasmuch as emitted radiations, reflected radiations and transmitted radiations together make up the normal radiation for the given temperature. This simple relationship between emission, reflection and transmission is applied and illustrated in the following articles. It is to be kept in mind that this

relationship is rigorously true only of a region in thermal equilibrium, where the radiation which falls upon the body and the radiation which comes from the body are both normal radiation.

Example. The appearance of a large uniformly heated tile kiln, as seen through a hole in the wall, is very striking. Even though the interior be partly free from tiles, nothing can be seen but a flood of soft yellow light. The radiation which is reflected, transmitted (if any) and emitted, from a tile, or from a portion of the hot gases, is normal radiation, so that identical radiations reach the eye from every portion of the interior. If the peep hole is large enough to cool the adjacent tiles, they become faintly visible; or if a beam of sunlight (which comes from a very much hotter source than the interior of the kiln) is reflected into the opening the tiles become visible as if they were in a dark chamber.

16. Kirchhoff's law.—*Emission and absorption are equal.* Consider a body in a region in thermal equilibrium. Let us designate by A the radiation falling upon the body, and by B the radiation coming from the body. Both A and B are normal radiation. A portion of B is *reflected* radiation; imagine this portion to be taken away from B and an identical amount subtracted from A. Another portion of B is *transmitted* radiation (through the body from objects behind it); imagine this portion to be taken away from B and an identical amount subtracted from A. The remaining portion of A is absorbed by the body, the remaining portion of B is emitted by the body, and these two portions are evidently identical to each other.

Emitted radiations and absorbed radiations are exactly equal to each other only when a body is surrounded by other bodies at the same temperature so that normal radiation falls upon the body. There is, however, a certain correspondence between emitting power and absorbing power of a substance when it is not surrounded by bodies at the same temperature. Thus, a hot gas emits certain wave-lengths, and the same gas absorbs those particular wave-lengths from a beam of white light which is passed through the gas. See Art. 82, page 131.

17. Selective emission, selective reflection and selective transmission. — A substance when heated to a given temperature usually gives out a radiation which is *not* normal for the given temperature, but which contains more of certain wave-lengths and less of other wave-lengths than normal radiation for the given temperature. Such a substance is said to exhibit *selective emission.*

A substance which exhibits selective emission must also exhibit, to some extent, selective reflection, or selective transmission, or both. This is evident when we consider that the total radiation coming from a body is normal radiation when the body is in a region in thermal equilibrium, as explained in Art. 15, so that an excess of certain wave-lengths in the emitted radiation means relatively less of those particular wave-lengths in the reflected or transmitted radiations, or in both.

The departure of the dotted line from the full line in Fig. 16 furnishes a good example of selective transmission. The radiations which were being studied by Lummer and Pringsheim came from an artificial black body, and certain wave-lengths were strongly absorbed by the water vapor and carbon dioxide in the atmosphere during the passage of the radiations from the artificial black body through the slit and prism of the spectroscope to the focal plane of the spectroscope, where the measurements of intensity of various wave-lengths were made (see Art. 25).

18. Ideal cases of selective action. — There are four ideal cases of selective action the consideration of which will help towards the understanding of the behavior of various substances which approximate to these ideal cases.

(*a*) *Bodies which do not reflect perceptibly.* — Such bodies do not transmit (that is, they do absorb) those wave-lengths which they emit in excess. In this case the transmitted and emitted radiations are complementary to each other.

(*b*) *Bodies which are opaque, that is, which do not transmit.* — Such bodies emit radiations which are complementary to the radiations which they reflect.

(*c*) *Bodies which do not emit perceptibly.* — Such bodies reflect best those wave-lengths which they do not transmit. That is to say, the reflected radiations and the transmitted radiations are complementary.

(*d*) *Opaque bodies which do not reflect* (*black bodies*). — Such bodies emit normal radiation.

The above statements refer to the behavior of bodies when the radiation which falls upon them is the normal radiation corresponding to the temperature of the body. The above mentioned peculiarities persist, however, to some extent, even when the incident radiation is not the normal radiation corresponding to the temperature of the body.

19. Existing cases of selective action. — (*a*) Gases do not reflect perceptibly, and they emit those wave-lengths in excess which they do not readily transmit (which they absorb).

Ruby glass which is red by transmitted light, and which shows no marked selective action by reflection, is green when it is heated to incandescence.

(*b*) Metallic copper is sensibly opaque. It is red by reflected light, but when incandescent it is green.

(*c*) There is no known case which corresponds closely to the third ideal case of selective action above mentioned. Fuchsine, of the emitting power of which nothing definite is known, transmits (in thin layers) red and violet light and reflects green light. The approximately complementary character of the transmitted and reflected radiations shows indeed that the emitting power of fuchsine in thin layers is relatively small.

Gold is yellow by reflected light and very thin gold leaf is green by transmitted light.

An example of great practical importance which falls under the ideal case (*c*) above is the following: A polished metal surface reflects nearly the whole of the light which falls upon it, and, of course, if the metal plate is of sensible thickness it is opaque. *Therefore a polished metal plate does not emit perceptibly.* It is well known, indeed, that a polished silver vessel containing water

remains hot for a longer time than a dull black vessel of the same size. In this case, however, the polished vessel gives off heat chiefly not by radiation, but by direct contact with the air. A polished metal vessel suspended in an almost perfect vacuum radiates its heat very slowly indeed. This is exemplified by the *Dewar bulb*, which is sold under various trade names, such as the "Hotakold Bottle" and the "Thermos Bottle." Figure 18 is a sectional view of a Dewar bulb with the protecting case removed. It consists of an inside glass vessel or bulb *BB*, and an outside glass vessel or bulb *OO*. These bulbs are hermetically sealed together at the top *LL*; the space between the two vessels is filled with a silvering solution, and both glass walls are thus coated with a brilliant layer of metallic silver; after which the silvering solution is removed, and the space between the walls of the two bulbs is exhausted by an air pump. Heat can escape from (or enter) such a bottle by ordinary conduction through the neck and stopper, and by radiation outwards (or inwards) across the vacuum space; this radiation is, however, very small, and therefore such a bottle keeps hot water hot or cold water cold for many hours.

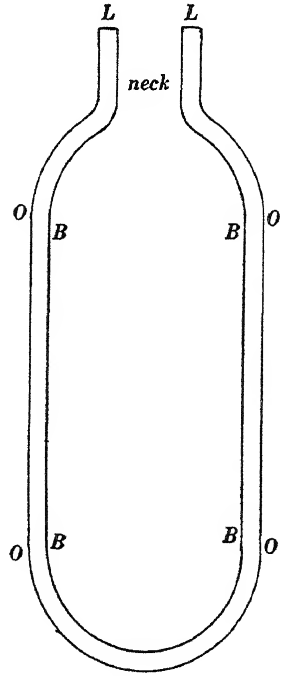


Fig. 18.

20. The bearing of the laws of radiation on the problem of the luminous efficiency of lamps.* — The ordinates of the curve in Fig. 19 show approximately the distribution of energy (values of

* A good discussion of lamp efficiency from the point of view of the theory of radiation is given by E. F. Roeber, *Transactions of the American Electro-chemical Society*, Vol. VIII, pages 243–259, 1905.

E) throughout the spectrum of kerosene lamp light. The total area under the curve represents the total energy radiated by the lamp, the shaded area represents the energy of the infra-red radiations, and the unshaded area between R and V

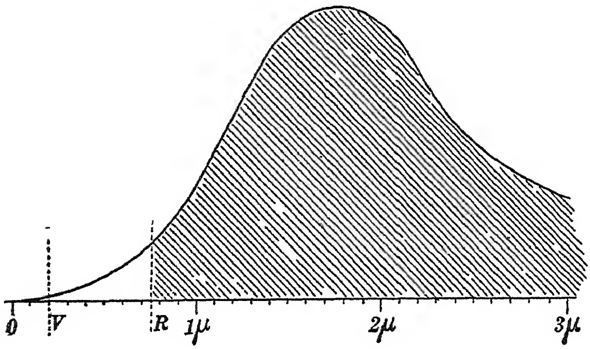


Fig. 19.

represents the energy of the luminous part of the radiations. It is evident from this figure that a small percentage, only, of the energy radiated by a kerosene lamp-flame lies within the visible spectrum.

One way to increase the luminous efficiency of a lamp is to increase to the utmost the temperature of the light-giving element. This increase of temperature increases E throughout, but the increase of E is especially great for the shorter wave-lengths. Thus, from Fig. 17, it is evident that a fairly large percentage of the energy of the sun's radiation lies within the visible spectrum. The most refractory substance known is, perhaps, metallic tungsten, and this metal is now extensively used for the filaments of electric glow-lamps.

Another way to increase the luminous efficiency is to seek for a substance which, at a given temperature, radiates the shorter wave-lengths greatly in excess of the amount which corresponds to normal radiation at the given temperature. The presence of a small percentage of cerium oxide in the mantle of the Welsbach lamp seems to bring about this desirable property of selective radiation to a very marked degree.

21. The artificial black body. — An opaque body which reflects very little of the radiation which falls upon it is black. Such a body when heated to a given temperature emits very nearly the normal radiation for that temperature, as explained above.

A *perfectly black body* would be an opaque body which reflects no portion of the radiation falling upon it. No known substance is perfectly black. *A small hole in the opaque wall of a large closed chamber is, however, perfectly black;* only an infinitesimal portion of the rays which enter the hole find their way out again, but are absorbed after repeated reflections from the interior walls of the chamber. If such a chamber be maintained at a uniform temperature, the radiations emitted from the hole will be the same as the radiation within, which is normal.

In the researches of Lummer and Pringsheim, which are referred to in Art. 13, the radiating body was a *hole* in the wall of a uniformly heated chamber.

22. White bodies. — A body which reflects approximately the same proportion of every wave-length of the radiation which falls upon it is called a *white body*. The light which is reflected by a white body is of the same composition (relative intensities of various wave-lengths the same) as the incident light. Thus, white paper is red in red light, green in green light, etc. A *perfectly white body* would be one reflecting the whole of the incident radiation, as opposed to a perfectly black body which would absorb the whole. No substance is perfectly white. In fact, there is no substance which reflects exactly the same proportion of the various wave-lengths of the incident radiation. Most white powders, for example, sugar, magnesium oxide, etc., reflect the longer wave-lengths to a slight excess, and have a yellowish tint. This yellowish tint may be neutralized by the admixture of a small amount of blue pigment, for example, ultra-marine blue, which reflects the shorter wave-lengths in excess. The use of bluing to whiten laundered articles is necessary on account of the fact that washed linen is slightly yellow.

Substances, like glass and ice, which transmit (and reflect)

every part of the visible spectrum equally well (approximately), are dazzling white when powdered. This is exemplified by the whiteness of snow. Nearly the whole of the light which falls upon a sheet of snow is reflected (diffusely), because of the repeated reflections by the successive particles as they are reached by the light as it penetrates deeper and deeper into the snow.

23. Surface color.* — A substance which shows marked selective reflection is said to have *surface color*. All such substances appear different in color by reflected and by transmitted light. Thus, gold is yellow by reflected light and gold leaf is green by transmitted light. The aniline dyes, especially when concentrated, show marked selective reflection. This is exemplified by the familiar bronze color by reflected light of aniline ink which is purple by transmitted light.

24. Absorption color. — Most colored substances, such as colored glass, colored solutions, etc., show color perceptibly by transmitted light only. Many colored substances which appear colored by reflected light, such as the pigments used in painting, really owe their color to selective transmission or absorption; the light which falls upon their grains is partly transmitted, becomes colored, and is reflected by numerous foreign particles and by breaks in the continuity of the grains. This is shown by the fact that a mixture of two pigments reflects only those wave-lengths

* The electromagnetic theory of light shows: (1) That a perfect electric conductor would totally reflect all radiations falling upon it; (2) that a substance, of which the structural elements (atoms or molecules) have a free period of undamped electrical vibration, would totally reflect wave-trains of that particular period; (3) that in proportion as these free vibrations are damped the substance would reflect less and absorb more of the wave-trains of that particular period.

An example of the first case is furnished by metals, which, especially for the longer wave-lengths, give almost complete reflection. Many substances, such as fluorite and fuchsine, reflect certain wave-lengths almost completely, and the inference is that these substances contain structural elements which have proper periods of electrical vibration which are in unison with these reflected wave-lengths. Repeated reflections from fluorite isolate that particular wave-train which is most completely turned back at each reflection, giving approximately a residuum of one wave-length.

which are reflected by *both* pigments unmixed, just as a pair of colored glasses transmits only those wave-lengths which can pass through both (see Art. 118, page 189).

25. Luminescence.* — Some substances, under certain conditions, emit radiations of the shorter wave-lengths greatly in excess of the normal amount corresponding to the temperature of the substance. This phenomenon is called, in general, *luminescence.*

Chemi-luminescence. — Many substances, while undergoing chemical action at low temperature, become luminescent. This phenomenon is called chemi-luminescence. Thus, phosphorus oxidizes slowly when it is exposed to the air and emits a pale white light.

Photo-luminescence. — Many substances become luminescent when exposed to the brilliant light of the sun, or to the light from an arc lamp. This phenomenon is called, in general, photo-luminescence. Some substances such as sulphate of quinine, kerosene, uranium glass, emit light of medium wave-length while they are exposed to radiation of a very short wave-length. This kind of a photo-luminescence is called *fluorescence.*

Some substances, impure calcium sulphide, for example, emit light for a long time after being exposed to brilliant sunlight. This kind of photo-luminescence is called *phosphorescence.*

A very interesting practical application of fluorescence is made at the zinc mines at Franklin Furnace, New Jersey. The ore from these mines is crushed and passed through an ore concentrator, and the presence of the valuable mineral, willemite, in the tailings is indicated by its brilliant fluorescence, the tailings being illuminated by light from an iron-arc lamp.

26. The bolometer and the radiometer. — In the experimental determination of radiation curves like those shown in Figs. 16 and 17, the radiation to be studied is passed through the slit of a spec-

* In the following statements no reference is made to thermo-luminescence, triebo-luminescence, etc. A very full discussion of the phenomena of luminescence may be found in Winkelmann's *Handbuch der Physik*, Vol. VI, pages 784–813.

troscope and spread out into a spectrum by means of a prism * or diffraction grating, and the thermal intensity of this spectrum at each wave-length is observed. Three devices have been used for measuring the thermal intensity at each point of such a spectrum, namely, (*a*)† the thermo-element, (*b*) the bolometer, and (*c*) the radiometer.

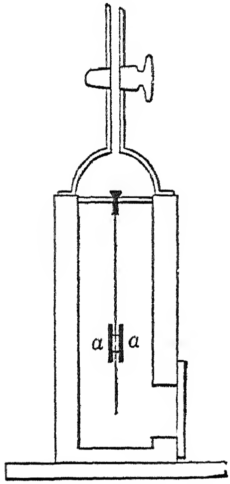


Fig. 20.

The bolometer, which was invented by Langley in 1880, consists of a very thin strip of blackened metal which is connected as one arm of a Wheatstone bridge. The bridge is carefully balanced, and when the thin strip of blackened metal is exposed to the rays at any part of the spectrum its temperature and therefore its electrical resistance is changed, and the intensity of the spectrum at the given point is indicated by the deflection of a galvanometer.

The radiometer, as used in the measurement of radiation, was invented by E. F. Nichols in 1896. It consists of two very small vanes of blackened mica *a* and *a*, Fig. 20, attached to horizontal cross-arms and suspended in a fairly good vacuum by a fine quartz fiber. The radiation to be indicated is allowed to fall upon *one* of these vanes. The vane is thus warmed, slightly, the few remaining molecules of air rebound with increased velocity from the face of the vane, and the reaction pushes the vane backwards, thus turning the suspended system about the fiber as an axis. A small mirror is attached to the suspended system in Fig. 20, and the deflection due to the radiation is observed by a telescope and scale.

* In case it is desired to investigate the infra-red portion of the spectrum, a rock-salt prism is generally used because glass is opaque to the infra-red rays.

† See *Electricity and Magnetism for Students*, by H. E. Hadley (London, Macmillan & Co., 1906), pages 359–382.

APPENDIX C.

PROBLEMS.

Chapter I.

1. The true period of rotation of Jupiter's second satellite is 85 hours and the velocity of the earth in its orbit is 30 kilometers per second. Find the apparent period of rotation of Jupiter's second satellite when the earth is moving directly away from Jupiter; and when the earth is moving directly towards Jupiter. Ans. (*a*) $85^h\,0^m\,30^s.62$, (*b*) $84^h\,59^m\,29^s.39$.

2. A hammer strikes a gong at intervals of two seconds. At what intervals, apparently, would the hammer strike to a listener approaching the source of the sound at a velocity of 25 meters per second? Ans. 1.86 seconds.

3. A sounding body makes 200 vibrations per second, that is, the period of the vibrations is $\frac{1}{200}$ second. What would be the apparent number of vibrations per second to a listener approaching the sounding body at a velocity of 25 meters per second? Ans. 215.3 vibrations per second.

4. Two bells *A* and *B*, 2,000 meters apart, are struck at intervals of ten seconds, that is, *A* is struck, then after ten seconds *B* is struck, then after ten seconds *A* is struck again, and so on. An observer hears bell *A*, then after 13.8 seconds he hears *B*, then after 6.2 seconds he hears *A*, and so on. What is the difference of the distances of the two bells from the observer? Ans. 1,254 meters.

5. It has been proposed to use a pen making a mark on a uniformly moving strip of paper, the pen being dropped upon the paper by a wireless telegraph signal from a distant vessel and raised off the paper by the action of the sound from a fog horn on the distant vessel, the fog horn being blown at the instant of send-

ing the wireless signal. Find the velocity at which the strip of paper must move in order that the length of the traced line in centimeters may represent the distance of the vessel in hundreds of meters. Ans. 3.4 centimeters per second.

6. A wheel having 500 teeth and 500 spaces between the teeth is used in Fizeau's method for determining the velocity of light. The wheel is speeded up slowly until it reaches a speed of 250 revolutions per second and while it is being speeded up the light disappears and reappears eight times. The distance from the wheel to the mirror M in Fig. 2 is 9.6 kilometers. What is the velocity of light? Ans. 300,000,000 meters per second.

Chapter II.

7. A belt, of which the mass per centimeter of length is 12 grams, is under a tension of 240×10^6 dynes. Find the velocity at which the belt must run in order that a bend once formed in the belt may stand still. Ans. 4,473 centimeters per second.

8. A water wave travels along a canal in which the normal depth of water is 6 feet, the width of the canal being 12 feet. The wave is 30 feet long and the water in the wave has a uniform velocity of 0.3 foot per second. Find the total energy of the wave counting both potential energy and kinetic energy. Ans. 379.5 foot-pounds.

9. A canal is 8 feet deep and 100 feet long, the ends being formed by rigid dams. The water in the canal is initially like Fig. 16, page 27. Find the number of seconds required for one complete vibration of the water in the canal when the gate is lifted. Ans. 12.5 seconds.

10. Consider a canal CD in Fig. 16, page 27. Imagine a float placed half-way between the central gate and one end of the canal, and imagine a pencil attached to this float and arranged to trace a line upon a strip of paper which moves past the pencil in a direction at right angles to the length of the canal. Plot a

curve similar to that which would be traced by the pencil upon the moving strip of paper.

11. Find the wave-length of a train of sound waves produced in air by a body making 16 vibrations per second; by a body making 256 vibrations per second; and by a body making 40,000 vibrations per second. Ans. 20.6 meters, 1.28 meters, and 8.25 millimeters respectively.

12. Twelve drops per second of water fall from a nozzle into a pool of water. The waves produced on the surface of the pool by the drops travel at a velocity of, say, four feet per second. What is the distance between successive waves of the wave-train produced by the drops? Ans. 4 inches.

13. Yellow light has a wave-length approximately equal to 59 millionths of a centimeter. What is the frequency of a wave-train of yellow light? Ans. 508×10^{10} vibrations per second.

14. A wave-train is produced on a very long stretched rubber tube by moving the end of the tube up and down at a definite frequency. How many times as much energy will be transmitted

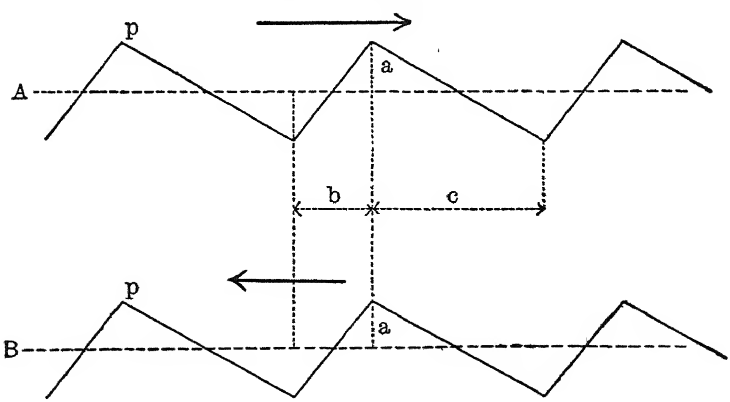


Fig. 15p.

along the tube per second if the end of the tube is moved up and down through twice as great a distance as before and at three times the frequency? Ans. 36 times as much.

15. Given two oppositely moving similar trains of waves *A*

and B, Fig. 15p; $a = 1\frac{1}{2}$ inches, $b = 2\frac{1}{2}$ inches, and $c = 5\frac{1}{2}$ inches. Draw these two wave-trains with the point p of the upper train directly over the point p of the lower train, as shown in the figure, and draw the resultant of the two trains for this position. Make five additional drawings showing A and B and their resultant $\frac{1}{8}$ of a period later, $\frac{3}{8}$ of a period later, $\frac{4}{8}$ of a period later, and $\frac{5}{8}$ of a period later, a period being the time required for the waves to travel over the distance $b + c$.

16. A train of sound waves coming from a distant body vibrating 256 times per second, strikes a wall perpendicularly and is reflected. A stationary wave-train results. At certain distances from the wall, or at certain points A, the air is subjected alternately to compression and rarefaction but does not move; and at certain distances from the wall, or at certain points B, the air moves to and fro but is not condensed or rarefied. What are the points A called, and what are the distances of this series of points from the wall? What are the points B called, and what are distances of this series of points from the wall? To which series of points does the surface of the wall belong? Ans. Distances to points A 64.5, 129, 193.5 centimeters, etc.; distances to points B 32.25, 96.75, 161.25 centimeters, etc.

17. The ear is affected only by variations of pressure, not by air movements. At what distance, or distances, from a reflecting wall will the sound of a distant body become inaudible when the body makes 125 vibrations per second and the sound strikes the wall perpendicularly? Ans. 66, 198, 330 centimeters, etc.

Chapter III.

18. A 2-inch glass cube CC, Fig. 18p, is placed as shown in front of a divided scale SS so that the divided scale may be seen partly through the cube at a and partly by looking over the cube at b; and the distance ab, as read off the scale, is 0.392 inch. Find the index of refraction of the glass of which the cube is made. Ans. 1.5.

19. A block SS, Fig. 19p (a), of a substance of which the index of refraction is to be determined has a polished face and it is submerged in carbon bisulphide LL in a glass vessel VV which

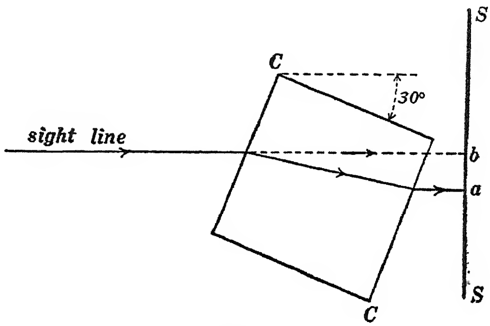


Fig. 18p.

has a flat front FF. A sheet of paper pp partly surrounds the vessel so as to give a uniform illumination. Under these conditions a sharply defined line is seen in the field of the telescope when the crystal is turned so that the critical angle of total reflection t is as shown. The block SS is supported at the end of a

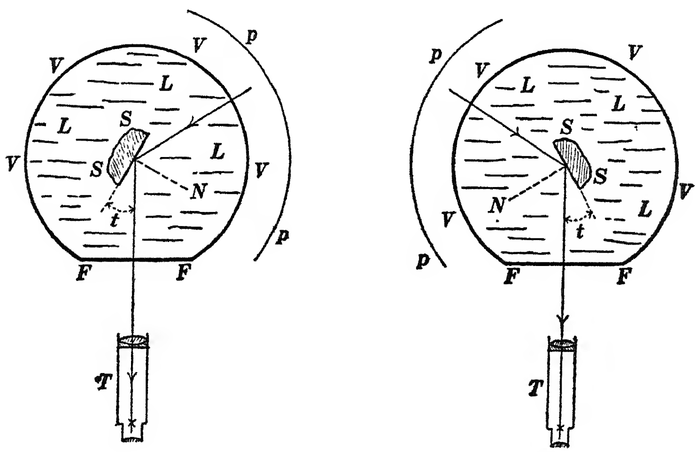


Fig. 19p.

vertical axis which projects above the surface of the carbon bi-sulphide, passes through the center of a horizontal divided circle

and carries a pointer which plays over the circle so that the angle which must be turned to bring the block *SS* from the position shown in (*a*) to the position shown in (*b*), Fig. 19*p*, can be observed. This angle is found to be 125° and the index of refraction of carbon bi-sulphide is 1.63. Find the index of refraction of the substance *SS*. Ans. 1.838.

20. The line *WW*, Fig. 20*p*, represents the position which

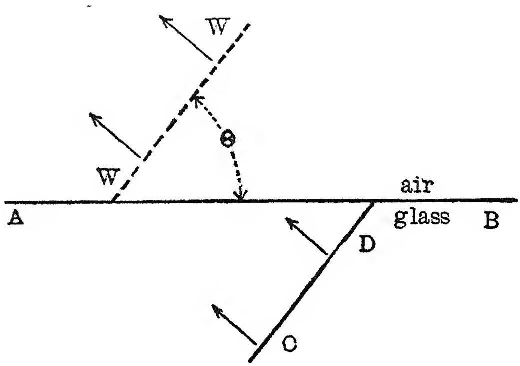


Fig. 20p.

would be reached by the wave *CD* at a given instant were it not for the passage from glass to air. Make a drawing showing Huygens' construction for the refracted wave in air at the given instant, the index of refraction of the glass being 1.5 and the angle θ being 60°.

CHAPTER IV.

21. A lens is made of glass of which the index of refraction is 1.6. The lens comes to a sharp edge, its diameter is 20 centimeters and its thickness at the center is one centimeter. Find its focal length. Ans. 83.3 centimeters.

22. A lens is made of glass of which the refractive index is 1.6. The radius of curvature of one surface of the lens is 20 centimeters, and the other surface of the lens is flat. What is the focal length of the lens? Ans. 33.3 centimeters.

See footnote on page 69.

23. A lens which comes to a sharp edge has a diameter of 10 centimeters and a thickness at the center of 0.6 centimeter, and its focal length is 22 centimeters. Find the index of refraction of the glass of which the lens is made. Ans. 1.947.

24. A converging lens has a focal length of 20 centimeters. An object is placed at a distance of 30 centimeters from the lens. Find (*a*) the distance of the image from the lens and (*b*) the ratio of diameter of object to diameter of image. Is the image real or virtual, erect or inverted? Ans. (*a*) 60 centimeters, and (*b*) 1:2.

25. An object is placed 15 centimeters from a converging lens of which the focal length is 20 centimeters. Find (*a*) the distance of the image from the lens, and (*b*) the ratio of diameter of object to diameter of image. Is the image real or virtual, erect or inverted? Ans. (*a*) − 60 centimeters, and (*b*) 1:4.

26. An object is placed 15 centimeters from a diverging lens, of which the focal length is 20 centimeters. Find (*a*) the distance of the image from the lens, and (*b*) the ratio of diameter of object to diameter of image. Is the image real or virtual, erect or inverted? Ans. (*a*) − 8.5 centimeters, and (*b*) 1.765 : 1.

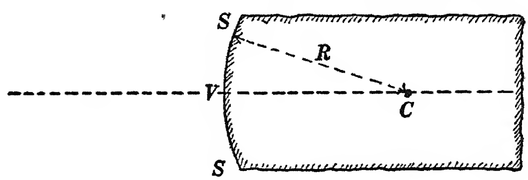


Fig. 28p.

27. The diameter of the object in problems 24, 25 and 26 is 8 centimeters. Draw to scale the geometrical construction locating the points of the image corresponding to extreme top and bottom of object in cach case.

28. A large block of glass of which the index of refraction is 1.5 has a polished spherical surface *SS*, as shown in Fig. 28*p*. The radius of curvature *R* is equal to 20 centimeters. (*a*) Find the distance from *V* to the point at which parallel rays from the left are focused in the glass by the spherical surface *SS*, and

(*b*) Find the distance from *V* to the point at which parallel rays coming from the right are focused by the surface *SS* in the air. Ans. (*a*) 60 centimeters, and (*b*) 40 centimeters.

29. Two thin lenses of the same diameter have focal lengths of 75 centimeters and 50 centimeters respectively. These two lenses are placed very close together and used as a single lens. What is the focal length of the combination? Ans. 30 centimeters.

Note. — In solving this problem consider that the retardation of the central part of a wave by the two lenses is equal to the sum of the retardations due to the lenses acting separately. By retardation is here meant the retardation relative to the edge portion of a wave, and the lenses being very close together the diameter of the wave after it has passed through the two lenses is sensibly equal to the diameters of the lenses.

30. Two thin lenses have focal lengths of 75 centimeters and − 120 centimeters, respectively. These lenses are placed very close together and used as a single lens. Find the focal length of the combination. Ans. 200 centimeters.

31. (*a*) Express the power of each lens in problem 29 and the power of the combination in diopters. (*b*) Express the power of each lens in problem 30 and the power of the combination in diopters. Ans. (*a*) 1.333, 2 and 3.33 diopters, and (*b*) 1.333, − 0.833 and 0.5 diopters.

32. Consider two simple lenses *A* and *B*, Fig. 32*p*, arranged

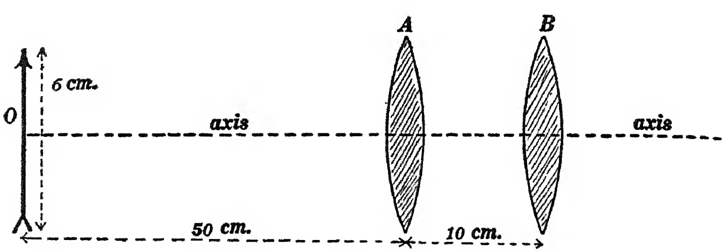


Fig. 32p.

as shown with respect to the object *O*. The focal length of *A* is 15 centimeters and the focal length of *B* is also 15 centimeters. Find by construction the position and diameter of the image of *O*.

Note.—Construct the image of O formed by A; and then construct the image of this image which is formed by B.

33. Using the same data as in problem 32 with the exception that lens B has a focal length of -15 centimeters, find by construction the position and size of the image of O.

34. An object O, a lens A and a lens B are arranged as shown in Fig. 34*p*. The diameter of O is 10 meters, the focal

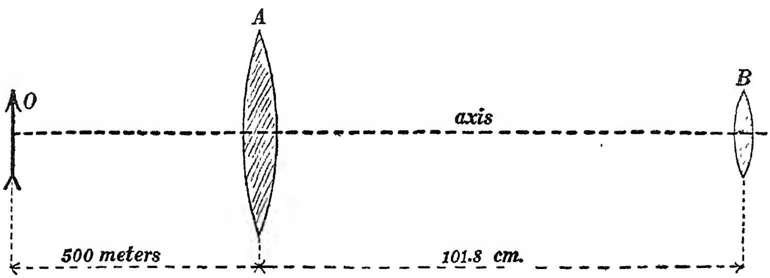


Fig. 34p.

length of A is 100 centimeters, and the focal length of B is 2 centimeters. Find the position and size of the image of O formed by the combination. Ans. The image is 8 centimeters beyond B and its diameter is 10.02 centimeters.

35. An object O which is 10 meters in diameter, a lens A and a lens B are arranged as shown in Fig. 35*p*. The focal

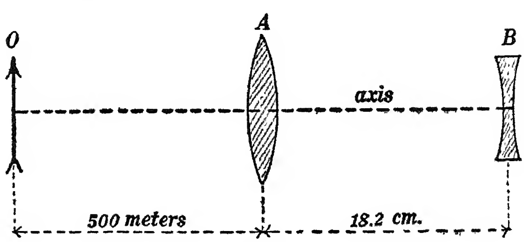


Fig. 35p.

length of A is 20 centimeters and the focal length of B is -2 centimeters. Find the size and position of the image of O formed by the combination. Ans. The image is 18.83 beyond B, and its diameter is 4.166 centimeters.

Chapter V.

36. A photographic lens of 15 centimeters focal length gives sharp images of very distant objects upon the ground glass of the camera when the slider carrying the lens is at marked position. Calculate the distance from this mark to the slider in order that an object two meters from the lens may be sharply focused on the ground glass. Ans. 1.2 centimeters.

37. A projection lantern takes a transparent slide 7×7 centimeters. The focal length of the lantern objective is 20 centimeters. It is desired to project on the screen an image of the slide two meters square. Required the distance from the lens to the screen. Ans. 592 centimeters.

38. A projection lantern is to be used at a distance of 3 meters from a screen and it is desired to enlarge a 7×7 centimeters slide to the size of one meter square. What focal length objective is required? Ans. 19 centimeters.

39. A far-sighted person sees distinctly an object at a distance of 200 centimeters or more. Find the power in diopters of a spectacle lens which will enable this person to see an object at a distance of 25 centimeters or more from the eye. Ans. 3.5 diopters.

Note.—In solving this problem consider the distance between the spectacle lens and the eye to be negligibly small.

40. Figure 40*p* shows a drawing on the face of a disk. This disk is rotated rapidly and illuminated by quick flashes of light which occur 4 times in each revolution of the disk. Make a drawing showing the appearance of the disk under these conditions.

41a. A pocket magnifier has 3 separate lenses of which the focal lengths are 1, 2 and 4 centimeters, respectively. What is the magnifying power of each: (*a*) when the eye is accommodated for a distance of 25 centimeters, and (*b*) when the eye is accommodated for parallel rays? Ans. (*a*) 26, 13.5 and 7.25 diameters; (*b*) 25, 12.5 and 6.25 diameters.

41b. An object 0.1 millimeter in diameter, actual size, is looked at through a microscope of which the magnifying power is 30 diameters. What is the diameter of a drawing of the object which when placed at a distance of 3 meters from the naked eye will appear the same size as the object as seen through the microscope? Ans. 3.6 centimeters in diameter.

42. The three lenses of the magnifier which is described in problem 41*a* are used together as one lens. What is the magnifying power of the combination with the eye accommodated for a distance of 25 centimeters, assuming the lenses to be indefinitely near together? Ans. 44.75 diameters.

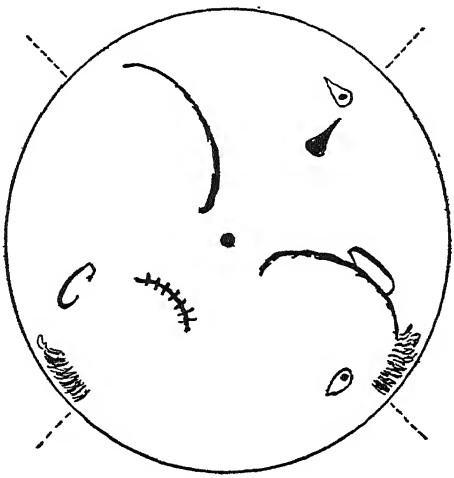


Fig. 40p.

43a. A compound microscope has an objective of which the focal length is 2 millimeters, an eye-piece of which the focal length is 10 millimeters, and the distance from center of objective to the plane of the image (*b* in Fig. 84, page 84) is 150 millimeters. Calculate the magnifying power of the instrument on the assumption that the observer's eye is accommodated for a distance of 25 centimeters. Ans. 1,924 diameters.

43b. Find the value of the distance *b* in Fig. 84, page 84, to give the magnifying power of 1,924, with an objective of 25 mil-

limeters focal length and an eye-piece of 10 millimeters focal length, eye being accommodated for 25 centimeters. Ans. 1,874 millimeters.

44a. The object-glass of the great telescope of the Lick Observatory has a focal length of 1,500 centimeters. What is the magnifying power of the telescope, using an eye-piece of which the focal length is 2 centimeters: (*a*) when looking at an infinitely distant object, and (*b*) when looking at an object 300 meters distant from the object-glass? Assume the observer's eye to be accommodated for parallel rays in each case. The observer's eye, if he choose to look directly at the object, is 315 meters from it. Ans. (*a*) 750 diameters, and (*b*) 829 diameters.

Note. — The distance Oi in Fig. 85, page 86, is equal to f only when the object is at a very great distance. It is really meaningless to calculate the magnifying power of a telescope to the degree of precision represented in the answers to this and the following two problems.

44b. In looking through the Lick telescope the "focus" is adjusted until the eye is accommodated for the distance of most distinct vision (25 centimeters.) What is the magnifying power of the telescope: (*a*) when looking at a very distant object, and (*b*) when looking at an object 300 meters distant from the object glass, other conditions being as specified in problem 44*a*? Ans. (*a*) 810 diameters, and (*b*) 895 diameters.

44c. An observer looks through the Lick telescope with one eye at a measuring stick 600 meters from the object-glass, with the other eye he looks directly at a scale of centimeters at a distance of 30 centimeters from the eye. What length of the scale will be apparently covered by one centimeter of the distant measuring stick, focal lengths being as specified in problem 44*a*? Ans. 0.400 centimeter.

Note. — While looking with one eye at a scale 30 centimeters from the eye the other eye unconsciously accommodates itself to 30 centimeters also.

Chapter VI.

45. A lens has 2 centimeters free diameter and 12 centimeters focal length. What is its numerical aperture? The object-glass of the great telescope of the Lick Observatory is 92 centimeters free diameter and 1,500 centimeters focal length. What is its numerical aperture? Ans. (*a*) $\frac{1}{6}$, (*b*) 0.061.

46. A photographic objective of which the focal length is 15 centimeters forms a satisfactory image over the whole of a photographic plate 16 × 22 centimeters. What is the field angle of the lens? Ans. 84.4 degrees.

47. The angular diameter of the sun as seen from the earth is a half degree. What is the diameter, in centimeters, of the image of the sun formed by the object-glass of the great telescope of the Lick Observatory? Ans. 13.05 centimeters.

48. A photographic lens with a numerical aperture of $\frac{1}{6}$ gives a good photograph with, say, $\frac{1}{50}$ second exposure. What exposure would be required with a lens working with a $f/16$ diaphragm or stop? With a $f/32$ stop? Ans. (*a*) 0.142 second, and 0.568 second.

Chapter VIII.

49. The points O and O' in Fig. 161, page 140, are the ends of a large tube in the middle of which is placed a shrill whistle which gives a tone of which the vibration frequency is 25,000 complete vibrations per second. The distance OO' is 3 feet and the distance from OO' to the plane AB is 50 feet. The point where the straight dotted line cuts the plane AB is a point of maximum loudness of sound. Find the distance from this point to the points where the first minimum of loudness occurs. Ans. 0.605 feet (velocity of sound 1,090 feet per second).

50. Make a sketch showing the shape of the curves on the plane AB, Fig. 161, page 140, along which the intensity of the sound from O and O' is either a maximum or a minimum; represent the appearance of these curves as seen from OO'.

51. A glass plate 0.00004 centimeter thick is illuminated by a beam of parallel rays of light making an angle of 30° with the normal to the plate. The index of refraction of the glass is 1.5. What wave-lengths of the light will be strengthened by interference within the region of the visible spectrum? Ans. $\lambda = 66 \times 10^{-6}$ centimeter and 39.6×10^{-6} centimeter.

52. The distance apart, center to center, of the slits in the grating *AB*, Fig. 173, page 153, is 0.0003 centimeter. The light *TT* is white light and the focal length of the lens *LL* is 50 centimeters. Calculate the distances from the point *F* to the two points *F′*, and from the point *F* to the two points *F″*, and so on, for violet light ($\lambda = 40 \times 10^{-6}$ centimeter) and for red light ($\lambda = 75 \times 10^{-6}$ centimeter). Ans. Distances *F* to *F′*, for violet light 6.738 centimeters, for red light 12.91 centimeters; *F* to *F″*, for violet light 13.83 centimeters, for red light 28.87 centimeters.

Chapter IX.

53. The intensity of illumination at a distance of four feet from a 16-candle lamp is sufficient for easy reading of ordinary book type. (*a*) Find the distance from a 20-candle lamp at which the lamp gives the same intensity of illumination; (*b*) express this intensity of illumination in spherical-candles of light falling on each square foot of illuminated surface; and (*c*) express this intensity of illumination in luxes. Ans. (*a*) 4.47 feet; (*b*) 0.0796 spherical-candle per square foot; (*c*) 12.21 luxes.

54. The glow lamp which is used as a standard in a Bunsen photometer has a candle-power of 16.8 candles in the direction towards the photometer screen. Another lamp *B* is placed at the other end of the photometer bar; and when the screen is adjusted to equality of illumination on both sides, it is 2.61 meters from the lamp *B*, and 1.80 meters from the standard lamp. What is the candle-power of *B* in the direction towards the screen? Ans. 35.3 candle-power.

55. The lamp B of problem 54 is placed at a distance of 0.70 meter from the center of a large mirror which reflects the light from B along the photometer bar towards the photometer screen; and when the screen is again adjusted to equality of illumination on both sides, it is 1.85 meters from the standard lamp and 1.82 meters from the center of the mirror. The lamp B presents towards the mirror in this case the same face that was presented towards the screen in problem 54. Find the factor by which the apparent candle-power of any lamp, when measured by the light reflected from the above mirror, must be multiplied in order to correct for the loss of light at the mirror. Ans. 1.13.

56. A beam of light consisting of parallel rays has a sectional intensity of 300 luxes. Find the conical intensity of the beam after it passes through a lens of which the focal length is 50 centimeters. Ans. 75 hefners.

57. An open-arc lamp is placed at a distance of five feet from a converging lens, and an image of the arc is formed at a distance of one foot beyond the lens. The light from the lamp has a conical intensity of 2,500 candles. Assuming that the luminous surface of the lamp is negligibly small, and ignoring loss of light at the lens by reflection and absorption, find the conical intensity of the beam beyond the image. Ans. 100 candles.

58. Two lamps A and B are placed at the ends of a Bunsen photometer bar, and the photometer screen is adjusted to give equality of illumination on its two sides. The screen is then one meter from lamp A and 3 meters from lamp B. A lens of which the focal length is 25 centimeters is placed 50 centimeters from lamp B, and the screen is left in its original position. Find how far lamp A must be placed from the screen to give equal illumination on the two sides of the screen, neglecting losses of light in the lens. Ans. 0.67 meter.

59. Calculate the spherical candle-power of the bare glow-lamp from the data given in Fig. 186, page 170. Ans. 13.33 spherical-candles.

60. Calculate from the data given in Fig. 187, page 171, the spherical candle-power of the glow lamp of problem 59 when it is provided with an aluminum cone shade, and express the amount of light absorbed by the shade as a percentage of the spherical candle-power of the bare lamp. Ans. 8.26 spherical-candles, 38 per cent. absorbed.

61. (*a*) How many 16 candle-power lamps are required to illuminate a lecture hall 50×75 feet with a moderately high ceiling? (*b*) How many 16 candle-power lamps would be required to give *exactly* the same average intensity of illumination in a room twice as long, twice as wide and twice as high with the same kind of walls and ceiling and the same kind of furniture? Why? Ans. (*a*) 47 lamps; (*b*) 188 lamps.

62. The mean coefficient of absorption of the walls, ceiling and furniture in the room specified in problem 61 (*a*) is 40 per cent. How many 16 candle-power lamps would be required to give the same intensity of the illumination in a room of the same size in which the mean coefficient of absorption of walls, ceiling and furniture is 55 per cent.? Ans. 65 lamps.

63. Let the brightness of daylight with the sun in the zenith be taken as unity. What is the brightness of daylight when the altitude of the sun is 75°, 60°, 45°, 30° and 15° above the horizon respectively, ignoring increase of atmospheric absorption with increase of zenith distance of the sun. Ans. 0.966; 0.866; 0.707; 0.5 and 0.259.

Note.—The relative brightness of daylight is inversely proportional to the area of country covered by a beam of sunlight of, say, one square kilometer in sectional area, and this is inversely proportional to the sine of the sun's altitude above the horizon.

64. A certain photographic lens gives a good photograph with an exposure of $\frac{1}{50}$ second when the sun is 75° above the horizon. What exposure would be required with the same lens when the sun is 5° above the horizon, ignoring increase of atmospheric absorption with increase of zenith distance of sun? Ans. 0.22 second.

65. A direct-current arc lamp gives the following distribution of light:

Angle from vertical,	10°	20°	30°	40°	50°	60°	70°	80°
Candle-power,	290	440	670	1,080	1,220	1,080	795	580

Caculate the intensities of illumination at points along a level open street distant $h \tan 10°$, $h \tan 20°$, $h \tan 30°$, $h \tan 40°$, etc., horizontally from the lamp: (*a*) when the height h of the lamp above the street is 15 feet, and (*b*) when the height h of the lamp above the street is 50 feet.

Express the intensities of illumination in "candle-feet," that is, in terms of the intensity of illumination produced by the beam from a standard candle falling *perpendicularly* upon a screen at a distance of one foot from the candle.

Plot two curves showing horizontal distances from the lamp as abscissas and intensities of illumination as ordinates. Sample answer. Intensity of illumination of the surface of the street at a distance of 26 feet horizontally from the lamp is 0.600 "candle-feet" when the lamp is 15 feet above the ground.

Sample answer: for 60° from vertical, 0.6 "candle-foot," lamp 15 feet above ground.

Note. — The distance d in feet of one of the points on the street from the lamp is equal to $\sqrt{h^2 + h^2 \tan^2 \theta}$, where θ is one of the angles given in the problem. The sectional intensity of any one of the beams above specified, at distance d feet from the lamp is equal to the candle-power of the beam divided by d^2, this result being expressed in "candle-feet." Furthermore, one unit sectional area of this beam at the given point on the street is spread over $1/\cos \theta$ units of area of ground, so that the intensity of illumination of the surface of the street at the given point is $\cos \theta$ times the sectional intensity (in "candle-feet") of the given beam at distance d feet from the lamp.

Chapter XI.

66. The intensity of the light which is transmitted through the analyzer of a polariscope is taken as unity when the analyzer is in the "parallel" position. What is the intensity of the light when the analyzer is turned 15°, 30°, 45°, 60°, 75° and 90° respectively from the "parallel" position? Ans. 0.933; 0.750; 0.500; 0.250; and zero.

67. A plate of Iceland spar 0.02 millimeter thick with the optic axis parallel to the surfaces of the plate is placed in a polariscope with the analyzer in the "crossed" position. The beam of polarized light consists of a bundle of parallel rays. What wave-lengths within the limits of the visible spectrum are cut off by the analyzer? Assume the ordinary and extraordinary indices of refraction to be the same for all wave-lengths, 1.658 and 1.486, respectively. Ans. 68.8, 57.3, 49.1 and 43.0 millionths of a centimeter.

68. A solution of cane sugar in a tube 50 centimeters long rotates the plane of polarization of sodium light through 8.31 degrees of angle. What is the amount of sugar in grams in each cubic centimeter of the solution? Ans. 0.025 gram per cubic centimeter.

Chapter XII.

69. The disk of a siren is driven at a speed which brings its tone into unison with the tone of a tuning fork. The speed of the disk is then observed to be 1,820 revolutions per minute, and the row of holes in the siren disk contains 20 equidistant holes. What is the frequency of vibration of the tuning fork? Ans. 606.7 vibrations per second.

Note.—The sound of a siren is very rich in overtones and it is sometimes very difficult to tell whether a given tone is in unison with the fundamental tone of the siren or one of its overtones.

70. Two tones of which the frequencies are 275 and 300 vibrations per second respectively, are overtones of an unknown fundamental. Of the various possible values of frequency of the fundamental, what is the largest? Ans. 25 vibrations per second.

Chapter XIII.

71. A string 100 centimeters long, weighing 0.1 gram per centimeter is stretched with a tension of 100;000 dynes. What is the frequency of vibration of its fifth overtone, that is, when the string vibrates in 5 segments? Ans. 25 vibrations per second.

72. To what tension must a string 75 centimeters long and weighing 0.02 gram per centimeter be stretched in order that the frequency of its fundamental mode may be 1,000 vibrations per second? Ans. 45×10^7 dynes.

73. Under given conditions, a string makes 256 complete vibrations per second when vibrating in its fundamental mode. What would be the number of vibrations per second of its fundamental mode if its length were doubled? If its tension were doubled? If its mass were doubled? Ans. (*a*) 128, (*b*) 360.96, (*c*) 181.56 vibrations per second.

Note. — The string in this problem is assumed to be perfectly flexible.

74. A certain guitar string makes 400 vibrations per second. The length of the string is 36 inches and the second fret is 4 inches from the end of the string. How many vibrations per second does the string make when it is pushed down against the second fret? Ans. 450 vibrations per second.

75. An organ pipe 10 feet long is open at both ends. What is the frequency of vibration of the fundamental tone of the pipe and what are the vibration frequencies of the respective overtones of the pipe? Ans. 54.5 vibrations per second.

76. An organ pipe 5 feet long is closed at one end. What is the vibration frequency of its fundamental tone and what are the vibration frequencies of the respective overtones of the pipe? Ans. 54.5 vibrations per second.

77. Sound travels 3.8 times as fast in hydrogen as in air. What would be the fundamental tone of an organ pipe 6 feet long, open at both ends and filled with hydrogen? Ans. 345.17 vibrations per second.

78a. A horn has a tube 4 feet long. Calculate the length of the auxiliary tube required to lower its pitch in the ratio of 9 : 8. Ans. 0.5 foot.

Note. — The air column in a horn vibrates as in an organ pipe open at both ends.

78b. A very small glass tube closed at one end contains a

small quantity of lycopodium powder. A Galton whistle (see page 232) is sounded near the open end of the tube, the air in the tube is set oscillating (as a stationary wave-train) by the sound so that the powder is heaped at the nodes of the stationary train with spaces between, and the measured distance over three of the spaces is 0.78 inch. What is the frequency of oscillation of the whistle? Ans. 25,000 complete vibrations per second.

CHAPTER XIV.

79. A bass voice sings the vowel *a* as in father on a note of which the frequency is 100 vibrations per second. A soprano voice sings the same vowel on a note of which the frequency is 700 vibrations per second. Show by a tabular arrangement of the overtones that the bass singer's vowel is more distinct.

80. What must be the relative proportions of hydrogen and air in the mouth cavity of a speaker to give the vowel *a* as in *part*, when the attempt is made to give the vowel *a* as in *paw?* Ans. 7 parts hydrogen and 6 parts air by volume.

Note.—The density of hydrogen is $\frac{1}{14}$, air being taken as unity. The frequency of vibration of the gas in an organ pipe or any cavity is inversely proportional to the square root of the density of the gas.

In ten units volume of mixed gas let there be a volumes of air and h volumes of hydrogen; then

$$a + h = 10$$

The mass of the a volumes of air is a (since the density of the air is taken as unity) and the mass of the h volumes of hydrogen is $h/14$. Let d be the density of the mixture, then $10d$ is the mass of the 10 volumes whence

$$a + \frac{h}{14} = 10d$$

CHAPTER XV.

81. How many beats per second are produced when two organ pipes open at both ends are sounded together, the lengths of the pipes being 48 inches and 50 inches, respectively? Ans. 5.45 beats per second.

82. Two organ pipes are sounded simultaneously. The frequency of one pipe is 250 vibrations per second and the number

of beats per second is 8. What is the frequency of the second pipe? Ans. 242 or 258 vibrations per second.

CHAPTER XVII.

83. At what velocity must a locomotive pass an observer in order that the sound of its whistle may be changed half a tone (frequency ratio of 16:15) as it passes? Ans. 32.1 feet per second.

84. Find the area of plaster-on-tile wall which must be covered by hair-felt 2.5 centimeters thick hung 8 centimeters from the wall in order to reduce t_1 to 2 seconds in the New Boston Music Hall when there is half an audience present; the total absorbing power of each unoccupied seat being taken as equivalent to 0.21 square meters of open window. Ans. 472 square meters.

Note. — Data for this problem are given on page 288.

85. The Packer Memorial Chapel of Lehigh University has a volume of 10,650 cubic meters. In this chapel there are 157 square meters of window glass, 1,200 square meters of wood sheathing, 1,940 square meters of hard plaster (including the hard tile floor), 375 square meters of extra heavy curtain ($p = 0.40$), 80 seats of cushions, 45 square meters of openings into organ, ante-room, etc., and plain wood settees for 770 persons. Calculate t_1 with and without audience. Ans. 2.57 seconds and 5.12 seconds.

Calculate the area of wood sheathing which, in the Packer Memorial Chapel, must be covered with hair-felt 2.5 centimeters thick placed 8 centimeters from the surface of the wood in order to reduce t_1 to 1.5 seconds with full audience. Ans. 679 square meters.

86. Assuming that the absorbing power of hair-felt 2.5 centimeters thick laid close against a wall is 0.50, calculate the amount of wood surface (and of hard wall surface in addition if necessary) which must be covered in the Packer Memorial Chapel to reduce t_1 to 1.5 seconds with full audience. Ans. 1,112 square meters.

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